Reply to Review for "A dynamic informed deep learning method for future estimation of laboratory stick- slip" by Yue et al.

[Review]:

This manuscript explores a novel model that integrates dynamic systems theory with the nonlinear fitting capabilities of deep learning. The HKAE model utilizes the Koopman operator and an autoencoder framework to reconstruct the dynamic behavior of laboratory slip systems, with a specific emphasis on shear stress variations. Through a synthesis of theoretical analysis and experimental validation, the HKAE model showcases exceptional performance in predicting complex nonlinear systems. This study underscores the HKAE model's advantages in handling intricate nonlinear dynamic systems and proposes promising directions for future research and applications. The following issues could enhance the manuscript's quality.

In Lines 70-71, the statement "our model envisions the future prediction as the continuous evolution of laboratory fault slip systems" lacks clarity and requires a detailed explanation.

In Section 2.1, simple simulation data was utilized. Did the model explore more intricate numerical simulations, and what were the specific outcomes? More details about the simulation should be given. Where is the laboratory data coming from? At least the references should be provided.

In Section 2.2, please elucidate the advantages of Koopman theory and delay embedding theory in managing the intricate dynamics of laboratory fault slip systems.

Line 97, "we're" should be "we are". No contraction is allowed in formal English writing. Line 340, as well, among others.

In Section 2.3, how do the functions of these three model modules impact the model's performance?

Line 305, "to shows" should be "to show". The authors should carefully check the whole paper regarding typos.

In Figure 9, RMSE and R2 should be included.

In Line 339, the model exhibits subpar performance in Exp. 4581. Is the model particularly sensitive to specific types of data?

In Section 4, what is the interpretability of the HKAE model? Are there specialized methods or techniques for elucidating the model's predictions?

In Section 5, the HKAE model has demonstrated robust predictive capabilities. What are the potential directions for future research?

The fonts in some figures are extremely large, while in others are too small to read. I highly recommend that the authors re-draw most figures to enhance their quality.

[Response]:

Thank you so much for your valuable comments, which are very helpful for improving our manuscript.

In response to your review comments, we will make the following responses:

（Black bold text: Reviewers' comments; Purple text: Our responses; Red text: changes in manuscript）

1. **In Lines 70-71, the statement "our model envisions the future prediction as the continuous evolution of laboratory fault slip systems" lacks clarity and requires a detailed explanation.**

[Response]:

Thank you for your comments. Here we want to emphasize that we do not define the problem as a time series forecast from a statistical point of view, but rather from a dynamical system perspective. Specifically, we take the shear stress time series as one of the observations of a laboratory slip system, and infer changes in the future state of the system through methods inspired by dynamical systems theory. To this end, we reconstruct the phase space of the system using delayed embedding theory and linearize its complex dynamics using Koopman theory to perform future inference and dynamical analysis.

To explain more clearly, we changed the statements to: "Instead of defining the problem as a statistical time series forecast task, we take the shear stress time series as one of the observations of a laboratory slip system, and infer changes in the future state of the system through methods inspired by dynamical systems theory. Generally, we reconstruct the phase space of the system using delayed embedding theory and linearize its complex dynamics using Koopman theory to perform future inference and further dynamical analysis."

2. **In Section 2.1, simple simulation data was utilized. Did the model explore more intricate numerical simulations, and what were the specific outcomes? More details about the simulation should be given. Where is the laboratory data coming from? At least the references should be provided.**

[Response]:

Thank you for your comments. The simulation data is conducted with code from Gualandi's work (2023) with these following equations:

$$\frac{\partial x}{\partial t} = \frac{e^{x}[(\beta_1 - 1)x(1+\lambda u) + y - u] + \kappa\left(\frac{y_0}{v_*} - e^{x}\right) - \frac{\partial u}{\partial t}\frac{1+\lambda y}{1+\lambda u}}{1 + \lambda u + \nu e^{x}}$$

$$\frac{\partial y}{\partial t} = \kappa\left(\frac{v_0}{v_*} - e^{x}\right) - \nu e^{x}\frac{\partial \mathrm{x}}{\partial \mathrm{t}}$$

$$\frac{\partial z}{\partial t} = -\rho e^{x}(\beta_2 x + z)$$

$$\frac{\partial u}{\partial t} = -\alpha - \gamma u + \frac{\partial \mathrm{z}}{\partial \mathrm{t}}$$

Where $[x, y, z, u] = [\ln\left(\frac{v}{v_*}\right), \frac{\tau_f - \tau_0}{a\sigma_{n0}}, \frac{1}{\lambda\beta\sigma_{n0}}(\phi - \phi_0), -\frac{1}{\lambda}\frac{p}{\sigma_{n0}}]$ represents the system state. Here we set the normal stress initial as $\tau_{n0} = 17.003\ Mpa$ and state vector initial as $[x_0, y_0, z_0, u_0] = [0.05, 0.0, 0.0, 0.0]$to generate fast-slow-switching slips as simulation examples. We supplied the equations and parameters used above during simulation in the revised version.

