Reply to Review for “A dynamic informed deep learning method for future estimation of laboratory stick- slip” by Yue et al.

[Review]:

Dear Editor,

The manuscript forecasts shear stress in a double-shear stick-slip experiment using an MLP-based autoencoder enhanced by a Koopman operator and a delay-embedded input. The authors chose this ML model due to its superior performance in capturing periodic phenomena, making it suitable for shear stress forecasting. I appreciate that two reviewers have provided insightful feedback and that the authors have made an effort to address all comments. However, rather than merely responding to the reviewers' remarks, I suggest integrating these valuable points and discussions into the main text.

My primary concerns are as follows:

1. As Reviewer #1 pointed out, the authors tend to overstate their results, despite (a) clear instances of underestimation or overestimation in multiple cycles, and (b) performance that is comparable to other models, such as LSTM, in many cases. Additionally, the interpretability argument presented by the authors is a general observation about the system’s dynamics rather than a concrete improvement in forecasting accuracy or model generalization.
2. The manuscript lacks a discussion on the model's applicability to other parameters, such as time to failure, as well as its relevance to real-world field studies. It is evident that stress values are only accessible in laboratory experiments and cannot be directly extrapolated to tectonic earthquakes. Therefore, what are the potential applications of this model in such scenarios? Furthermore, while shear stress exhibits a highly periodic behavior in double-shear tests, rough fault stick-slip experiments (Goebel et al., 2012; Dresen et al., 2020) have demonstrated that roughness evolution significantly influences stress cycles. The authors should include a discussion on this aspect.

Additionally, I have a few minor comments:

1. Figures 8 and 10 contain numerous plots. For instance, in Figure 10, if the first two columns represent different views of the same plot, they should be visually linked using lines or another connecting element. And use some kind of numbering gor them.
2. Figure 10: What does a negative R2 value indicate?

[Response]:

Thank you so much for your valuable comments, which are very helpful for improving our manuscript.

In response to your review comments, we will make the following responses:

（Black bold text: Reviewers' comments; Purple text: Our responses; Red text: changes in manuscript）

1. **However, rather than merely responding to the reviewers' remarks, I suggest integrating these valuable points and discussions into the main text.**

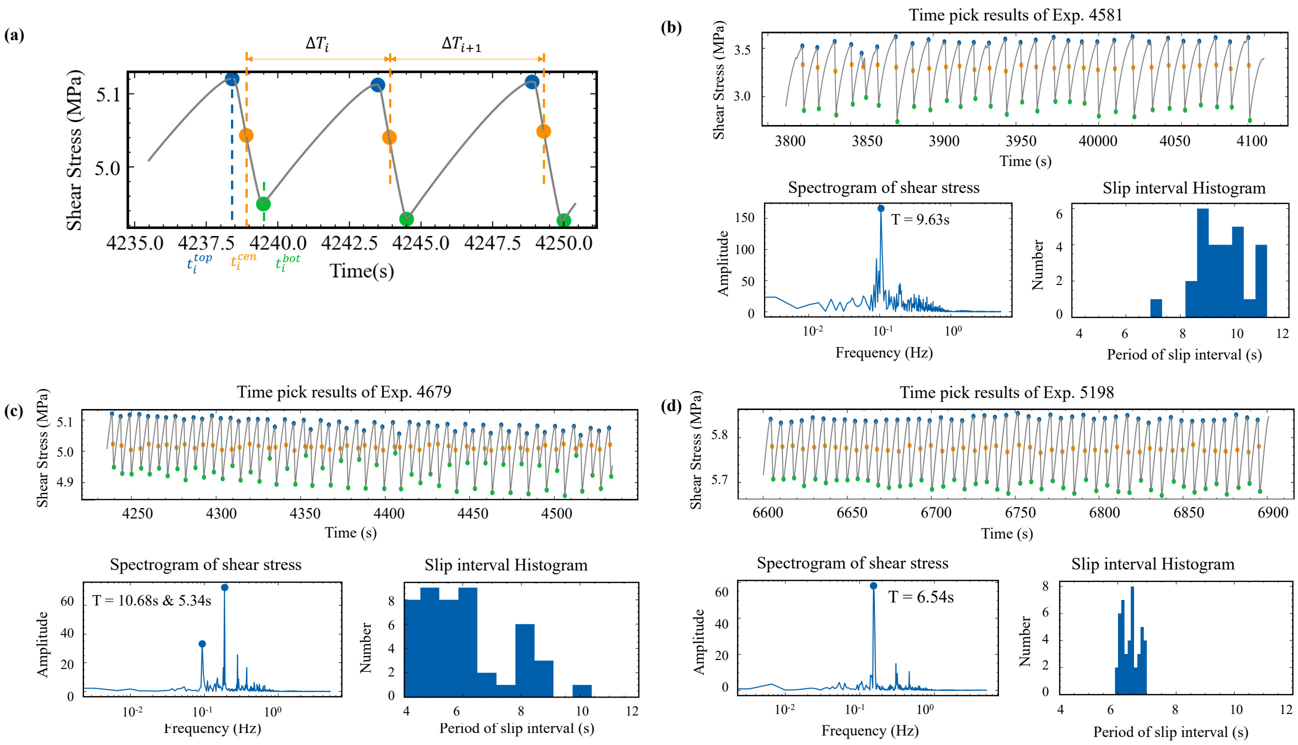
[Response]:

