Response to Reviewer #2 for Geoscientific Model Development: Manuscript gmd-2024-148 By Liu et al.

We sincerely thank Reviewer #2 for thoughtful and constructive feedback. We have carefully considered each comment and made every effort to implement all the suggested changes. The notes below address each comment in detail. Please note that Reviewer's comments are shown in bold type and our responses in plain type.

Reviewer #2

Summary & General Comments

The article presents an overview of the development of a strongly coupled 4D-Var assimilation system where an aerosol atmospheric component, the total mass concentration of black carbon (BC), is added to the 4D-Var control vector. The article contains a detailed explanation of how the necessary linear models have been developed, by extracting and recoding the BC-related aerosols physical modelling codes and by formulating a specific B-matrix model (and control-vector conversion) for the BC mass concentration. Rather technical validation results are displayed showing the correctness of the TL and AD models, along with preliminary experimental results. The article finishes with an outlook mostly referring to a Part II where the authors intend to discuss comprehensive experimental results on the impact of assimilating BC in a strongly coupled formulation, on the other atmospheric fields (wind, temperature, pressure, humidity).

Response: We sincerely appreciate the reviewer's thorough summary and insightful comments on our study. The reviewer has correctly captured the key aspects of our work, including the development of the strongly coupled 4D-Var assimilation system with BC added to the control vector, the extraction and recoding of BC-related aerosol processes, and the construction of the background error covariance model for BC mass concentration. Furthermore, we appreciate the reviewer's recognition of the validation of the TL and AD models, as well as the preliminary experimental results presented in this work. As noted, we plan to present a more comprehensive analysis of the impact of BC assimilation on meteorological fields in Part II. Thanks again for the valuable feedback.

This article Part I is overall clearly structured, with each section well introduced. As stated by the authors, the aim of the paper is to present the methodology without entering into a complete, comprehensive evaluation of experimental results in full, long-period 4D-Var assimilation experiments. Taking into account that strongly coupled atmosphere-aerosol chemistry assimilation systems have been very little presented so far in open literature (to the reviewer's knowledge), the authors' choice to propose such an introductory Part I can be supported. Nevertheless, the paper focuses too strongly on technical sanity checks (such as the results of tests of TL and AD models which are standard and well-known tests when developing variational codes) which for themselves bring no innovative information.

Conversely, the paper lacks explanations on specific scientific challenges that would strengthen the scientific interest of the paper:

Response: We sincerely appreciate the reviewer's constructive feedback and recognition of the clear structure and methodological focus of this Part I paper. Indeed, as the reviewer correctly pointed out, our intention is to present the methodology and necessary technical developments rather than providing a full, long-period evaluation of 4D-Var assimilation experiments, which will be the focus of Part II.

Regarding the reviewer's concern about the strong emphasis on technical sanity checks, we would like to clarify that while TL and AD model verification is indeed a standard step in variational data assimilation, it is particularly crucial in our study due to the complexity of incorporating aerosol-related processes into a strongly coupled system. Ensuring the correctness of the TL and AD models is essential for establishing a solid foundation for the strongly coupled 4D-Var assimilation system.

We acknowledge the reviewer's suggestion to provide a more detailed explanation of the specific scientific challenges. We have carefully considered the reviewer's concerns regarding Section 4.4 in the original manuscript, as well as the comments from the other two reviewers. Based on these valuable suggestions, we have thoroughly revised this section, which is now presented as Section 5.3 in the revised manuscript.

In the updated version, we have clearly introduced the objective of the four experiments, which is to investigate the impact of different BC assimilation strategies on both BC and atmospheric variables. We have renamed the four experiments as DA_BC, DA_MET, DA_MET_then_BC, and DA_MET_BC_simult. The revised Table 3 now provides a clear description of the four experiments. We have also compared the BC analysis increments obtained from the DA_BC, DA_MET_then_BC, and DA_MET_then_BC, and DA_MET_BC_simult experiments, noting that the BC analysis increments from the DA_MET experiment are very small. Additionally, we compare the atmospheric analysis increments caused by BC assimilation in DA_BC, DA_MET_then_BC (DA_MET_then_BC - DA_MET), and DA_MET_BC_simult (DA_MET_BC_simult - DA_MET).