We mainly focus on the actual laboratory data, while the simulation data is used as a conceptual and pure sample, to make the further dynamic analysis easier to understand. More complex numerical simulation data, e.g., weak cyclicity data (Wang et al., 2021) we consider to discuss in our further work.

The laboratory data comes from Laurenti's work (2022), the original source is http://psudata.s3-website.us-east-

2.amazonaws.com/. We have annotated in Section 2.1 as "The experimental data from the biaxial shear equipment comes from the PSU laboratory (Laurenti et al., 2022)" and in Code and data availability.

Reference:

[1] Gualandi, A., Faranda, D., Marone, C., Cocco, M., Mengaldo, G., and Bendick, R.: Deterministic and stochastic chaos characterize laboratory earthquakes, Earth and Planetary Science Letters, https://doi.org/10.1016/j.epsl.2023.117995, 2022

[2] Laurenti, L., Tinti, E., Galasso, F., Franco, L., and Marone, C.: Deep learning for laboratory earthquake prediction and autoregressive forecasting of fault zone stress, Earth and Planetary Science Letters, https://doi.org/10.1016/j.epsl.2022.117825,2022

[3] Wang, K., Johnson, C. W., Bennett, K. C., and Johnson, P. A.: Predicting fault slip via transfer learning., Nature Communications, https://doi.org/10.1038/s41467-021-27553-5, 2021

3. **In Section 2.2, please elucidate the advantages of Koopman theory and delay embedding theory in managing the intricate dynamics of laboratory fault slip systems.**

[Response]:

Thank you for your suggestions. We have further elucidated the advantages in the revised version.

(1) The advantage of delay embedding theory is that using only the shear stress observations to reconstruct the phase space of laboratory system. Considering the limited observation of fault system, it's potential to use stress series (or stress proxy like displacement observations) to recover the system behaviours.

(2) The advantage of Koopman theory is to linearize the dynamics from complex laboratory fault system, then support the analysis of system behavior using linear analysis tools (e.g., singular value decomposition), which will offer the interpretability and insights from the dynamical system perspective.

4. **Line 97, "we're" should be "we are". No contraction is allowed in formal English writing. Line 340, as well, among others.**

[Response]:

Thank you for your suggestions. We have adjusted these informal expressions in revised version.

5. **In Section 2.3, how do the functions of these three model modules impact the model's performance?**

[Response]:

Thank you for your comments. Since the HKAE is designed inspired by dynamical systems theory, the modules are more closely connected, making it more difficult to discuss a thorough separation. We discuss the roles of the modules theoretically and perform alternative ablation experiments.

(1) The delay embedding module reconstructs the phase space of system. Without delayed embedding to provide embedding coordinates, the latter two modules are unable to construct stable mappings and approximate Koopman operators from univariate data alone. Without delayed embedding to provide embedding coordinates, the latter two modules are hard to construct stable mappings and approximate Koopman operators from univariate data alone (Brunton et al., 2021).

(2) The mapping learning module performs the necessary nonlinear mapping, which, if removed, becomes a linear moving-average-like method. We've tested the ARIMA for the most challenging fast-slow-switching

slips and the results shows that the linear methods do not perform well (Figure 1).

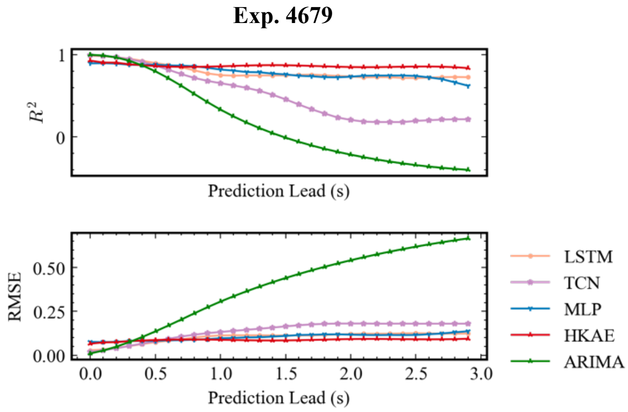

Figure 1. Ablation study using historical 10s to predict future 3s. Green line illustrates the performance of ARIMA, representing linear methods without nonlinear mapping.