Thank you for your well-founded comments. In our previous discussions with reviewers, we were deeply inspired. We have compiled valuable insights from those discussions and incorporated them into the revised manuscript, primarily including:

1. Time-frequency analysis of stress variations across different slip behaviors, slip interval statistics, and their relationship with prediction performance.
2. Performance differences of HKAE under varying slip modes and prediction horizons, as well as the impact of embedding dimensions on prediction accuracy.
3. Influence of the three model modules (especially the Koopman Operator configuration) on overall model performance.

Detailed modifications can be found in the revised version we submitted (mainly in Sec. 2, 3.1 and 5).

For point #1: Section 2 & 3



Revised Figure 6: (a) Calculation of the slip interval ΔT. (b)-(d) slip intervals histogram and spectrogram of 3 experiments.

We apply ED to the learned Koopman operator (Eq. (4)) and obtain the decomposed complex eigenvalue (Table 2), which represents eigendynamic modes characterized by distinct amplitudes and periods. The results are in general agreement with the dominant period obtained from frequency spectral analysis of different slip experimental stresses (Figure 6b-d). …

For point #2: Section 4

In addition, we find that the dimension of delayed embedding affects the prediction performance and prediction preference of HKAE to a larger extent. For example, in the slow-slip, low system dimension scenario, the overall prediction of HKAE with low embedding dimension is significantly better than that with high embedding dimension. For the fast-slow alternating slip and fast slip scenarios, the high-embedding dimension tends to obtain long-term stable prediction results, while the low-embedding dimension is able to obtain more accurate short-term prediction results (Figure 3 in the Supplementary Information). This suggests that the embedded dimension of HKAE need to be adjusted according to the slip activity state in practical applications.

For point #3: Section 4

To address these questions, informed by dynamic system theories, we pioneered a dynamic informed method, the HKAE, to predict the future shear stress of laboratory fault slips. The HKAE model is designed on the basis of delay embedding theory and Koopman theory and leverages the nonlinear fitting capabilities of neural networks and the systematic perspective of dynamic theories. The advantages of the HKAE include (1) multiscale modelling of laboratory slip systems under limited observations and (2) evolution mode and insights into laboratory slip from a dynamic systems perspective. The rationality of the HKAE architecture design was further verified in the ablation experiments of the three modules, especially the setup of the Koopman Operator module (Figure S6 in Supplementary Information).

2. **The authors tend to overstate their results, despite (a) clear instances of underestimation or overestimation in multiple cycles, and (b) performance that is comparable to other models, such as LSTM, in many cases.**

[Response]:

Thank you for your comments.

