Response to the reviewer 2

November 28, 2023

Summary and High Level Discussion

The paper explores different stabilization methods for level-set equations and the impact of reinitialization on the accuracy of the solution.

The methods are demonstrated on an idealized geometry (union of a rectangle and a semidisk) to mimic a fjord with a semi-circular ice front. The front velocity is prescribed. The authors nicely present how the different stabilization approaches and the frequency of the reinitialization affect the accuracy of the position of the level set.

The paper addresses a very important topic in ice-sheet modeling and it is easy to read. However, I have major concerns which prevent me from recommending the paper for publication in its present form.

- My main concern is that despite the title and the presentation of the work, there is little about ice front migration in this manuscript. In fact, the geometry and the prescribed velocity are too simplified to be representative of an ice front migration problem. In addition to the very simplified description of the fjord, the prescribed front velocity is aligned with the fjord axis, which is at odds with the fact that the calving component of the front velocity is typically assumed to be orthogonal to the ice front.
  Response: Thank you for your insightful review of our manuscript. We appreciate your concerns about the limited representation of ice front migration and the simplicity of the chosen geometry and prescribed velocity. It is important to clarify that this paper is not specifically about calving; rather, its focus is on the treatment of moving boundaries in ice sheet modeling. The deliberate design of our control experiments aims to isolate errors introduced by the numerical treatment of stabilization and reinitialization of the levelset. As demonstrated in the manuscript, these aspects can significantly impact ice front migration if not carefully chosen. We would like to emphasize that the consideration of calving comes after the numerical method is well-tested, which is the step we are taking here.

  The standalone advection level-set equations have been extensively studied in the literature, and this paper adds little to what is already available. On the contrary, I would have found the paper very valuable if the authors

targeted a more realistic ice sheet problem as well, where the level-set velocity was computed using ice flow equations (e.g., the Shallow shelf Approximation) for the ice velocity and at least one of the calving laws typically used in the literature.

Response: To the best of our knowledge, research on level-set stabilization and reinitialization in glaciology is scarce, with existing best practices being largely domain-dependent. Notably, level-set equations in other fields primarily address multiphase problems, which substantially differ from ice flow problems. We would greatly appreciate if the reviewer could provide references if we missed important studies. Additionally, considering the ongoing inter-comparison project, CalvingMIP, which focuses on calving and incorporates more realistic ice front geometry, and has already provided an overview over the calving-front implementations in ice-sheet models. Indeed, only two models were found to be currently using the level-set methods, and those are exactly the two presented in this paper, i.e. ISSM and Úa. As part of the CalvingMIP project, the level-set method has now been implemented in the fEthish ice sheet model.This underscores a discernible interest within the ice-sheet modeling community to gain deeper insights into the implementation of calving in models utilizing this approach, and we are therefore addressing an identified need within the community.

- Another concern I have is that the authors do not explain what reinitialization method they are using, despite the fact that the effect of reinitialization is one of the main topics of the paper. When they introduce the reinitialization they reference two papers they co-authored but I could not find any detail there either. Further, plots in figure 2 show a loss of symmetry, which is likely due to the reinitialization procedure, but the authors do not offer any explanation of why that is happening. I worry that there might be an issue with the reinitialization procedure which would affect the results and possibly the paper conclusions.

  Response: We acknowledge your valuable observation regarding the need for a more detailed explanation of the reinitialization method, and we will provide a thorough description of the method in the revised manuscript. Furthermore, it is important to note that the observed loss of symmetry in Figure 2 can be attributed to a mesh effect stemming from the structured triangular mesh, where all the triangles align diagonally from the top-left to the bottom-right. This effect diminishes when larger reinitialization intervals are employed. We will run additional experiments on 200 m and 500 m resolutions, and address these concerns comprehensively in the revised manuscript to bolster the robustness and clarity of our findings.

- Finally, the forward and backward diffusion stabilization considered in this paper aims at keeping the level-set function close to the distance function, so that no reinitialization is needed. This is qualitatively confirmed by their results. However, the authors miss this point in the discussion of the

results. Also, the authors do not provide any reference for this stabilization method.

Response: Thank you for bringing this to our attention. The detailed derivations and formulations for the discussed aspects can be found in the Úa Compendium (`https://github.com/GHilmarG/UaSource/blob/master/UaCompendium.pdf`). In the revised manuscript, we will include the appropriate references to enhance the transparency and traceability of our work.