(3) The Koopman operator achieve linear evolution of system in the latent space, and offer dynamic interpretability. We change the Koopman operator into a linear layer of equal size but with bias, and test the "10-3s" predictions for the most challenging fast-slow-switching slips. The statistical metrics illustrate that HKAE performs better than the model without Koopman evolution module. More importantly, removing this module makes it impossible to analyze the dynamical patterns of the system.

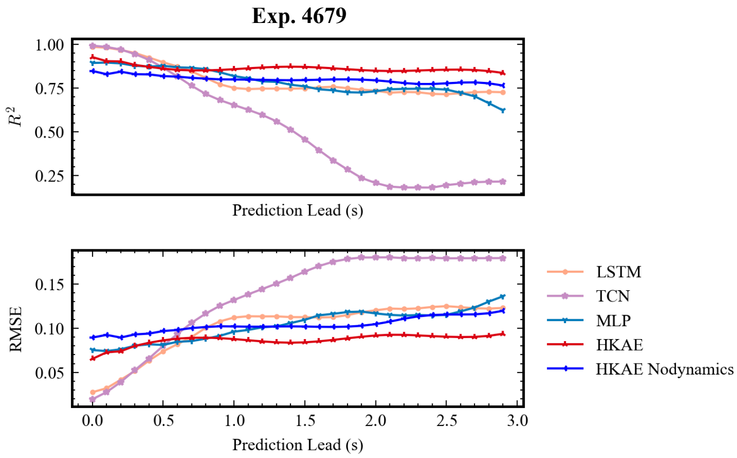

Figure 2. Ablation study using historical 10s to predict future 3s. Blue line represents the HKAE model whose operator in Koopman Evolution module is replaced by a normal linear layer with activation.

Reference:

[1] Brunton, S. L., Budišić, M., Kaiser, E., and Kutz, J. N.: Modern Koopman Theory for Dynamical Systems., SIAM Review, 229–340, https://doi.org/10.1137/21m1401243, 2022.

6. Line 305, "to shows" should be "to show". The authors should carefully check the whole paper regarding typos.

[Response]:

Thank you for your suggestions. We have checked the typos in revised version.

7. In Figure 9, RMSE and R2 should be included.

[Response]:

Thank you for your suggestions. We have added statistical metrics in the figure.

8. In Line 339, the model exhibits subpar performance in Exp. 4581. Is the model particularly sensitive to specific types of data?

[Response]:

Thank you for your comments. Theoretically, the model is designed entirely from dynamical systems theory, which is a generalized structure and therefore not sensitive to specific data.

Theoretically, the model is designed entirely from dynamical systems theory, which is a generalized structure and therefore not sensitive to specific data. Objectively, HKAE's results on the Exp.4581 are poor because of its high prediction difficulty, but compared to other methods, HKAE actually has a more competitive edge (Figure 9 in manuscript). We infer the true dimension of the laboratory slip system will also have a certain impact on the performance of HKAE. We analyze slow slip experiments (Exp. 5198) that are statistically average in terms of competitiveness in performance, and find that they are significantly better able to achieve better performance in low embedding dimensions (Figure 3).

Taking into account your suggestions for more complex simulation data, we will further discuss the sensitivity of HKAE to data with different characteristics in our follow-up work.

