Response to reviewers of the manuscript

"*A 3D-Var Assimilation Scheme for Vertical Velocity with the CMA-MESO v5.0*"

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for Geoscientific Model Development

Response to Reviewer #1

Reviewer #1

This is my second review of this manuscript. The authors re-run the batch experiments with increased assimilation times, and the experimental design became more reasonable. I appreciate the authors' modifications to the manuscript. I recommend publication with some minor changes and residual explanations, which I have highlighted below.

Response: We gratefully thank the reviewer for your valuable feedback that we have used to improve the quality of our manuscript. Our response and changes/additions to the manuscript are given in the blue text below.

Scientific and major points:

1、The author's response indicates that the pseudo-vertical velocity observation values used for assimilation are generally small, usually less than 1 m/s, which closely matches the magnitude of the background field. Additionally, the frequency distribution of these observations suggests that numerous pseudo-vertical velocities are assimilated for each analysis time, numbering in the thousands. The case study (Figure 8 of the revised manuscript) demonstrates the positive impact of these observations on precipitation adjustments, could you please provide me with the value of the horizontal wind analysis increments resulting from assimilating such a large number of observations?

Response: Thank you for your valuable comments. Figure R1 presents the horizontal wind increments for the case study in our manuscript at the analysis time 0600 UTC on July 9, 2020. At the lower level of the model (Fig. R1 (a)), there are obvious horizontal wind (up to a maximum of about 10 m s^{-1}) and convergence (less than $-4\cdot10^{-4}$ s^{-1}) increments. At the same time, there are divergence or weak wind convergence increments in the middle level of the model (Fig. R1 (b)), and such a configuration of the horizontal wind field facilitates the model to produce certain vertical velocities in the middle and lower levels, thus further facilitating convection at these locations.

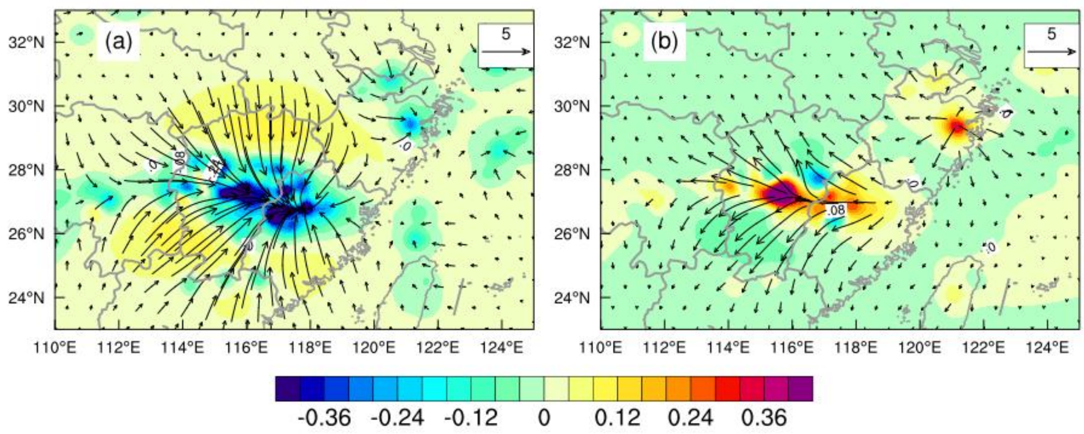

Figure R1. The analysis increments of horizontal wind (vector; unit: m s^{-1}) and horizontal wind divergence (color; unit: 10^{-4} s^{-1}) at the (a) 13th (~850 hPa) and (b) 23th (~500 hPa) levels of the model at 0600 UTC on July 9, 2020.

2、The authors emphasize the propensity for generating some false precipitation forecasts when pseudo-vertical velocity observations are assimilated, as evidenced by the outcomes of batch experiments. Is this caused by the use of a large horizontal influence radius?

Response: Yes, we agree. In our study, a relatively large scale factor of 400 km was used for several reasons. First, the derived w pseudo-observations are mostly small values, i.e., smaller than 1 m s^{-1}. Second, the observation operator for w, the adiabatic Richardson equation, combines the continuity equation, adiabatic equations, and the hydrostatic relation, making it more suitable for large-scale systems.

Line 215: "Forecasts with higher ETS (close to 1) and FSS (close to 1) and lower BIAS (closer to 1), demonstrate better forecast skills." The statement about the BIAS score is not rigorous. A lower BIAS score does not indicate better forecast skills.
Response: Thank you for your valuable comments, and we apologize for our less rigorous statement. The statement "Forecasts with higher ETS (close to 1) and FSS (close to 1) and lower BIAS (closer to 1), demonstrate better forecast skills." has been revised to "Forecasts with higher ETS (close to 1) and FSS (close to 1) and closer BIAS to 1, demonstrate better forecast skills." (Line 227 in the revised manuscript).