Our main conclusions from this analysis are as follows: The preliminary results obtained from this set of four experiments indicate that different BC assimilation strategies have little impact on BC analysis increments but significantly affect the analysis increments of atmospheric variables. When only BC observations are assimilated, the influence of BC on atmospheric variables is more pronounced, whereas the simultaneous assimilation of meteorological observations moderates this influence. This suggests that in BC assimilation, meteorological observations can help constrain the uncertainty introduced by BC observations on atmospheric variables, thereby improving the reliability of the assimilation results. Moreover, these results demonstrate the successful implementation of the newly developed CMA-GFS-AERO 4D-Var system and highlight it as an effective approach for investigating the feedback of BC data assimilation on meteorological forecasts.

In the future, we will conduct batch experiments using CMA-GFS-AERO 4D-Var to gain deeper insights into the role of BC assimilation in numerical weather prediction and further refine the system for broader applications.

Additionally, in response to another reviewer's suggestion, we have adjusted the radius of influence for BC observations to 2 km, 10 km, and 20 km for urban, rural, and remote stations, respectively, according to Elbern et al. (2007). Consequently, all experiments in Section 5.3 have been redone using the updated radii, and the corresponding figures and text have been revised

accordingly to reflect the new results.

For more details on the analysis, please refer to Section 5.3 of the revised manuscript. We once again appreciate the reviewer's valuable comments.

1. Compared to the BC physics available in the original CUACE codes, how much has the BC physics for the CMA-GFS-AERO codes been adapted in terms of the representation of the physical processes, such as transport, chemical transformation and the interaction with radiative processes ? Taking this comment one step further, has there been any kind of simplification made when developing the BC physics modules for the linear models, any step of regularization of a non-linear formulation, or any omission of specific complex processes whose linearization was felt too difficult (at least for this v1.0 of the system) ? More explicit explanations should be provided, likely in Sections 3.1 and 3.2.

Response: Thanks for the insightful comment. The AERO-BC module was created by extracting BC-related codes from the CUACE model, with its functionality aligning with the BC aerosol processes in the CAM module of CUACE. In other words, the physical processes for BC in AERO-BC are identical to those in the CAM module, with no changes made. The main differences lie in the engineering aspect: (1) while the CAM module was originally written in Fortran 77, the AERO-BC code has been rewritten in Fortran 90; (2) since CAM in CUACE deals with six types of aerosols, the code structure is somewhat complex and redundant, whereas AERO-BC focuses solely on BC, resulting in a simpler and more streamlined structure. These updates improve code readability and enhance computational efficiency, without affecting the underlying physical processes.

In constructing the tangent linear (TL) and the adjoint of AERO-BC, no simplifications were made to the AERO-BC processes. Specifically, no regularization was applied to the nonlinear equations, nor were any complex processes, which were difficult to linearize, omitted. As a result, the TL and the adjoint of AERO-BC fully include all processes related to emission flux, vertical diffusion, and aerosol physical processes (e.g. hygroscopic growth, coagulation, nucleation, condensation, dry deposition/sedimentation, and below-cloud scavenging).

Additionally, we have added these clarifications in Sections 3.1 and 3.2 of the revised manuscript.

2. More explanation of why the strongly coupled case provides significantly different results on the analysis fields of the "traditional" atmospheric fields, compared with no BC assimilation or with the weakly coupled case, is missing in Section 4.4. Two striking results are displayed but eventually with very little physical interpretation while both seem to be systematic results:

Response: We sincerely appreciate the reviewer's insightful comment. In the original manuscript, our expression in Section 4.4 was not sufficiently clear, which may have caused confusion. We apologize for any misunderstanding. After carefully considering the reviewer's comments, we have completely rewritten this section, which is now presented as Section 5.3 in the revised manuscript. In the updated version, we have added the necessary explanations to improve clarity and address the reviewer's concerns.

a. Adding BC in the modelling and assimilation system rather than omitting this component induces a positive analysis increment on temperature. So question here: should one

understand that adding BC in the forecast trajectory *anyway* will slightly increase temperature via the radiative effect of absorption? Is this effect then very systematic ? Is it local or even global ?