(1) For multiple cycles (i.e., intercyle in main text), we conclude that HKAE outperforms other methods, mainly from the goodness-of-fit of the slip intervals (i.e., Figure 10 a-2, b-2). Under statistical prediction with multiple cycles, HKAE is not numerically superior under all prediction leads. However, we believe that it is clearly **more meaningful to be able to predict precise slip intervals under multiple cycles of prediction**. In this case, HKAE clearly outperforms the other methods in that its prediction of the slip interval is more stable as the number of prediction steps increases, whereas the other methods show varying degrees of oscillation. In addition, HKAE also outperforms the other methods in terms of the statistics of the prediction results for the whole slip interval, as shown by the larger R^2 of ΔT although in the fast slip experiments, the R^2 of ΔT is negative, a phenomenon that occurs in all the prediction methods, and we have added a description of this limitation in the main text.

(2) For the performance of HKAE relative to other methods under different cases, we measure three main dimensions:

- First under the condition of intra-cycle 10s predicting 3s, Figure 8 shows the comparison of the prediction scores of different methods. Here we use the embedding dimension ($d = 100$) across experiments. Under the experiment of embedding dimension test (Figure S3) it can be found that better performance can be obtained under Exp. 4581 ($d = 60$) and Exp. 5198 ($d = 5$) in Figure 8. Secondly, under this experimental condition, we emphasize its prediction results in the stress-rise and stress-fall phases, and from the performance of the example represented by Figure 9, the prediction results of HKAE are superior, **especially in the stress-release phase**.

➢ Under the condition of inter-cycle 20s prediction for 10s, Figure 10 demonstrates the comparison of the prediction scores of different methods. Again, HKAE does not show a significant advantage in terms of statistical metrics (hence our use of the word comparable in the abstract), but it is able to predict slip intervals more accurately under these prediction conditions, which is certainly **a more meaningful characterization of the model's effectiveness**.

Based on your comments, we have made the following adjustments in the main text to make the content more objective.

For # Abstract

The HKAE performs dynamic modelling of laboratory fault systems and provides a continuous estimation of the future state of the system. It has been used in experiments with different slip behaviours and has the ability to predict shear stress variation during a slip cycle and slip activity during long-term seismic cycles. The HKAE outperforms traditional statistical methods while achieving results comparable to cutting-edge deep learning methods across multiple prediction scales. This is particularly evident in its accurate prediction of the stress release phase and precise estimation of the slip interval. More importantly, through dynamic theory and operator analysis in latent space, the HKAE provides insights into the stability of laboratory slip systems rather than full end-to-end black-box predictions.

For # Conclusion

In addition to the modelling performance, the analysis of the execution process of the HKAE can provide dynamic diagnostics for the laboratory slip system operating behind the shear stress observations, such as those of system trajectory behaviour, characteristic dynamic modes, and system stability. HKAE prediction results and dynamical system analyses show that slip behaviour, especially the long-term future prediction of fast-slip stress states, remains challenging.

3. **Additionally, the interpretability argument presented by the authors is a general observation about the system's dynamics rather than a concrete improvement in forecasting accuracy or model generalization.**

[Response]:

Thank you for your comments.

Interpretability is critical for applying deep learning in geosciences. Here, interpretability is defined relative to purely data-driven models: while time-series models like TCN and LSTM remain black-box, HKAE provides dynamical system-level insights, including dominant evolutionary modes and stability of system dynamics et al. We argue that such dynamical insights constitute a valid form of interpretability. Dynamical analysis of observational data is essential for understanding predictability and stability in Earth systems [1,2], particularly in fault slip scenarios where recent studies have adopted dynamical perspectives to reveal key behavioral patterns [3,4].

Unlike post-hoc explanation methods (e.g., SHAP, feature importance) that focus on variable correlations, HKAE's interpretability originates from first-principles dynamical theory. To prevent ambiguity, we explicitly defined this distinction in Section 2.3 of the revised manuscript.

References:

[1] Miller, S. A., Nur, A., & Olgaard, D. L. (1996). Earthquakes as a coupled shear stress-high pore pressure dynamical system. Geophysical Research Letters, 23(2), 197-200.