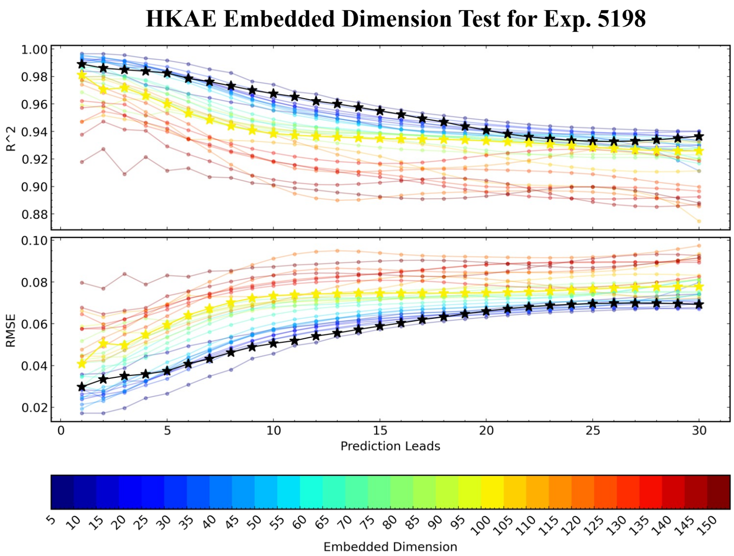

Figure 3. Embedded dimension test for slow slip system. Black line indicates the embedded dimension calculated by Cao et al., while yellow line represents the embedded dimension used in the manuscript.

9. In Section 4, what is the interpretability of the HKAE model? Are there specialized methods or techniques for elucidating the model's predictions?

[Response]:

Thank you for your comments. To enhance the expression of HKAE interpretability, we have provided a more detailed explanation in the revised version.

The interpretability of HKAE comes from the delay embedding theory and Koopman theory. The encoder of HKAE mapped the phase space of input shear stress into a latent space, where we think the evolution of the system is linear. And the linear dynamics are controlled by the matrix-like approximate Koopman operator. Thus， the characteristic of system can be discussed. The Similar interpretations can be found in works by Lusch et al. (2018), Azencot et al. (2020) and Ouala et al. (2023).

The amplitude and of dynamic are shown in Figure 7 and introduced in Section 3.1. Section 4 further discuss the dynamics evolution characteristics (Figure 11a). Then Figure 11c and 11d discussed the latent variables evolution under the approximate koopman operator, which indicates that the HKAE can obtain components that can be linearly modeled in the latent space (the first 7 subplots), but there will still be some components that are difficult to describe with linear dynamics. There could be two reasons for this result: one is the limitation of modeling due to insufficient observations, where some of the system's dynamics need to be explained as nonlinear forcing (Brunton et al., 2017); the other is the complexity of the system's dynamic characteristics. We've been considering discuss it in our further work.

Reference:

[1] Lusch, B., Kutz, J. N., and Brunton, S. L.: Deep learning for universal linear embeddings of nonlinear dynamics., Nature Communications, https://doi.org/10.1038/s41467-018-07210-0, 2018

[2] Azencot, O., Erichson, N. B., Lin, V., and Mahoney, MichaelW.: Forecasting Sequential Data using Consistent Koopman Autoencoders, International Conference on Machine Learning. PMLR, 2020.

[3] Bounded nonlinear forecasts of partially observed geophysical systems with physics-constrained deep learning

10. In Section 5, the HKAE model has demonstrated robust predictive capabilities. What are the potential directions for future research?

[Response]:

Thank you for your suggestion. We'd like to further discuss 2 directions:

(1) One of the potential directions is considering the time-vary dynamics in the system. There're time-vary dynamics since we analyze the time-frequency analysis of the shear stress of Exp. 4679. It' s challenging for HKAE to model the time-vary dynamics since the Koopman operator is trained to become a "global operator" (Brunton et al., 2022; Liu et al., 2023). The global operator is less likely to make accurate predictions when there are large changes in the dynamical features.

(2) The second one is to optimize the modeling framework in the presence of only a single observation to obtain more accurate estimates of the system dynamics. For example, considering the laboratory system as a forced system (Brunton et al., 2017), whose force may be processed acoustic emissions.

Reference:

[1] Brunton, S. L., Brunton, B. W., Proctor, J. L., Kaiser, E., and Kutz, J. N.: Chaos as an Intermittently Forced Linear System, Nature Communications, https://doi.org/10.1038/s41467-017-00030-8, 2017.

[2] Brunton, S. L., Budišić, M., Kaiser, E., and Kutz, J. N.: Modern Koopman Theory for Dynamical Systems., SIAM Review, 229–340, https://doi.org/10.1137/21m1401243, 2022.

[3] Liu, Y., Li, C., Wang, J., and Long, M.: Koopa: Learning Non-stationary Time Series Dynamics with Koopman Predictors, arXiv: Learning, 2023.

11. **The fonts in some figures are extremely large, while in others are too small to read. I highly recommend that the authors re-draw most figures to enhance their quality.**

[Response]:

Thank you for your suggestion. We have adjusted the figures and re-drawn the figures in revised version.