Response: Thanks for the insightful comment. Currently, the radiative effect of BC on atmospheric temperature is not yet considered in the CMA-GFS-AERO forward model, and this will be a key area of focus for our future work. In the revised manuscript, Section 3.3.5 introduces how CMA-GFS-AERO 4D-Var incorporates cross-covariances between BC and atmospheric variables through the background error covariance matrix. The adjoint model then propagates the impact of BC observations onto atmospheric variables, leading to corresponding analysis increments. Additionally, Section 5.2 presents single-observation experiments that systematically analyze the generation of temperature analysis increments at observation times when BC observations are assimilated at the initial, middle, and end of the assimilation time window. These results align with the theoretical framework described in Section 3.3.5. Therefore, in the present version of the CMA-GFS-AERO 4D-Var system, the positive analysis increment on temperature is primarily due to the assimilation of BC observations in the 4D-Var system, rather than the radiative heating effect of BC.

b. Why precisely is strongly coupled assimilation of BC causing an overall decrease of the amplitude of the analysis increments by an order of magnitude ? What are the damping retro-actions ?

Response: We sincerely appreciate the reviewer's insightful comment. As we mentioned in our previous response, after carefully considering the reviewer's comments, we have completely rewritten this section. In this revision, we have refined the explanation of the experiments and their results to improve clarity. Please refer to Section 5.3 in the revised manuscript for the updated expression. Below, we present the reasons why the analysis increments of atmospheric variables are smaller when both atmospheric and BC observations are assimilated simultaneously: "... The differences in analysis increments of the four atmospheric variables caused by BC assimilation between DA MET BC simult and DA BC/DA MET then BC may be due to the fact that information fusion reduces the impact of individual observation. As mentioned above, DA MET then BC is similar to DA BC in that, in the process of BC assimilation, only BC surface observations are incorporated into the assimilation system. At this stage, the system relies solely on BC observations to correct the initial field. In the absence of atmospheric observations, BC observations play a dominant role, leading to larger analysis increments of atmospheric variables. In contrast, in DA MET BC simult, both operational meteorological observations and BC surface observations are assimilated simultaneously. In this scenario, atmospheric observations provide more comprehensive or reliable information, which may reduce the dominant influence of the BC observations on the analysis increments of atmospheric variables. As a result, a more balanced adjustment of atmospheric variables is achieved in DA MET BC simult ... "

We once again appreciate the reviewer's valuable comments.

The article is fairly clearly written though some specific checking of English phrase construction could be worthwhile. In the specific comments below, a few particularly unclear phrasings are stressed, which deserve further attention and rewrite by the authors. In the bibliographical section, 5 references relate to documents in Chinese. It is unclear to the reviewer what GMD's policy about references in languages other than English is. It might be appropriate that the authors confirm that they can commit to make available translated texts, should they be asked by future readers.

Response: Thanks for the valuable feedback. We appreciate your comments on the clarity of the article, and we have reviewed the phrasing as suggested to improve the overall readability. Regarding the specific comments below, a few unclear phrasings have been revised as suggested. Concerning the references in Chinese, we understand the importance of ensuring accessibility for international readers. We confirm that, should future readers request, we will make efforts to provide translated texts for the Chinese references cited in the manuscript.

In conclusion, my recommendation is to accept the paper, as a Part I component to be complemented by a Part II, after revision. The goal of the revision, following the comments above, should be to strengthen the scientific explanations of the implementation of BC in the 4D-Var framework as well as to strengthen the physical interpretation of the experimental results displayed in Section 4. A further recommendation could be to extend the paper's title from "System description" to "System description and preliminary experimental results".

Response: We sincerely appreciate the reviewer's positive recommendation and constructive suggestions. We are grateful for the recognition of our work as a Part I study and fully acknowledge the importance of strengthening both the scientific explanations of BC implementation in the 4D-Var framework and the physical interpretation of the experimental results in Section 4 (now presented as Section 5 in the revised manuscript).

Following the reviewer's comments, we have carefully revised the manuscript to enhance these aspects. Regarding the suggestion to extend the paper's title, we acknowledge its potential benefits in better reflecting the content of the study. After careful consideration, we have updated the title from "System description" to "System description and preliminary experimental results" in the revised manuscript.

We sincerely appreciate the reviewer's valuable feedback, which has significantly contributed to improving the clarity and completeness of our study.

Specific Comments & Typos

Section 1.

line 69: what does "PM" stand for ?

Response: Thanks for pointing this out. "PM" stands for "particulate matter". PM_{2.5}, also known as fine particulate matter, refers to the particulate matter with an aerodynamic diameter of 2.5 micrometers or less.