Minor points:

Line 90: Add "field" after "background".

Response: Thank you for your suggestion. It has been revised as suggested (Line 90 in the revised manuscript).

Line 151: Add "component" after "horizontal wind".

Response: Thank you for your suggestion. It has been added as suggested (Line 158 in the revised manuscript).

Line 152: From Figure 2, the convergence of u wind is not extending to ground, but

1000 hPa?

Response: Yes, our previous statement was not precise enough. According to the reviewer's suggestion, we have revised the sentence to "there is a convergence of u wind that extends to the 1000 hPa" (Line 159 in the revised manuscript).

Line 174: Change "vertical velocities" to "pseudo-*w* observations".

Response: Thank you for your suggestion. It has been revised as suggested (Line 185 in the revised manuscript).

Lines 176-179: Add the definition of variables Z and H. In addition, replace the character H with another symbol to distinguish it from the observation operator symbol in the manuscript.

Response: Thank you for your suggestion. The definition of symbol Z has been added in the revised manuscript (Line 189 in the revised manuscript). Following the reviewer's suggestion, we have revised the symbol H to H_{ei}, and the definition of symbol H_{ei} has been added in the revised manuscript (Line 189 in the revised manuscript).

Line 209: Add "observations" after "(VR)".

Response: Thank you for your suggestion. It has been added as suggested (Line 221 in the revised manuscript).

Line 212: In this section, the assessment for batch experiments is not limited to convective precipitation. It is recommended to delete the word "convective".

Response: Thank you for your suggestion. Deleted as suggested (Line 224 in the revised manuscript).

Line 235: Change "0600-1200 UTC" to "0600 to 1200 UTC".

Response: Thank you for your suggestion. It has been revised as suggested (Line 247 in the revised manuscript).

Line 281: "while a horizontal wind divergence".

Response: Thank you for your suggestion. It has been revised as suggested (Line 301 in the revised manuscript).

Line 282: Delete "effectively".

Response: Thank you for your suggestion. Deleted as suggested (Line 302 in the revised manuscript).

Line 285: Change "using" to "based on".

Response: Thank you for your suggestion. We have revised it as the reviewer suggested (Line 304 in the revised manuscript).

Line 288: “assimilation (DA-W) experiments” such a statement may lead to ambiguity. In fact, the control experiment in the article also involves observation assimilation. Here, it would be better to highlight that the DA-W experiment assimilates vertical velocity.

Response: Thank you for your suggestion. Following the reviewer’s suggestion, this unrigorous description has been revised to “Two sets of experiments were configured, including CTRL and DA-W experiments with different assimilation iterations. Both sets of experiments assimilated aircraft measurements, radiosondes, and other observations (for a comprehensive list, refer to Fig. 3 (b)) at 1-hour intervals during a 3-hour data assimilation period. In addition, the pseudo-w observations are also assimilated in the DA-W experiments.” (Lines 306–309 in the revised manuscript) to make it clearer.

Thanks again for the reviewer’s time and valuable comments.

Response to Reviewer #2

Reviewer #2

The authors have addressed most comments, and the quality of the manuscript improves a lot. I suggest publishing it after a minor revision.

The authors sincerely appreciate the valuable time the reviewer has dedicated to reviewing our manuscript. Below, we have outlined the detailed corrections made to our previous draft.

Minor revision:

Line 66: "assimilating w observations as a control variable" is not accurate, replace it with "assimilating w observations with control variable w".

Response: Thank you for your suggestion. We have revised it as you suggested (Line 65 in the revised manuscript).

Lines 70-71: "This operator ensures … the 3DVar cost function". This sentence is a little bit confusing to me. The observation operator links the model state variables (not control variable) to the observed variables. How about "This operator ensures adherence to physical constraints and links the w observations to other model state variables for the minimizing the 3D-Var cost function".

Response: Thank you for your suggestion, and we apologize for our less rigorous statement. We have revised the text as you suggested. Please see Lines 69–70 in the revised manuscript.

Lines 86-92: The Cv, d, R, x, xb and B are either vectors or matrixs, so they should be bold and non-italic. Please check it throughout the manuscript.

Response: Thank you for your careful consideration. Following the reviewer's suggestion, we have updated the symbol Cv, d, R, x, xb and B to be bold and non-italic throughout the revised manuscript.