[2] Runge, J., Bathiany, S., Bollt, E., Camps-Valls, G., Coumou, D., Deyle, E., ... & Zscheischler, J. (2019). Inferring causation from time series in Earth system sciences. Nature communications, 10(1), 2553.

[3] Gualandi, A., Faranda, D., Marone, C., Cocco, M., & Mengaldo, G. (2023). Deterministic and stochastic chaos characterize laboratory earthquakes. Earth and Planetary Science Letters, 604, 117995.

[4] Gualandi, A., Dal Zilio, L., Faranda, D., & Mengaldo, G. (2024). Similarities and differences between natural and simulated slow earthquakes. Geophysical Research Letters, 51(14), e2024GL109845.

4. **The manuscript lacks a discussion on the model's applicability to other parameters, such as time to failure, as well as its relevance to real-world field studies. It is evident that stress values are only accessible in laboratory experiments and cannot be directly extrapolated to tectonic earthquakes. Therefore, what are the potential applications of this model in such scenarios?**

[Response]:

Thank you for your comments.

(1) Our decision to exclude TTF prediction stems from two considerations:
   - Existing works on laboratory earthquake forecasting predominantly predict both TTF and stress states, with high consistency between the two predictions [1-3].
   - As a dynamics-inspired method, HKAE prioritizes direct system state variables (shear stress) over engineered features like TTF. Shear stress explicitly characterizes the system's physical state, whereas TTF represents a derived phenomenological metric.

(2) While stress measurements are inaccessible in tectonic settings, recent studies demonstrate that GNSS or seismogram time series can serve as proxies for stress state variations [4-6], analogous to acoustic emission-stress correlations in laboratory earthquakes. Neural network architecture of HKAE could adapt to field data by modifying input-output mappings structures (e.g., add a network branch to learn the relationship between stress and acoustic emissions). In particular, recent work has shown that machine learning algorithms can detect states such as displacement of active faults directly from continuous waveforms [6].

We have added these ideas in Discussion. In our next work, we will consider the prediction of actual measurable metrics such as TTF on the one hand, and test the prediction of accessible data from tectonic seismic on the other.

References:

[1] Rouet-Leduc, B., Hulbert, C., Lubbers, N., Barros, K., Humphreys, C. J., & Johnson, P. A. (2017). Machine learning predicts laboratory earthquakes. Geophysical Research Letters, 44(18), 9276-9282.

[2] Shokouhi, P., Girkar, V., Rivière, J., Shreedharan, S., Marone, C., Giles, C. L., & Kifer, D. (2021). Deep learning can predict laboratory quakes from active source seismic data. Geophysical Research Letters, 48(12), e2021GL093187.

[3] Wang, C., Xia, K., Yao, W., & Marone, C. (2025). Generalizable deep learning models for predicting laboratory earthquakes. Communications Earth & Environment, 6(1), 219.

[4] Rouet-Leduc, B., Hulbert, C., & Johnson, P. A. (2019). Continuous chatter of the Cascadia subduction zone revealed by machine learning. Nature Geoscience, 12(1), 75-79.

[5] Johnson, C. W., & Johnson, P. A. (2024). Seismic features predict ground motions during repeating caldera collapse sequence. Geophysical Research Letters, 51(11), e2024GL108288.
[6] Johnson, C. W., Wang, K., & Johnson, P. A. (2025). Automatic speech recognition predicts contemporaneous earthquake fault displacement. Nature Communications, 16(1), 1069.

We suggest that the HKAE can achieve competitive modelling of seismic activity and diagnose the dynamic behaviour of regional seismic systems by incorporating dynamic system theory. … In addition, stress is not directly accessible under tectonic seismic environmental conditions, which increases the difficulty of applying HKAE under real conditions. However, recent studies have shown that time-series observations, such as GNSS and seismometers, exhibit the feasibility of serving as a proxy for the state change of stress in tectonic earthquakes, and this relationship is similar to that between acoustic emission signals and stress changes in laboratory earthquakes (Johnson et al., 2024; 2025). HKAE has advantages for data fusion due to its flexible neural network architecture implementation. Therefore, the generalizability of the model can be improved by integrating external data such as historical acoustic emissions or other measurable laboratory observations by means such as adding coding branches.