We have added an explanation of PM_{2.5} in the revised manuscript as follows:

"Black carbon (BC) aerosol, a major component of the fine particulate matter (PM_{2.5}) defined by an aerodynamic diameter of 2.5 micrometers or less, primarily originates from the incomplete combustion of biomass and fossil fuels (Kuhlbusch, 1998)..."

Section 2. None.

Response: We sincerely appreciate the reviewer's feedback and patience.

Section 3.

line 169: "The transport processes for ψ_{bc} are the same as <u>those</u> for the variables <u>associated</u> with the different water species ..."

Re-phrase "water-matter" everywhere in the paper (not sure this is a good English wording, though it is understandable)

Response: According to the reviewer's good instructions, we have changed "that" to "those" in this sentence, and "water-matter" has also been revised to "variables associated with the different water species" throughout the manuscript.

lines 175-179 (end of section 3.1):

1. the whole text should be re-written, splitting it into two separate sentences.

Response: According to the reviewer's good instructions, we have re-written the text in the revised manuscript as follows:

"Besides, according to the vertical distribution of BC in the MERRA-2 (Modern-Era Retrospective analysis for Research and Applications, Version 2) reanalysis data (https://daac.gsfc.nasa.gov), we observed that the BC mass mixing ratio decreases rapidly after entering the stratosphere, reaching values of about 10^{-12} kg/kg. This is 2-3 orders of magnitude smaller compared to the surface. To improve computational efficiency and balance memory usage with the effectiveness of BC forecasting, we set the height of ψ_{bc}^n in the CMA-GFS-AERO model to 65 levels (approximately 30 hPa), which corresponds to the middle layer of the stratosphere...."

2. An additional explanation of how the absence of BC above model level 65 is dealt with in the models should be added. What happens regarding vertical transport for instance ? What's the impact in the adjoint code ?

Response: Thanks for the insightful comment. We have added an explanation regarding the treatment of BC above model level 65. We assume that BC concentrations above this level are negligible, given their small magnitude in the stratosphere. For vertical transport, this approximation does not have a significant impact on the model. In the adjoint code, BC concentrations above model level 65 are also treated as negligible, and this does not significantly affect the adjoint calculations.

We have added the explanation in the revised manuscript as follows:

"...Regarding the absence of BC above model level 65, we handled vertical transport by assuming that any BC concentrations above this level are negligible. This approximation does not significantly affect the model's performance, as the BC mass mixing ratio is very small in the upper layers. Correspondingly, in the adjoint code, BC concentrations above model level 65 are also treated as negligible, and this does not significantly affect the adjoint calculations."

lines 231-234: "Firstly ..." and later "Secondly ..." => reformulated these two sentences such that there is a verb. Perhaps, try with "Firstly, the distribution weights ... are calculated.". The same construction would apply to the next sentence.

Response: Thanks for the insightful comment. Following the recommendation, we have revised the two sentences in the revised manuscript as follows:

"...<u>Firstly, the distribution weights (ω^n) of each size bin of ψ_{bc}^n in the background field are</u>

<u>calculated</u> ...<u>Secondly</u>, the analysis increment of ψ_{bc}^n ($\delta \psi_{bc}^n$) is calculated based on the analysis increment of C_{bc} (δC_{bc}), following the equation..."

lines 274-277: reformulate that sentence (much too long). Make two separate ones.

Response: Thanks for pointing this out. We have revised the sentence into two separate ones in the revised manuscript as follows:

"...If the observation height is less than the height of the first model layer and the difference between the two heights is less than 300 meters, the BC concentration at the first model layer is regarded as the equivalent BC observation. <u>However, if</u> the difference between the two heights is greater than or equal to 300 meters, the data from that site is discarded."

line 282: I don't think that a sentence should start abruptly by "And ...". Simply remove this word with no loss of clarity of the text ?

Response: Thanks for pointing this out. We agree that a sentence should not start abruptly by "And …". Following the recommendation, we have removed this word in the revised manuscript without affecting the clarity of the text.

lines 286-288:

1. The vertical correlation model of the background error *is* expressed as ...

Response: This has been revised as suggested.

2. Some additional explanation of how this formula has been obtained is required (by analogy to the water species case ? by specific experimental trials ? from external works and then add a reference ?)

