Response

Dear editor,

We appreciate the time and effort you and the anonymous reviewers have dedicated to reviewing our work. We have carefully considered each point raised and have made substantial revisions to the manuscript accordingly. A detailed response to the comments is provided below.

RC#1

They developed a new vertical lapse-rate model to enhance the performance of VMF1/VMF3-based ZHD and ZWD interpolation. Generally speaking, the manuscript is well-written, but some minor revisions may improve the quality of the paper.

[1] *Since the tropospheric delays should be mitigated in many space observations, "in GNSS applications" can be removed from the article title, as the model's application scope can be broader.*

Response: We appreciate this insightful comment and agree that the model's application extends beyond GNSS. As tropospheric delay affects various space-based geodetic techniques, including GNSS, DORIS, VLBI, and InSAR, we have revised the title to reflect this broader applicability. The new title is "A New Vertical Reduction Model for Enhancing the Interpolation Accuracy of VMF1/VMF3 Tropospheric Delay Products"

[2] The authors provided a detailed introduction to the accuracy and applications of VMF1/VMF3 ZTD, but does not seem to address the urgency of improving the accuracy in the introduction part. Thus, the authors are encouraged to revise the introduction part.

Response: This is a good suggestion. We have revised the manuscript, first, we emphasized the importance of ZHD modeling in GNSS data processing: "Given the distinct dynamic characteristics of the ZHD and ZWD, GNSS data processing typically involves correcting for the ZHD while treating the ZWD as an unknown parameter. As hydrostatic and wet mapping functions differ, the error in ZHD cannot be absorbed in the estimated ZWD, which in turn affects the accuracy of ZTD and station height estimations. (Boehm et al., 2006; Tregoning and Herring, 2006; Kouba, 2009). Furthermore, an accurate ZHD is necessary for converting ZTD to precipitable water vapor (PWV) in GNSS meteorology (Bevis et al., 1992; Wang et al., 2017; Zhu et al., 2024), a 1 cm error in ZHD corresponds to a 1.5 mm error in the retrieved PWV."

Second, we presented the urgency of improving the accuracy of VMF1/VMF3 products: "While ZTD derived from VMF1/VMF3 products generally exhibit superior accuracy compared to those derived from empirical tropospheric models, discrepancies in ZHD and ZWD have been documented in certain studies. Specifically, the RMSE of ZHD estimated by grid-wise VMF1/VMF3 using the recommended interpolation method can reach 5 cm when compared with reference ZHD values obtained from radiosonde measurements in some regions. Similarly, the

RMSE of ZWD can also attain substantial magnitudes. These findings underscore the potential limitations of currently widespread methods under specific geographical or atmospheric conditions (Sun et al., 2021b, a)."

[3] Is it necessary to introduce Equation (7) if it is not used in the modeling and evaluation?

Response: We re-visited the manuscript, and this equation has been deleted in the new manuscript.

[4] "Since the horizontal resolution of the reference data coincides with the VMF1 and VMF3 products, temporal interpolation and horizontal geospatial interpolation were not needed to carry out for the model evaluation." Should be corrected to "Since the temporal and horizontal resolution of the reference data coincides with the VMF1 and VMF3 products, temporal interpolation and horizontal geospatial interpolation were not needed to carry out."

Response: Amended.

[5] The time resolution, accuracy, data availability (percentage of usable data) and quality control of the GNSS ZTD product should be introduced detailly.

Response: This is a good suggestion. The manuscript has been revised: "A rigorous quality control procedure was implemented to ensure the quality of the reference ZTDs. To mitigate the impact of known midnight discontinuities present in the IGS ZTD time series, only odd-numbered UTC epochs (i.e., 1, 3, ..., 23) were retained, thus avoiding potential offsets in the reference data. Initially, the IGS ZTD time series, originally at 5-minute intervals, was resampled to a 2-hour interval. Subsequently, epochs with a standard deviation exceeding 4 mm, as indicated within the IGS ZTD products, were excluded. Finally, following these two filtering stages, stations with fewer than 5000 remaining ZTD epochs were removed from consideration to capture the seasonal variation of the tropospheric delay."

[6] What does γ in equation (16) mean, please specify.

Response: Amended. γ is the ZWD decay parameter defined by Dousa. See:

DOUSA J, ELIAS M. An improved model for calculating tropospheric wet delay[J]. Geophysical Research Letters, 2014, 41(12): 4389-4397.

RC#2

The paper addresses a critical aspect of GNSS applications. The proposed new model for ZTD was significantly better than the ones deduced by traditional methods using ERA5 and IGS-ZTD as reference. However, I'm curious if the new model could hold significant promise for enhancing GNSS positioning accuracy. With additional details on methodology, expanded validation, the work could set a strong foundation for practical implementation. I recommend the paper for publication with minor revisions.

Detail Comments:

1. Due to this study focuses on GNSS applications, I suggest to add the experiment for the application of the new model in the GNSS navigation. The new model could be also assessed more comprehensively, which could further highlight the significance of the new model enhancing the GNSS navigation.