Figure 2: As I mentioned in previous review, adding a constraint to the vertical propagation of w assimilation would have an impact. It would be better if some discussions are given here even though it is not implemented.

Response: Yes, we think this is a good suggestion. We have added the statement "It is worth noting that there are currently no constraints on the w assimilation impact propagation in the vertical direction. However, it is better to set limits to prevent excessive increments at higher model levels, thus leading to more realistic forecasts." in the revised manuscript (Lines 160–162) as you suggested.

Lines 184-185: It should be "a series of continuous 10-day runs … were".

Response: Thank you for your suggestion. It has been revised as suggested (Lines 196–197 in the revised manuscript).

Line 215: it is a little bit confusing "lower BIAS" here, better to be "closer BIAS to

1”.

Response: Thank you for your suggestion. It has been revised as suggested (Line 227 in the revised manuscript).

Lines 221-222: delete “compared to the CTRL experiment”.

Response: Thank you for your suggestion. Deleted as suggested (Line 233 in the revised manuscript).

Line 228: better to modify “has a better adjustment effect” to “has a positive impact” to be clearer.

Response: Thank you for your suggestion. It has been revised as suggested (Line 240 in the revised manuscript).

Lines 260-264: Could you discuss more on it. A deeper discussion could benefit more to this paper.

Response: Thank you for your suggestion. The statement has been revised to “In Fig. 8(a), line A-B represents the observed main precipitation belt. Fig. 9 shows the sections along the line A-B for the CTRL-4CY and DA-W-4CY experiments at 0700 UTC on July 9, 2020. The DA-W-4CY experiment effectively enhances the w values across the entire model layers. This enhancement is achieved by generating increments of wind convergence (less than -4×10^{-4} s^{-1}) at the lower (the 13th) level of the model, while inducing divergence or weak wind convergence increments at the middle (the 23th) level of the model (Fig. 10). Such a configuration of the horizontal wind field enables the model to generate specific vertical velocities in the middle and lower levels, leading to a decrease in water vapor below 850 hPa compared to the CTRL-4CY experiment (Fig. 9(c)). Simultaneously, positive increments of water vapor are observed in the middle and upper layers of the model. Consequently, upward movement enhances the vertical transport of water vapor, promoting water vapor saturation and facilitating cloud formation, ultimately resulting in rainfall.” (Lines 272–280 in the revised manuscript) to have a deeper discussion on it.

We would like to thank you again for taking the time to review our manuscript.

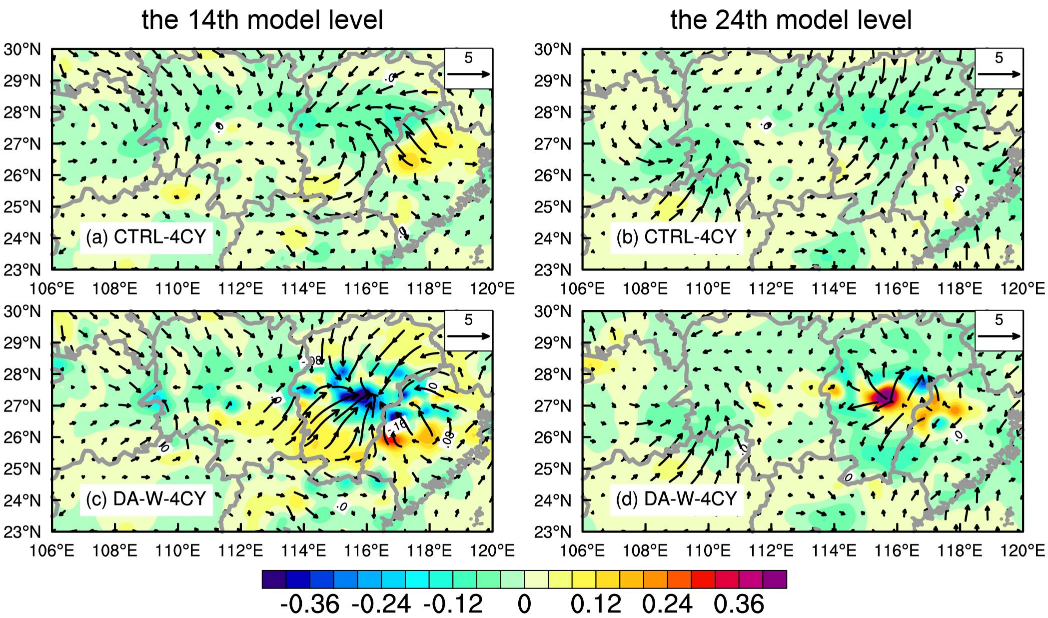

Figure 10 of the revised manuscript. The analysis increments of horizontal wind (vector; unit: m s^{-1}) and horizontal wind divergence (color; unit: 10^{-4} s^{-1}) of the (a, b) CTRL-4CY and (c, d) DA-W-4CY experiments at the (a, c) 13th (~850 hPa) and the (b, d) 23th (~500 hPa) model levels at 0600 UTC on July 9, 2020.