5. **Furthermore, while shear stress exhibits a highly periodic behavior in double-shear tests, rough fault stick-slip experiments (Goebel et al., 2012; Dresen et al., 2020) have demonstrated that roughness evolution significantly influences stress cycles. The authors should include a discussion on this aspect.**

[Response]:

We sincerely appreciate the reviewer's valuable comments on the evolution of complex stress changes. In response, we have strengthened the Discussion section (Section 5) to explicitly address the implications of heterogeneous fault conditions (e.g., roughness effects) on stress predictability.

This work intentionally focuses on three fundamental slip regimes (fast, slow, and alternating fast-slow slips) to establish baseline dynamics under bi-shear experiments. These regimes provide essential benchmarks for evaluating HKAE's core capabilities (Section 3). We fully agree that fault roughness amplifies nonlinear stress variations, as evidenced by prior studies (Goebel et al., 2012; Dresen et al., 2020). Such complexity warrants dedicated investigation beyond the current triaxial shear experimental framework. We added related points in Discussion.

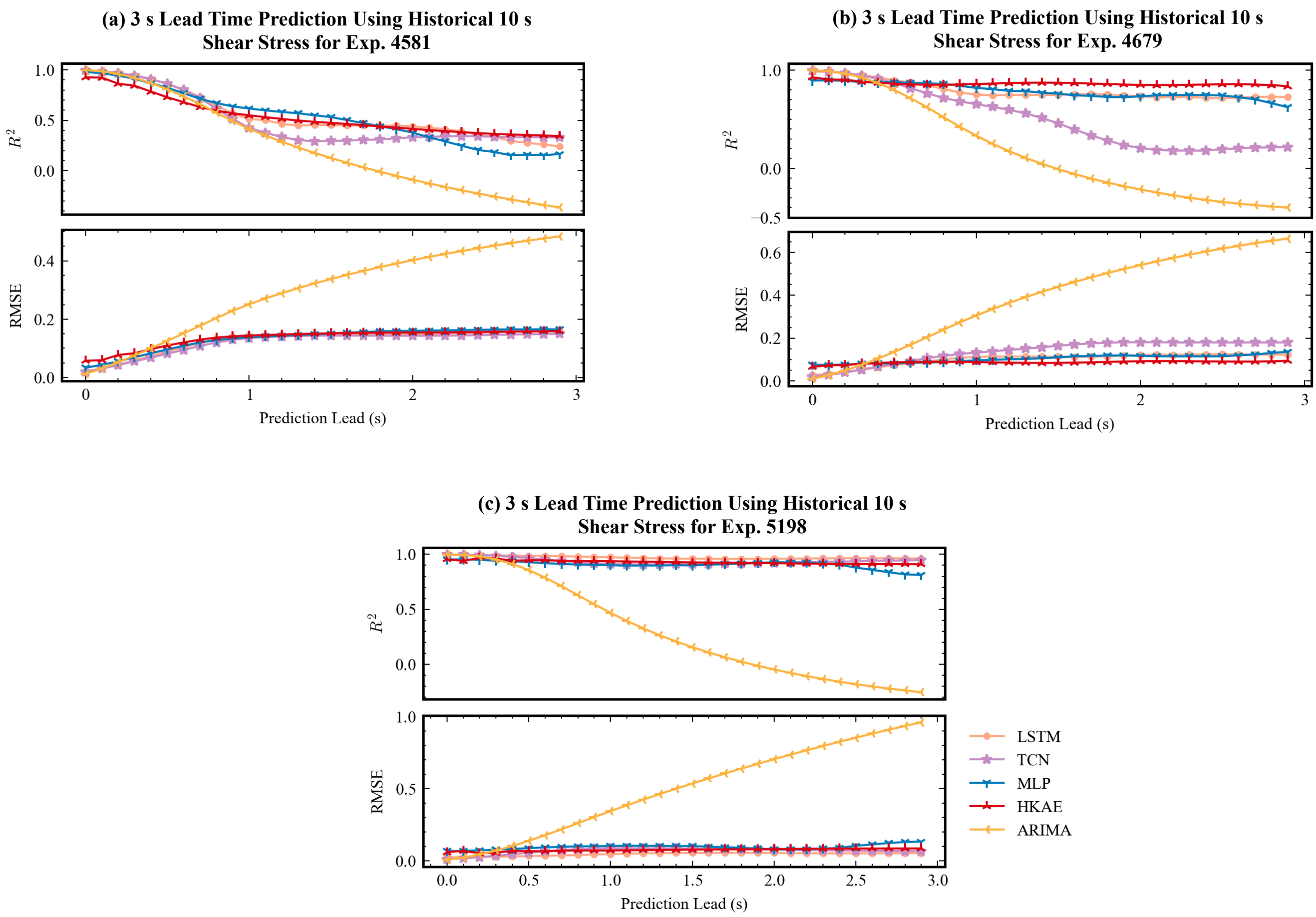
We suggest that the HKAE can achieve competitive modelling of seismic activity and diagnose the dynamic behaviour of regional seismic systems by incorporating dynamic system theory. Currently there are two main challenges in the application of HKAE to actual tectonic conditions. One is that the stress state changes of actual tectonic earthquakes may be complex. In order to verify the modelling capability of HKAE, we tested it using a typical double-shear experiment representing slip fast and slow with alternation. It is shown that the stress changes are more complex under rough fault viscous slip experimental conditions (Dresen et al., 2020). Although recent studies have shown that slip Time-To-Failures (TTFs) under high roughness can be predicted using machine learning (Karimpouli et al., 2023), slip dynamical system properties under high roughness remain currently undiscussed, which may affect the future predictive performance of HKAE under such more complex conditions of slip. …