Response: Thanks for the comment. We appreciate the suggestion for additional clarification regarding the derivation of the vertical correlation model for background error. This formula was derived through a combination of theoretical considerations (Bergman, 1979) and experimental tuning, with particular reference to the methodology used for humidity in the CMA-GFS 4D-Var system.

In the revised manuscript, we have added the following clarification:

"...The vertical correlation model of the background error is derived through a combination of theoretical considerations (Bergman, 1979) and experimental tuning, with particular reference to the methodology used for humidity in the CMA-GFS 4D-Var system. It is expressed as..."

Section 4.

lines 315-316: link the two sentences together and remove the start with "And ..." (just use "... and ...")

Response: Thanks for pointing this out. Following the recommendation, we have revised the sentence to remove the "And ..." at the beginning. The revised sentence now reads as follows:

"... The first 9 days were used as the spin-up time to establish a realistic BC distribution ... "

line 322: again, what does "PM" stand for ?

Response: Thanks for pointing this out. "PM" stands for "particulate matter". For the definition of $PM_{2.5}$, please refer to our previous response. Additionally, PM_{10} , also known as inhalable particulate

matter, refers to the particulate matter with an aerodynamic diameter of 10 micrometers or less. We have added an explanation of PM_{10} in the revised manuscript as follows:

"...and particulates (OC, BC, $PM_{2.5}$ and PM_{10}), where PM_{10} refers to the inhalable particulate matter with an aerodynamic diameter of 10 micrometers or less. These data were processed into grid-point emission data applicable to the CUACE model through the EMIPS emission source processing system..."

lines 364-365: replace "filed" by "*field*". Note that the same typo appears several times later in the text, so the simplest is to make a systematic search and replace.

Response: We apologize for the misspelling of the word "field", and this has been revised throughout the manuscript as suggested.

lines 390-391: The sentence "This phenomenon indicates that ..." definitely <u>requires a</u> <u>complete reformulation</u>. It is currently simply not understandable ! What do you want to explain ?

Response: We sincerely apologize for the lack of clarity in this sentence. What we want to explain is that compared with variables such as potential temperature and specific humidity in the CMA-GFS-AERO model, the tangent linear approximation for BC is quite effective, making it well-suited for constructing a 4D-Var system. We have revised it in the manuscript as follows:

"...<u>This phenomenon indicates that, in comparison to variables such as potential temperature and</u> specific humidity in the CMA-GFS-AERO model, the tangent linear approximation for BC is quite effective, making it well-suited for constructing a 4D-Var system."

line 395: the caption of figure 4 mentions "simple physics" => what does "simple physics" refer to ? Do the authors refer to specific simplified physics involved in the 4D-Var models (TLM, ADM) ? If this is the case, then more explanations should be provided earlier in the text, for instance in section 3.2 (and also check my general comment above)

Response: We sincerely apologize for the confusion caused by the phrase "simplified physics" in the caption of Figure 4. This was an incorrect expression, and we have removed it in the revised manuscript. Additionally, we would like to clarify that in constructing the tangent linear and adjoint of AERO-BC, the physical processes in AERO-BC were not simplified. For further details, please refer to our response to the general comment above.

line 406: Remind explicitly that the time step is 300s as it's of interest here for the reader to promptly be able to convert time steps into a forecast time length

Response: We sincerely appreciate the reviewer's valuable suggestion. In the revised manuscript, we have explicitly clarified that the time step is 900 s (300 s in the outer loop and 900 s in the inner loop). The revised text is as follows:

"...Following Eq. (16), we conducted five experiments with the integration time equal to 1, 6, 12, 24, and 36 steps with the time step of 900 s..."

line 460 and Figure 6. Is the propagation of the BC increment by the wind the generally dominant effect ? Is this what the authors quite generally have been observing in their results ? Or conversely does this statement only apply to the very simplified context of the

single-point observation experiments ? (this is what I would derive from the results later in section 4.4 when the effect of full observations strongly-coupled assimilation is shown). Nevertheless, an additional sentence here could be clarifying, in order to avoid misinterpretation with other results shown later on.

Response: Thanks for the insightful comment. Regarding line 460 and Figure 6 in the original manuscript, in the context of the single-point observation experiments, the propagation of BC increments is indeed primarily dominated by advection due to the limited observational constraint. When more comprehensive observations are assimilated, advection remains a key factor in BC increment propagation. However, its dominance is less pronounced compared to the single-point experiment, as other processes, such as vertical mixing and deposition, also contribute to BC distribution adjustments.

