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Dear editor and reviewer,

Re:” **Three-Dimensional Analytical Solution of Self-potential from Regularly Polarized Bodies in Layered Seafloor Model**”

Thank you for taking time to review our paper in such detail. We appreciate the chance to revise this paper.

Through your review, we have identified several deficiencies in the manuscript. In response to the reviewers' comments, we have made extensive revisions. We have supplemented the content of the formula derivations, adding more detailed steps to enhance understanding. And we revised all the figures, incorporating new three-dimensional potential distribution plots to more effectively present the analytical solution results. We also added substantial explanatory content to improve the article's readability. In Section 4, we provided a more detailed description of the experimental process.

In this response letter, your comments are highlighted in blue, and our responses are in black.

The authors introduced a 3D analytical solution for forwarding modeling of self-potential from sulfide deposits. Although the topic is interesting and the overall flow of the paper, from the formulation to comparison with 2D analytical solution and experimental results, is great, I can hardly say the paper is well written in general and there seems to be a lot of room for improvement of the manuscript.

The motivation of the current model development is not compelling, especially given that (1) the system has not been proven to be applicable to the field (even though the authors attempted to validate the model with laboratory experiments, it does not necessarily mean it can be used in the field) and (2) complicated situations are likely to be more easily dealt with numerical tools/methods and there seem to be some published already.

Formulation is poorly explained especially given that deriving analytical solution for the system should be the essential part for a model development paper. I found it difficult to follow how the equations are derived. There can be several reasons for this, including (1) the papers cited (Li et al., 2005, He, 2012) for seemingly essential equations are not publicly available and not explained well, (2) the authors do not seem to use mathematical symbols in an organized way, including seemingly interchangeable and thus confusing use of uppercase/lowercase and/or Greek/English letters and/or vector/scalar expressions, and (3) figure to help the reader to understand formulation is lacking and/or not referenced.

Especially, please consider making clearer explanations on (1) what is proved and how this is done in Section 2.1, (2) the mirror image method in general potentially with a specific diagram/schematic in Section 2.2 and (3) derivation of one of formulation from Table 1 in Section 2.3.

Thanks for your suggestions.

Laboratory settings allow for controlled conditions where variables can be systematically tested and understood, providing a strong foundation for subsequent field tests. We plan to extend our research to field applications in future studies, following this step-wise approach. This methodology aligns with established scientific practices where theoretical models are first validated in controlled environments before being tested in more complex, real-world scenarios. Studies have shown that theoretical models validated in laboratory settings can be successfully adapted for field applications. For instance, Ishido and Mizutani (1981) validated SP methods using laboratory simulations before successful field deployment (Ishido, T., & Mizutani, H. (1981). Experimental and theoretical basis of electrokinetic phenomena in rock-water systems and its applications to geophysics. Journal of Geophysical Research: Solid Earth, 86(B3), 1763-1775). Andre et al. developed a sandbox experiment to monitor the evolution of a self-potential anomaly associated with redox processes. These precedents provide a strong basis for our approach (Revil, A., Su, Z., Zhu, Z., & Maineult, A. (2023). Self-Potential as a Tool to Monitor Redox Reactions at an Ore Body: A Sandbox Experiment. *Minerals*, *13*(6), 716.).

While numerical methods are indeed powerful and versatile, they also have limitations. Numerical solutions often involve approximations and are influenced by boundary conditions and source configurations. For instance, Stoll and Bryan (1970) highlighted the complexities in solving stiffness matrices and the limitations posed by artificial boundary conditions in numerical methods (Stoll, R. D., & Bryan, G. M. (1970). Wave attenuation in saturated sediments. The Journal of the Acoustical Society of America, 47(5B), 1440-1447). Our model, based on the mirror image method, offers a strict analytical solution that avoids these numerical approximations, providing exact results under defined conditions.

Therefore, analytical solutions provide exact results, which are critical for validating numerical models and ensuring their accuracy. They can serve as benchmarks for numerical simulations, enabling us to identify and correct deviations in numerical approaches. In scenarios where the analytical model is applicable, it offers faster computations compared to iterative numerical methods.

For the derivation process, we have supplemented with a more comprehensive derivation and final formula. We have corrected the citations for the referenced papers. Equation (7) (equation (10) in revised manuscript), which represents the formula for the scalar potential caused by a constant electric dipole, can be supported by several academic sources (For example, here's a detailed derivation in UTA *https://web2.ph.utexas.edu/~vadim/Classes/2017f/dipole.pdf#:~:text=URL%3A%20https%3A%2F%2Fweb2.ph.utexas.edu%2F~vadim%2FClasses%2F2017f%2Fdipole.pdf%0AVisible%3A%200%25%20*). This should provide a more solid theoretical foundation and address any concerns about the validity of the equation. We also recognize that providing a reference to an unpublished document for an important formula is inappropriate. Considering that this is a classical formula for an electric dipole in electromagnetic physics, we believe it is more appropriate not to include a reference here.