6. **Figures 8 and 10 contain numerous plots. For instance, in Figure 10, if the first two columns represent different views of the same plot, they should be visually linked using lines or another connecting element. And use some kind of numbering gor them.**

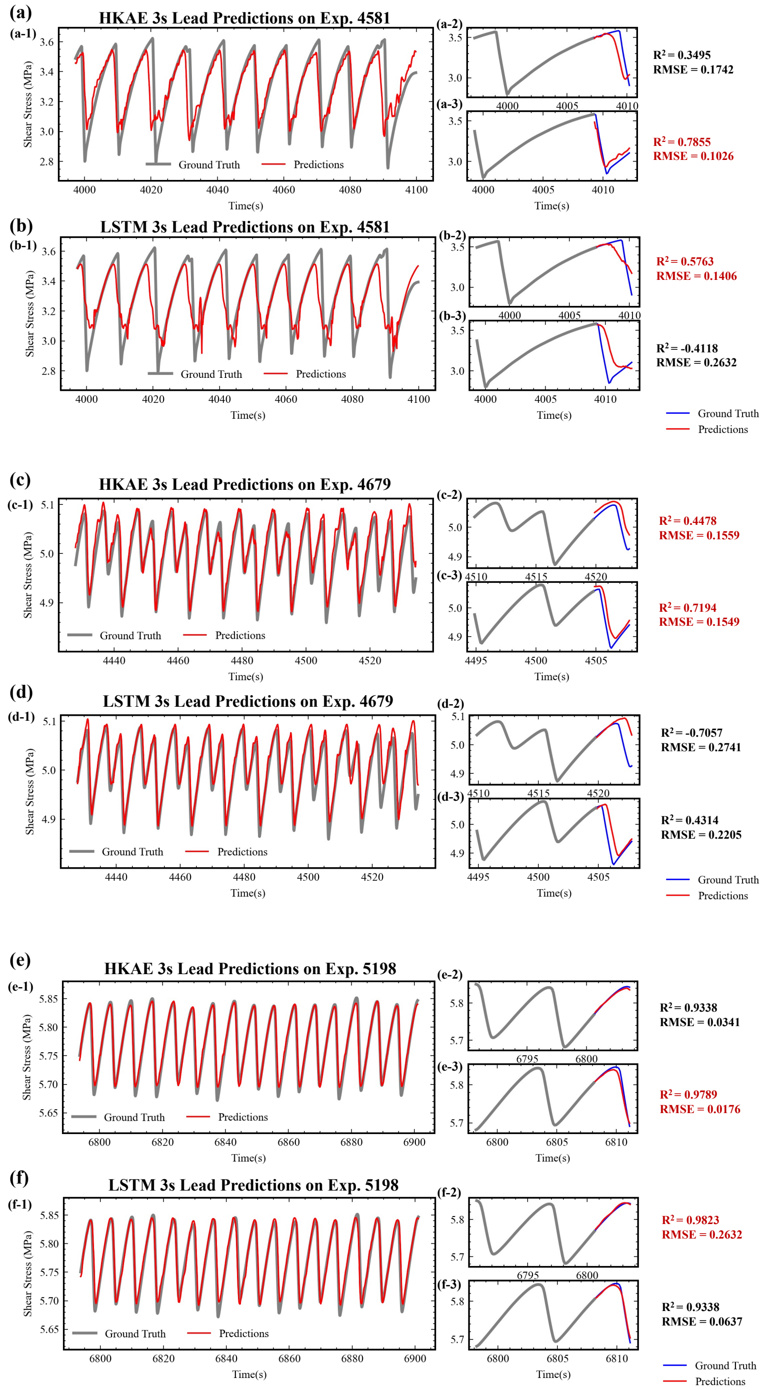
[Response]:

