# Response letter to the referee #3

The authors have addressed the majority of my concerns and I think the paper is very close to being publication worthy. The one area outstanding is on the theory side. My point was not that one weakly nonlinear theory reference (Gear and Grimshaw) should be replaced by another, but that the whole idea of using weakly nonlinear theory as a quantitative tool is flawed. Between myself and Kevin Lamb there have been at least ten papers showing the various ways in which KdV type theories fail, and that the exact DJL theory can (sometimes) succeed where they fail. As I mentioned, there is even a monograph I wrote on the open source uses of the DJL theory. So please include something to even out the discussion. It isn't just weakly nonlinear theory or regional models, there is a lot more in between.

***Response*:**

We sincerely thank the reviewer for this valuable comment regarding the theoretical framework discussion. We fully acknowledge that the original manuscript failed to reflect contemporary advances in solitary wave theory. In this revision, we have substantially revised the introduction to directly address these concerns by:

(1) Explicitly stating that weakly nonlinear (KdV-type) theories are fundamentally flawed as quantitative tools for realistic ISW dynamics particularly in the NSCS, supported by citations to Lamb (1999) and Stastna and Lamb (2008) demonstrating errors in phase speed and wave width estimates;

(2) Elevating DJL theory as the rigorous alternative that succeeds where KdV fails, highlighting its exact formulation without weak-amplitude/long-wave approximations;

(3) Clearly positioning modern 3D non-hydrostatic models (MITgcm, SUNTANS, FVCOM) as the essential bridge between theoretical paradigms and operational systems.

Detailed modifications are listed below (Section 1, Lines 26-47):

"Internal solitary wave (ISW) research has historically relied on theoretical frameworks to describe nonlinear wave dynamics. Weakly nonlinear theories, epitomized by the Korteweg-de Vries (KdV) equation (Benney, 1966) and its extensions (Grimshaw et al., 2010), employ asymptotic expansions to decouple vertical structure from horizontal evolution. While providing valuable conceptual insights, these approximations exhibit systematic quantitative deficiencies for large-amplitude ISWs particularly in the northern South China Sea (NSCS), where vertical displacements of ISWs exceed 200 m (Huang et al., 2017; Alford et al., 2015). Specifically, KdV-type theories might overestimate phase speeds and underestimate wave widths (Lamb, 1999; Stastna and Lamb, 2008), limiting their utility as predictive tools. Concurrently, the exact Dubreil-Jacotin-Long (DJL) theory emerged as a mathematically complete alternative, solving the stratified Euler equations without amplitude or wavelength approximations (Stastna and Legare, 2024). The DJL equation computes ISW structure and propagation speed through an eigenvalue problem that intrinsically accounts for isopycnal displacement effects, providing high-fidelity solutions even for complex stratifications. Nevertheless, both KdV and DJL approaches share inherent constraints that they describe steady-state waves or slow shoaling dynamics (Lamb and Xiao, 2014), but cannot resolve transient 3D processes and define entire ISW lifecycles in realistic oceans.

To overcome these limitations, high-resolution numerical solvers have become indispensable for simulating ISW dynamics. By the early 21st century, two-layer analytical models (Holloway et al., 1997) and depth-averaged 2D hydrostatic approaches (Du et al., 2008) proved inadequate for capturing non-hydrostatic effects and strong nonlinearity in regions like the NSCS. This spurred development of high-resolution 3D non-hydrostatic solvers capable of resolving critical processes including generation, propagation, and dissipation of ISWs (Simmons et al., 2011). Contemporary open-source frameworks like SUNTANS (Zhang et al., 2011), MITgcm (Vlasenko et al., 2005; Alford et al., 2015), and FVCOM (Lai et al., 2019) now enable realistic simulations of ISW generation, propagation, and dissipation through advanced numerical schemes validated against modern observational arrays. These advances form the foundation for our ISWFM-NSCS model, which bridges the gap between theoretical paradigms and operational forecasting in the NSCS basin."