As indicated by the title of Section 2.1, this section demonstrates that the self-potential generated by a regularly polarized sphere can be equivalently represented by the self-potential generated by an electric dipole. Through this demonstration, we can directly use the self-potential formula for an equivalent electric dipole when calculating the self-potential of regularly polarized bodies.

We have made substantial revisions and additions to the derivation process and descriptions in order to clarify our derivation process. Here, we provide a supplementary explanation of the entire derivation process. When there are two media, a source point will generate another image point on the opposite side of the interface between the two media. We derived the potential of this image point using boundary conditions. In the case of three layers of media, the source point generates image points on the opposite sides of both interfaces, and these image points will, in turn, generate new image points in the other media. For example, a source point in a seawater-air combination generates an image point (image point 1) in the air, which subsequently generates another image point (image point 1-1) in the seafloor medium. Similarly, a source point in a seawater-seafloor combination generates an image point (image point 2) in the seafloor, and this image point generates another image point (image point 2-1) in the air. This process continues iteratively. During the derivation, we found that the self-potential expressions for different image points can be categorized into four types. These four types of dipoles are distinguished by different dipole moments (as shown in Table 1 of the main text). These image points themselves do not have physical significance but are mathematically differentiated.

We acknowledge the confusion caused by the inconsistent use of mathematical symbols. We have revised the manuscript to use mathematical symbols in an organized and consistent manner, distinguishing clearly between uppercase/lowercase, Greek/English letters, and vector/scalar expressions. This should enhance readability and reduce confusion.

Specific comments

L9. “It is significant to fast and precise forward modeling and inversion for SMS explorations.”. I think English grammar here is not correct.

We have revised it to: “This approach is significant for achieving fast and precise forward modeling and inversion in SMS explorations."

L22. What is “AUV”?

AUV is an abbreviation for autonomous underwater vehicle. AUV is equipped with advanced navigation and positioning systems, allowing for precise localization in complex seafloor environments. By integrating various sensing devices, such as natural potential instruments and magnetometers, they can conduct comprehensive surveys and acquire diverse sets of data.

We have recognized that using abbreviations here is inappropriate, and we have made corrections in the main text.

L34. What do you mean by “affection of the field source”?

Our description here is unclear. In the numerical forward modeling of the self-potential method, using the finite element method as an example:

$$Kx = f$$

where K is the stiffness matrix, x is the potential distribution, and f is the source term.

The source points (such as polarization bodies or charge distributions) indeed affect the overall numerical solution process, but this influence primarily manifests in the right-hand side term (source term) of the system equation, rather than directly altering the stiffness matrix. The stiffness matrix is mainly determined by the physical properties and structure of the medium. For an anomalous body with uneven surface polarization intensity, the right-hand side term (source term) will change due to this uncertainty. This is one of the limitations of numerical forward modeling.

We have revised this section in the main text.

L34. “which occur difficulty in solving the Poisson equation.” I think the authors should correct English.

We have revised it to: “Compared to numerical methods, analytical solutions are strict formulas that can overcome the difficulties in solving the Poisson equation.”

L58. What is the definition of H?

We apologize for the oversight; this should be the seawater depth (D) instead of H.

L79. rho_0 must be a typo of r_0?

"We apologize for our writing error; it should be r_0 at this point. We have corrected this in the main text.

L82. “So we get the potential distribution along the surface of a uniformly polarized sphere is equivalent to an electric dipole.” I do not understand what this means.

Through the derivation of the above formulas, we conclude that under the conditions of R=r and P_0=M, the self-potential distribution of a uniformly polarized spherical body is the same as that of a dipole. For clarity, we have made modifications in the main text:” Therefore, we can conclude that the potential distribution of a uniformly polarized spherical body is the same as that of a dipole.”

L92. Is the use of permittivity correct here? In other words, do permittivity and conductivity have the same units?

Use of σ_1 in the Ocean:

In the ocean, the medium is conductive. The conductivity of seawater is typically denoted by σ_1. This is appropriate because the electric field behavior in a conductive medium is influenced by its conductivity. The formula (11) involving σ_1 is used to describe the scalar potential and electric field distribution in seawater, which aligns with the medium's conductive nature.

Use of ε_0 in the Air:

In the air, the medium is not conductive but rather dielectric. The permittivity of free space is denoted by ε_0.When the electric field extends into the air, the potential expression uses ε_0 because the permittivity influences the behavior of electric fields in a dielectric medium.