Thank you for your comments.

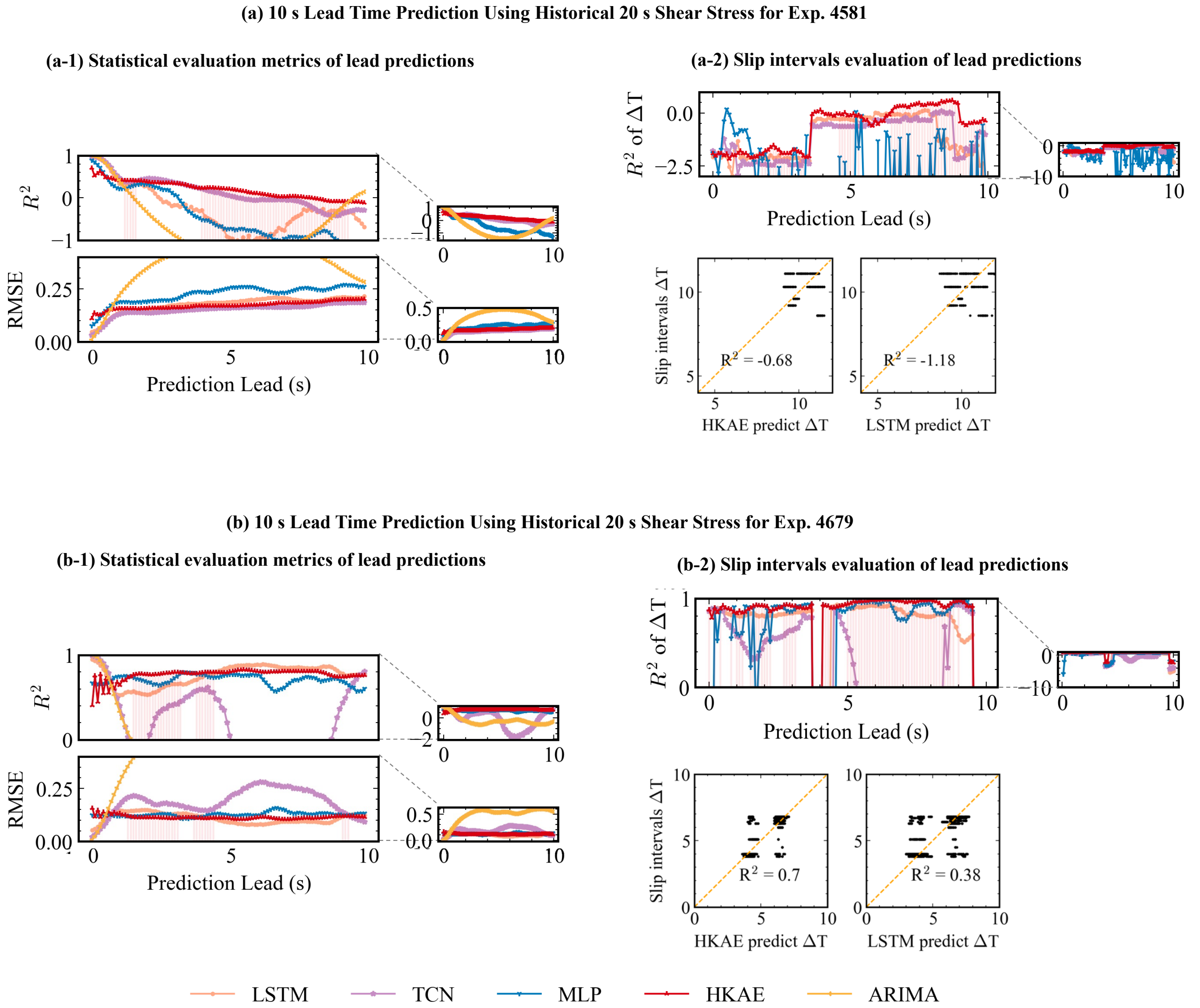
We have adjusted the presentation of Figure 8, 9 and 10 in the revised version. We add the numbers of panels to quickly localization.