To clarify this, we have added the following sentence at the end of the discussion of Figure 6:

"...<u>In this idealized single-point observation experiment, the propagation of BC increments is</u> primarily dominated by advection due to the limited observational constraint. When more comprehensive observations are assimilated, advection remains a key factor, but its dominance is less pronounced as other processes also influence the adjustment of BC distributions (see Section <u>5.3</u>)."

line 470. "Figure 7 depicts ... *at the initial time of* the assimilation window ..." Response: Thanks for pointing this out. We have revised it as suggested.

lines 516-519: should be totally re-written as they are not clear at present. A proposal : "These results suggest that the assimilation of meteorological observations has a small impact on the BC analysis increments. Furthermore, weakly and strongly coupled assimilation seem to lead to similar BC analysis increments."

Response: We sincerely appreciate the reviewer's suggestion. In the revised manuscript, we have completely rewritten this section (now presented as Section 5.3 in the revised manuscript), and the content in lines 516-519 has been updated accordingly. The new version provides a clearer and more precise explanation.

line 533: "there are certain degrees of analysis increments ..." => this wording is very obscure, *please reformulate*.

Response: Thanks for the suggestion. We have revised the sentence to improve clarity. The updated wording is as follows:

"When only BC surface observations are assimilated (DA_BC), analysis increments of temperature (Fig. 10a), pressure (Fig. 10d), east-west component of horizontal wind (Fig. 10g), and relative humidity (Fig. 10j) are present in North China and Eastern China..."

lines 552-558: only to mention that this is the part of section 4.4 that explicitly describes what seem to be interesting physics-related results of assimilating BC, already in these preliminary experiments. This is the part where more physical interpretation of these results is expected, on the feedback mechanisms in strongly-coupled assimilation and about the warm bias on the temperature analysis increment. (Refer to my general comments)

Response: Thanks for the insightful comment. As we mentioned in our response to the general

comments, we have carefully considered the reviewer's concerns regarding Section 4.4 of the original manuscript and have thoroughly rewritten it in Section 5.3 of the revised manuscript.

Regarding the feedback mechanisms in strongly coupled assimilation, we have also added discussions in Section 3.3.5 of the revised manuscript. Specifically, CMA-GFS-AERO 4D-Var incorporates cross-covariances between BC and atmospheric variables through the background error covariance matrix. The adjoint model then propagates the impact of BC observations onto atmospheric variables, leading to corresponding analysis increments.

Additionally, regarding the warm bias on the temperature analysis increment, Section 5.2 presents single-observation experiments that systematically analyze the generation of temperature analysis increments at different observation times when BC observations are assimilated at the initial, middle, and end of the assimilation time window. These results align with the theoretical framework described in Section 3.3.5.

We appreciate the reviewer's valuable suggestions, which have helped improve the clarity and depth of our analysis.

Section 4.5.

Only to mention that I am supportive of this section explaining the computational figures of the enhanced 4D-Var system.

Response: We sincerely thank the reviewer for the supportive comment on this section. This section aims to provide a clear description of the computational performance of the CMA-GFS-AERO 4D-Var system, highlighting its high efficiency and good scalability. We are glad that the reviewer finds this explanation satisfactory.

Section 5.

line 614: "surface BC observations"

Response: This has been revised as suggested.

line 615: "through batch tests"

Response: This has been revised as suggested.

Acknowledgments.

line 633: "The development of <u>the</u> CMA-GFS-AERO 4D-Var system is a systematic project" => what do you mean by "systematic project" ?

Response: Thanks for pointing this out. By "systematic project", we mean that the development of the CMA-GFS-AERO 4D-Var system involved the collaboration of many colleagues from various disciplines and teams. To make this clearer, we have revised the sentence as follows:

"...<u>The development of the CMA-GFS-AERO 4D-Var system is a collaborative effort involving</u> contributions from many colleagues. We sincerely thank the entire team for their cooperation..."

References

Bergman, K. H.: Multivariate Analysis of Temperatures and Winds Using Optimum Interpolation, Mon. Wea. Rev., 107, 1423-1444, https://doi.org/10.1175/1520-0493(1979)107<1423:MAOTAW>2.0.CO;2, 1979.