Given this context, the use of σ_1 for the ocean and ε_0 for the air is intentional and correct. It reflects the different physical properties of the two media — conductivity for seawater and permittivity for air.

And permittivity and conductivity do not have the same units. The unit of permittivity is farads per meter (F/m) and the unit of conductivity is siemens per meter (S/m). In our paper, distinguishing between these two physical quantities is to illustrate the uniqueness of solving the natural potential in a two-layer air-seawater model.

L137. What do you mean by “the dipole moment is 1 D.”?

In some papers, the electric dipole moment is expressed in Debye (D). We have recognized that using D here is inappropriate, and we have revised it to the more standard unit of Coulomb meters (C·m).

L138. 10-10 must mean 10^-10. What are units and adjustment term exactly?

Your understanding is correct; it should be 10^{-10} here. This oversight was due to a formatting error in the LaTeX typesetting of our paper. We will provide further clarification on this statement.

In the formulas for the three-layer model, the potential calculation is an iterative process. We set the iteration condition such that the iteration stops when the difference between the potential values of successive calculations is less than 10^{-10} millivolts. In practice, 10^{-10} is a very small value that adequately ensures both computational efficiency and accuracy.

L140. Do “exceptions” mean negative/positive peaks?

Your understanding is correct; this refers to a horizontal electric dipole producing potential signals of equal magnitude but opposite sign. We have revised the expression in the main text: The magnitude of the absolute values is equal.

L142. Why tilted dipole is related to any depth?

Our previous expression was inaccurate. An inclined polarized electric dipole generates self-potential signals of different absolute values on either side of the dipole. The self-potential signals increase along the polarization angle of the dipole.

L147. The authors should describe exactly what redox reaction is occurring.

In the main text, we have supplemented the specific description of the redox reaction. The prevailing explanation for the origin of the self-potential in seafloor hydrothermal sulfide deposits is the geobattery model formed by the redox reactions of sulfide minerals. We simulated this redox reaction using an iron-copper composite sphere. Here, we have added a graph showing the variation of the self-potential signals of 10 electrodes placed directly above the inclined sphere. In this graph, it can be observed that the self-potential polarities on either side of the sphere's polarization direction are opposite, with similar trends in their variation. We selected the self-potential results at 5000 seconds to plot the self-potential slices in Figure 8 of the main text. At this time, the self-potential is relatively stable.

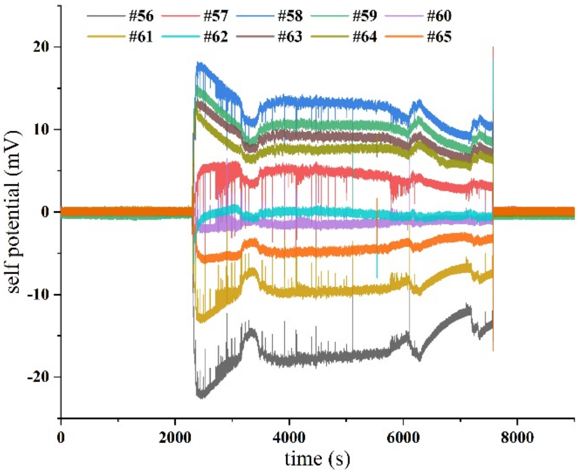

Figure 1. Temporal self-potential curves for electrodes located above the sphere.

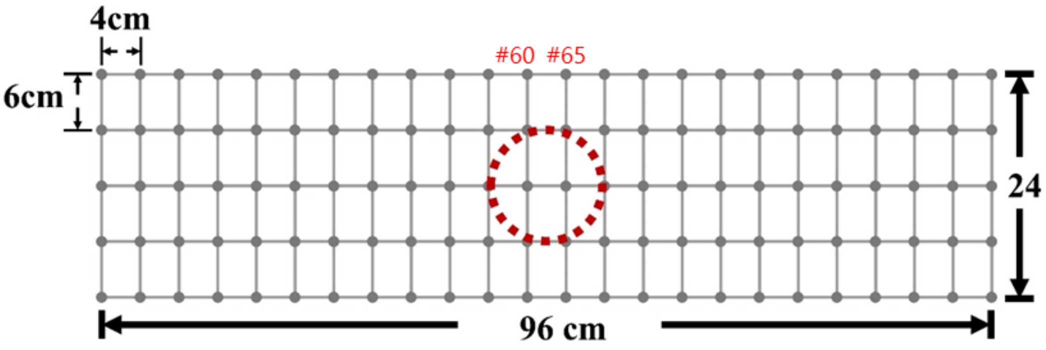

Figure 2. Sketch of the experimental observation electrodes, with the red dashed line

indicating the projection position of the sphere. Electrodes 60 and 65 are marked.