Response to Reviewer #3

Reviewer #3

In this study, a 3D-Var data assimilation scheme for vertical velocity, based on the adiabatic Richardson equation is developed within the high-resolution CMA-MESO model, enabling the update of horizontal winds and mass fields of the model's background. The manuscript is well written, the experiments are well design, and the conclusions are well supported by the results. However, some details should be clarified and the manuscript also should be further improved according to the comments given below. I think it is suitable for publication after moderate to major revisions.

We sincerely appreciate the time and effort invested by you in evaluating our manuscript. We have carefully considered your valuable suggestions and have made extensive revisions to our previous draft to address the issues you raised. The detailed corrections are outlined below for your review. However, before receiving your review comments, the manuscript had already undergone one round of revisions. Some of these modifications may address your concerns. Hence, in subsequent responses, we will refer to the initial revised manuscript as "the revised manuscript v1". We appreciate your thorough feedback and believe that these revisions have significantly improved the quality of our work. Thank you once again for your insightful suggestions.

Major comments:

1. The main verification method in this study is precipitation score, attention also needs to be paid to how precipitation is distributed at one or two typical moments.

Response: Thank you for your valuable comments. Based on the previous feedback from other reviewers, we have conducted a new set of batch experiments. Both the CTRL and DA-W experiments were initialized at 0000 UTC daily from July 1 to July 10, 2020, and run until 1200 UTC each day. The first 3 hours were considered as a "spin-up" period. In the CTRL experiments, observations from aircraft measurements, radiosondes, and other sources (for a comprehensive list, refer to Fig. 3 (b)) were assimilated from 0300 to 0600 UTC with a 1-hour assimilation interval (radial velocity observations are available at each analysis time, while other data sources are only available at 0300 and 0600 UTC). The CTRL-1CY experiment indicates assimilation at 0300 UTC only, while the CTRL-2CY experiment represents assimilation at 0300 and 0400 UTC, and so on (the number preceding the experiment name "CY" represents the assimilation iterations). The DA-W experiments are similar to the CTRL experiments, but include the assimilation of w pseudo-observations (w pseudo-observations are available at each analysis time).