Response: This is a very good suggestion. We acknowledge the reviewer's valuable suggestion regarding the application of the proposed model within GNSS navigation. We concur that such an evaluation is crucial for demonstrating the model's practical utility.

Tropospheric delay constitutes a significant error source in GNSS code and phase observations, directly impacting positioning accuracy. As described by Equation (5) in the manuscript, the hydrostatic and wet components of the tropospheric delay are modeled using distinct mapping functions. At low elevation angles, the divergence between these mapping functions becomes pronounced, leading to hydrostatic/wet mapping separation errors that affect GNSS-derived station heights and ZTD estimations.

For precise GNSS positioning, ZWD is usually estimated as a time-varying unknown parameter, while ZHD is corrected directly. Studies have demonstrated that, with a 5-degree elevation cutoff, hydrostatic/wet mapping separation errors can induce height errors approximately one-tenth the magnitude of the ZHD error. Consequently, achieving 1 mm height accuracy necessitates ZHD accuracy on the order of 10 mm. In standard Single Point Positioning (SPP), where ZWD is often corrected directly, inaccuracies in ZWD also propagate into the estimated coordinates.

Furthermore, accurate ZHD modeling is essential for GNSS meteorology applications. As previously discussed, ZHD influences the accuracy of ZTD estimation. Moreover, ZHD directly affects the retrieval of Precipitable Water Vapor (PWV) from ZWD. Assuming accurate ZTD estimation, a 1 cm error in ZHD translates to a 1.5 mm error in the retrieved PWV.

Based on the above discussion, we conclude that existing research has sufficiently demonstrated the impact of improved tropospheric modeling accuracy on GNSS applications. This research has validated the improvement achieved by the proposed model in tropospheric delay prediction, with a magnitude significant enough to exert a substantial influence on GNSS positioning and GNSS meteorology. The new model is very easy to implement, however, a sophisticated precise point positioning (PPP) software package, developed in C++ and incorporating the proposed model, is currently under development. Potential issues in other modules may temporarily hinder the comprehensive evaluation of the model's performance. We intend to thoroughly investigate the reviewer's suggestion following the release of a stable software version. In the interim, a MATLAB implementation of the newly developed model has been made publicly accessible for testing and evaluation purposes.

2. The Introduction has reviewed the detailed development of "VMF1/VMF3 ZTD". However, it has no information about why we need "A New Reduction Model for Enhancing the Interpolation Accuracy of VMF1/VMF3 Tropospheric Products in GNSS applications". If the officially recommended ZTD interpolation method is enough accurate, a new vertical reduction model may be not making much sense.

Response: This is a very good suggestion. We have revised the manuscript to emphasized the importance of developing a new lapse rate model for the grid-wise VNMF1/VMF3 model. “While ZTD derived from VMF1/VMF3 products generally exhibit superior accuracy compared to those derived from empirical tropospheric models, discrepancies in ZHD and ZWD have been documented in certain studies. Specifically, the RMSE of ZHD estimated by grid-wise VMF1/VMF3 using the recommended interpolation method can reach 5 cm when compared with reference ZHD values obtained from radiosonde measurements in some regions. Similarly, the RMSE of ZWD can also attain substantial magnitudes. These findings underscore the potential limitations of currently widespread methods under specific geographical or atmospheric conditions (Sun et al., 2021b, a).”.

3. There are many reanalysis data. Why do you choose ERA5 for the development of the new ZTD vertical reduction model?

Response: This is a very good suggestion. Many countries and organizations are dedicated to developing high-quality reanalysis datasets. Examples include NCEP/NCAR and MERRA-2 from the United States, ERA5 from Europe, JRA-55 from Japan, and CRA40 from China, etc. Each of these datasets has its own strengths and characteristics. We chose to use ERA5 data in our research. While ERA5 is widely recognized for its excellent data quality, our primary reason for this choice is its consistency with our research subject: the VMF1/VMF3 grid products released by TU Wien. These products are also based on data from the European Centre for Medium-Range Weather Forecasts (ECMWF), the same source as ERA5. Using ERA5 ensures better consistency between our newly developed model and the VMF1/VMF3 grid products. Although we haven't yet compared the modeling results using other datasets, we plan to explore these options in future research."

4. For the GNSS ZTD data do you have done quality control? There may data gaps or jumps in the data, which strategy do you use for them?

Response: This is a very good question. IGS ZTD data have some gaps/jumps, which may affect the evaluation results. We apologize for missing the quality control information in the original manuscript, and here is a revised one: “A rigorous quality control procedure was implemented to ensure the quality of the reference ZTDs. To mitigate the impact of known midnight discontinuities present in the IGS ZTD time series, only odd-numbered UTC epochs (i.e., 1, 3, ..., 23) were retained, thus avoiding potential offsets in the reference data. Initially, the IGS ZTD time series, originally at

5-minute intervals, was resampled to a 2-hour interval. Subsequently, epochs with a standard deviation exceeding 4 mm, as indicated within the IGS ZTD products, were excluded. Finally, following these two filtering stages, stations with fewer than 5000 remaining ZTD epochs were removed from consideration to capture the seasonal variation of the tropospheric delay.”.