Revised Figure 8: Genenral evaluation for 3 s lead prediction using historical 10 s shear stress, with R^2 and RMSE used as evaluation metrics. (a)-(c) for Exp. 4581, Exp. 4679 and Exp. 5198 respectively.



Revised Figure 9: 3 s leading prediction details during different phases of stress variation for different laboratory datasets. (a), (c), and (e) show the HKAE results, whereas (b), (d), and (f) show the LSTM results. The left panels (x-1) present the total 3 s leading predictions for the test set. The right panels (x-2, x-3) illustrate the predictions in the 3 s horizon, with R2 and RMSE used as evaluation metrics. Predictions with higher metrics are highlighted in red.



Revised Figure 10: General evaluation for 10 s lead prediction using historical 20 s shear stress, with R^2, RMSE and R^2 of event intervals (Eq. 18) used as evaluation metrics. (a)-(b) for Exp. 4581, Exp. 4679 respectively. (a-1) and (b-1) illustrate the metrics variation with prediction leads. (a-2) and (b-2). The prediction results of the sliding prediction process for the sliding intervals were counted and compared with the real sliding intervals (Figure 6b-d).

7. **Figure 10: What does a negative R2 value indicate?**

[Response]:

We thank the reviewer for raising this important point.

A negative R^2 indicates that the prediction results of MORNING are worse than the mean value of the observed data used directly. The negative R^2 in Fig. 10 evaluates the results of the ΔT prediction of the slip period for the HKAE and LSTM for the 10s lead case. From the figure, it can be observed that in the fast slip experiment (Exp. 4581), the slip period occurs in segments of 8-12 s. The negative R^2 values for both HKAE and LSTM suggest that neither of the currently employed methods may be able to capture the evolutionary trend of the fast slip system, although the negative R^2 values for HKAE are slightly smaller than those for LSTM.

We add the reasons for the occurrence of negative R^2 values in Sec. 3.3.

To further assess the ability of the prediction method to model the event cycle, we counted the predictions for the slip intervals in the prediction window in the sliding prediction experiment, and assessed the goodness of fit between the predictions and the true intervals (Figure 10a-2, b-2). Considering the event cycle predictions over the entire prediction window, the HKAE also has a advantage over the LSTM, as demonstrated by the fact that its cycle predictions are closer to the identity line. However, the R^2 scores of slip intervals are negative in Exp.4581, which indicates that both HKAE and LSTM have limited ability to capture the evolutionary features of fast slip systems.