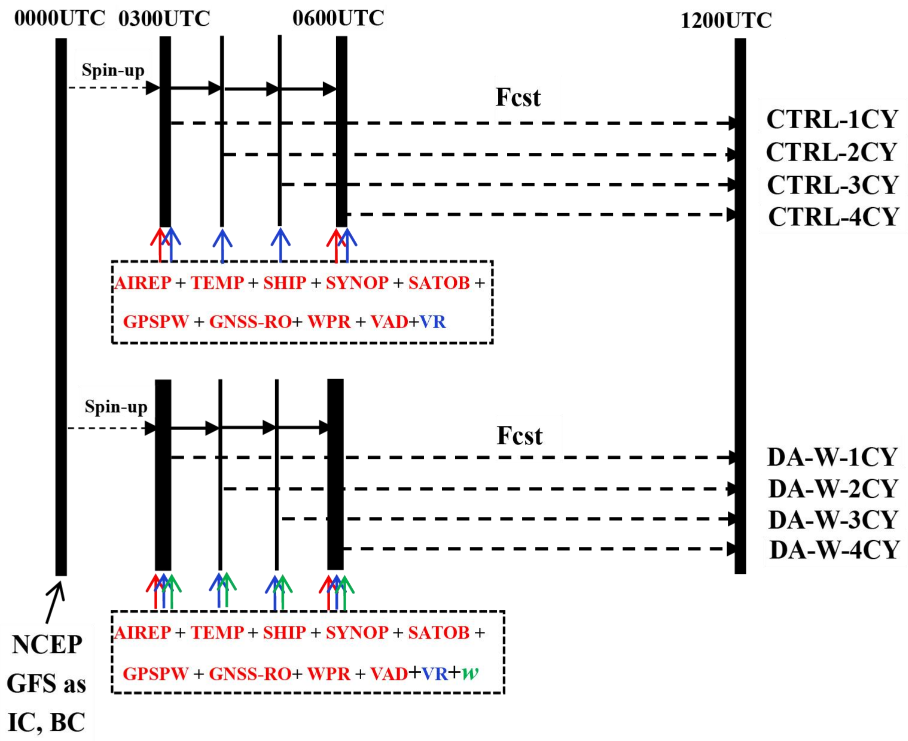

Figure 3 (b) of the revised manuscript v1. Illustration of the numerical experimental scheme for the CTRL and DA-W batch experiments. Both experiments utilize NCEP GFS data as the initial condition (IC) and boundary condition (BC). The abbreviation "Fcst" represents forecast. The assimilated data comprises conventional observations from aircraft measurements (AIREP), radiosondes (TEMP), ships (SHIP), and ground stations (SYNOP). In addition, cloud-track-wind (SATOB), precipitable water derived from the Global Positioning System (GPSPW), refractivity radio-occultation data from the Global Navigation Satellite System (GNSS-RO), wind profiler radar (WPR), velocity-azimuth display (VAD) wind, and the radar radial velocity (VR) are assimilated. The pseudo-*w* data is also assimilated for the DA-W experiments.

In addition, following the previous reviewer's suggestion, we have replaced the individual test with a case in the batch experiments, specifically the case initialized on July 9, 2020. For this case, we have also included the distribution of its 6-hour accumulated precipitation as Fig. 8 in the revised manuscript v1.

