

1 Localization method

In order to ‘cut-off’ the spurious correlation in the background covariance matrix \mathbf{P}^f , a correlation matrix \mathbf{L} filled with local supports is introduced. Local support is a term meaning that the function is only non-zero in a local region and is zero elsewhere. The local support function is set as:

$$5 \quad \mathbf{S}_{i,j} = \frac{\mathbf{D}_{i,j}}{L_{thres}} \quad (1)$$

$$\mathbf{L}_{i,j} = \begin{cases} 1 - \frac{5}{3}\mathbf{S}_{i,j}^2 + \frac{5}{8}\mathbf{S}_{i,j}^3 + \frac{1}{2}\mathbf{S}_{i,j}^4 - \frac{1}{4}\mathbf{S}_{i,j}^5, & \mathbf{S}_{i,j} < 1 \\ -\frac{2}{3}\mathbf{S}_{i,j}^{-1} + 4 - 5\mathbf{S}_{i,j} + \frac{5}{3}\mathbf{S}_{i,j}^2 + \frac{5}{8}\mathbf{S}_{i,j}^3 - \frac{1}{2}\mathbf{S}_{i,j}^4 + \frac{1}{12}\mathbf{S}_{i,j}^5, & 1 \leq \mathbf{S}_{i,j} < 2 \\ 0, & \mathbf{S}_{i,j} \geq 2 \end{cases} \quad (2)$$

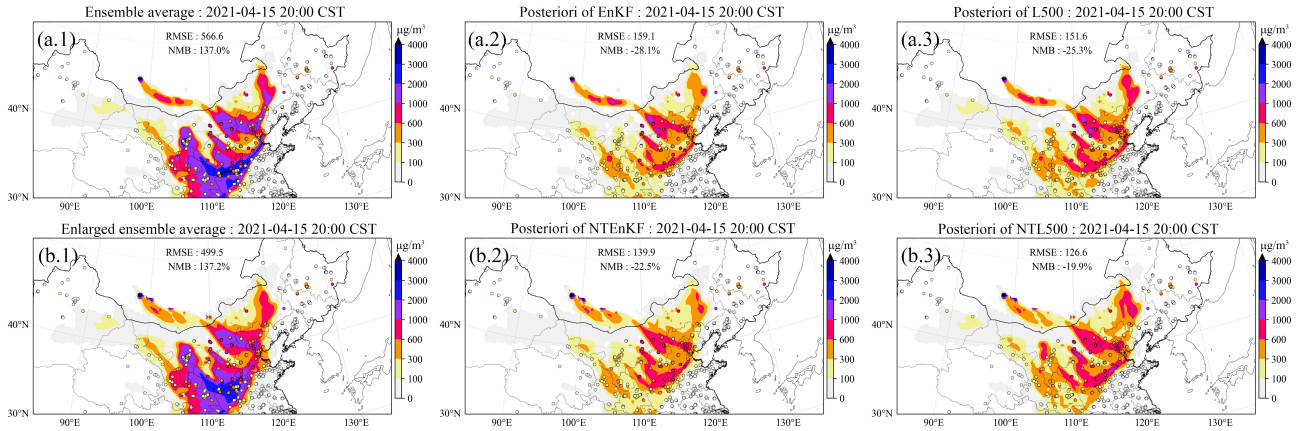


Figure S1. Spatial distribution of ground-based BR-PM₁₀ observations (scatter) and simulated dust plume (SDP) on surface from central time ensemble model mean **(a.1)**, the posterior SDP updated by EnKF **(a.2)**, the posterior SDP updated by EnKF with localization distance of 500 km **(a.3)**, central and neighboring time ensemble model mean **(b.1)**, the posterior SDP updated by NTEKF **(b.2)**, the posterior SDP updated by NTEKF with localization distance of 500 km **(b.3)** at 20:00, 15th April 2021 (CST). BR-PM₁₀:baseline-removed PM₁₀

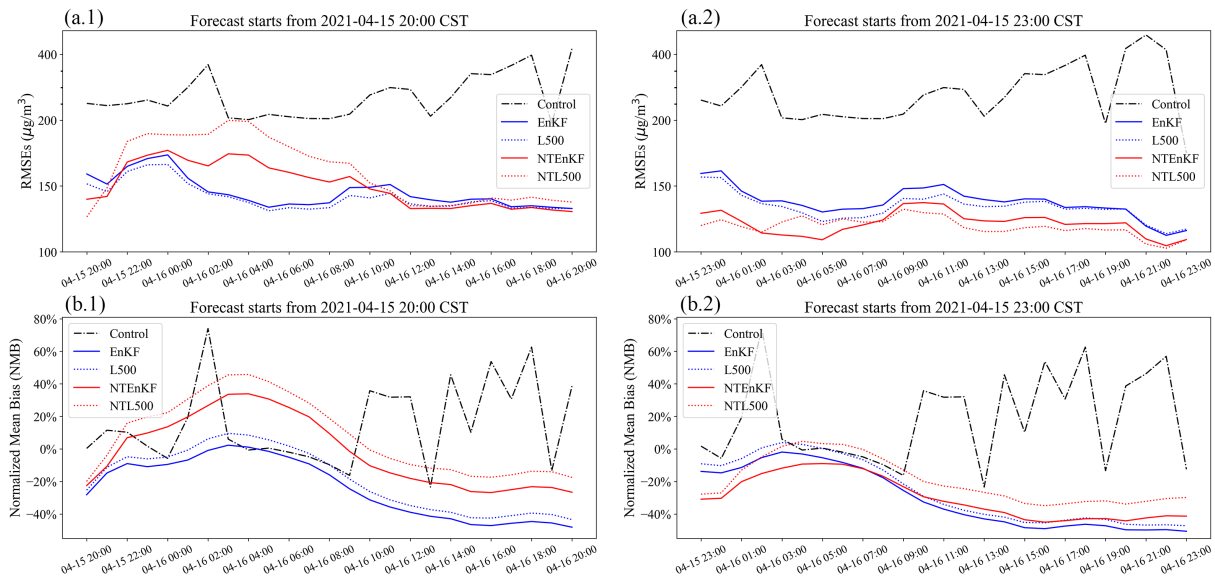


Figure S2. Variation of 24 hours forecast RMSEs starting from 20:00 (a.1), 23:00 (a.2) and normalized mean bias (NMB) starting from 20:00 (b.1), 23:00 (b.2) on 15th April 2021.