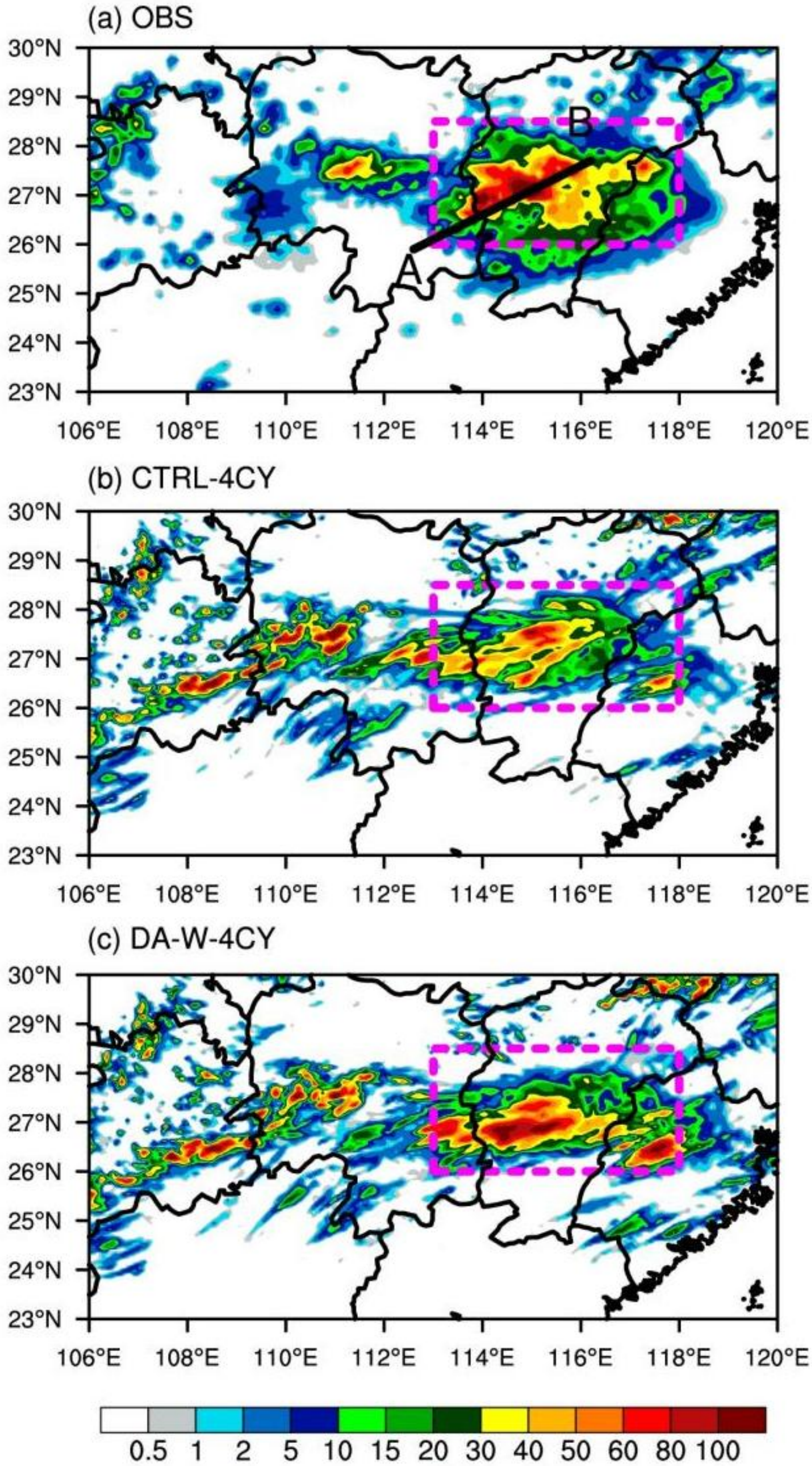

Figure 8 of the revised manuscript v1. The 6-hour (0600-1200 UTC) accumulated precipitation (units: mm) on July 9, 2020 for (a) observations (OBS), (b) CTRL-4CY, and (c) DA-W-4CY experiments. The areas enclosed by dotted purple lines indicate regions with observed strong rainfall.

2. Also, how does the assimilated vertical velocity affect the horizontal wind fields and how does it affect other thermodynamic fields such as temperature and humidity?

It is suggested to add one or two figures of quantitative and qualitative verification about the thermodynamic fields.

Response: Thank you for your suggestion. In this study, the adiabatic Richardson equation, which links w observations to model state variables u, v, and Π (the dimensionless pressure), is used as the observation operator for w. Therefore, assimilating w observations directly generates analysis increments for u, v, and Π variables, and then the analysis increments of temperature and specific humidity are calculated from the analysis increment of Π based on the physical constraint equations (state equation of moist air and hydrostatic balance equation). A single w observation experiment is conducted in our manuscript as Fig. 1, the pseudo-observation of w is positioned at an altitude of 5448.6 m (23th model level, approximately 500 hPa) with a value of 1 m s^{-1}. In order to quantify the size of the increments, the following description "The increments of horizontal wind at the lower and middle model levels can reach 0.060 and 0.077 m s^{-1}, respectively." (Lines 151–152 in the revised manuscript) has been added in the revised manuscript. In addition, the statements "(~ -8.7×10^{-6}–2.6×10^{-5} K in Fig. 1 (d))" and "(~ -7.0×10^{-8}–6.4×10^{-8} kg kg-1 in Fig. 1 (c))" (Lines 155–156 in the revised manuscript) to quantify the increments of temperature and humidity fields.

3. In particular, how the assimilation of affects vertical velocity fields itself, related verifications are also suggested.

Response: Thank you very much for your comments and professional advice. Since w is not an analysis variable, it is not updated at the analysis step. In our study, the adiabatic Richardson equation is used as the observation operator to assimilate pseudo-w observations, which leading to the update of the horizontal winds and pressure fields as the w assimilated. Although w is not directly updated at the analysis time, due to the assimilation of vertical information updating the horizontal wind field, w will be adjusted during subsequent model integrations through constraints imposed by the dynamic fields.

4. Since this study mainly focused on vertical velocity, vertical distribution plot of analysis increments in single observation test is suggested.

Response: Thank you for your suggestion. The vertical cross-sections of the analysis increment have been included as Fig. 2 in the revised manuscript v1. Additionally, the following statement has been added in the revised manuscript v1 (Lines 156–160): "From the vertical cross-section of the analysis increment for each variable (Fig. 2), it can be seen that the increase in specific humidity is primarily concentrated in the lower layer below the observation location, while the increases in the other three variables are distributed throughout the entire layer. Regarding the increase in horizontal wind component u, below the single point observation, there is a convergence of u wind that extends to the 1000 hPa. Above the single point observation, there is a divergence of u wind that extends to approximately 150 hPa".

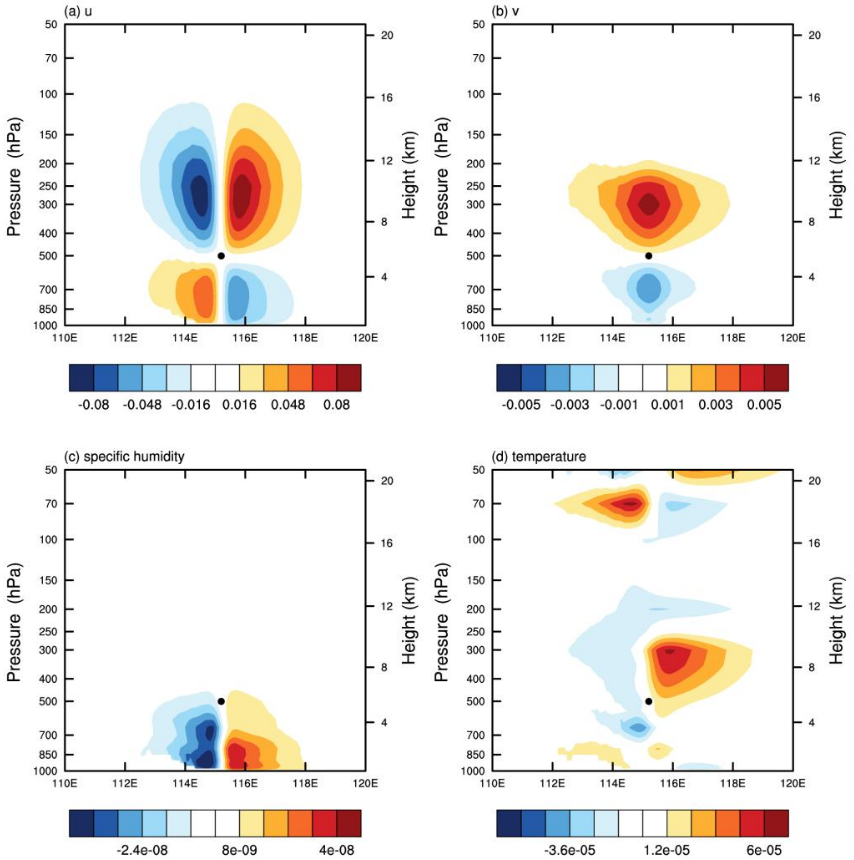

Figure 2 of the revised manuscript v1. The analysis increments of (a) zonal wind u, (b) meridional wind v (unit: m s^{-1}), (c) specific humidity (unit: kg kg^{-1}), and (d) temperature (unit: K) in a vertical cross-section at 38.0° N at 1500 UTC on July 4, 2020, for the single observation experiment. The solid black dots in the figure represent the locations of the single observation.

5. The treatment of the observation operator in this study is more like a static initialization based on equilibrium equations. If the model also assimilates the horizontal wind field at the same time, how should the analytical field of the final wind field take advantage of the different information. Does the difference between the two information create spurious gradients?

Response: We appreciate your valuable comment. To illustrate this issue, a comparison of the horizontal wind increments for two experiments (experiments CTRL-4CY and DA-W-4CY) in our revised manuscript v1 is presented as Figure R2. Please refer to the response to question 1 for a detailed description of experiments CTRL-4CY and DA-W-4CY. The CTRL-4CY experiment involves assimilation of horizontal wind data from other sources (radiosonde data) as mentioned by the reviewer. The DA-W-4CY experiment, on the other hand, assimilates vertical velocity in addition to the CTRL-4CY experiment, thereby adjusting the model's horizontal wind fields. From Figure R2, the horizontal wind analysis increments produced by the CTRL-4CY experiment are comparable to those of the DA-W-4CY experiment, but at locations with pseudo-w observations (as shown in Fig. R3), the DA-W-4CY experiment exhibits larger horizontal wind increments, accompanied by noticeable convergence and divergence patterns. At the same time, at locations without pseudo-*w* observations, the horizontal wind analysis increments of the DA-W-4CY experiment are comparable to those of the CTRL-4CY experiment.

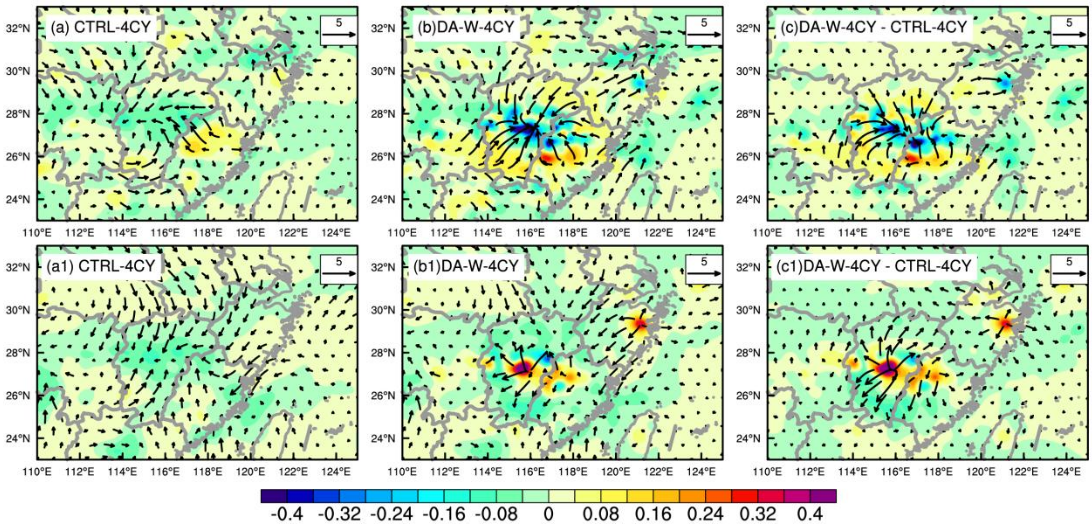

Figure R2. The analysis increments of horizontal wind (vector; unit: m s^{-1}) and horizontal wind divergence (color; unit: 10^{-4} s^{-1}) of the (a, a1) CTRL-4CY and (b, b1) DA-W-4CY experiments at the (a, b) 13th (~850 hPa) and the (a1, b1) 23th (~500 hPa) model levels at 0600 UTC on July 9, 2020. (c) and (c1) are the difference between the CTRL-4CY and DA-W-4CY experiments.

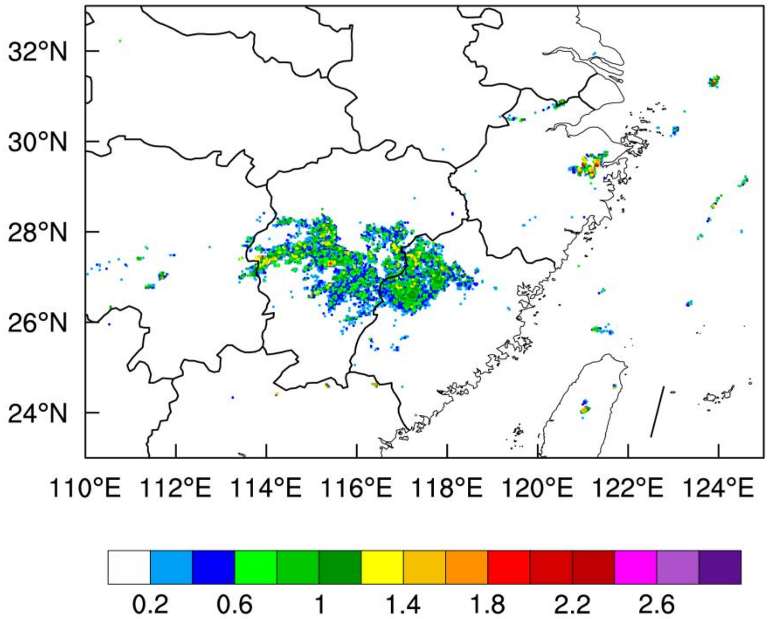

Figure R3. The derived maximum *w* pseudo-observations (units: m s^{-1}) at 0600 UTC on July 9, 2020.

Figure R4 illustrates the evolution of gradients during the minimization process for the CTRL-4CY and DA-W-4CY experiments. It can be seen that the DA-W-4CY experiment generates larger gradients than the CTRL-4CY experiment at the initial iteration step, but it rapidly converges, and even completes the minimization process more quickly than the CTRL-4CY experiment.

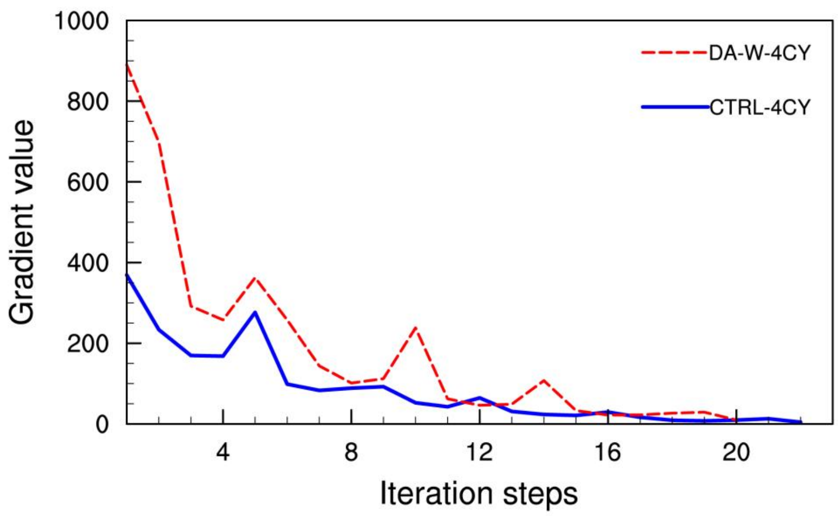

Figure R4. The evolution of gradient values with the iteration steps at the analysis time 0600 UTC July 9, 2020 for the CTRL-4CY and DA-W-4CY experiments.

Minor comments:

1. This study uses radar reflectivity-derived vertical velocities as observations, in my opinion, radar radial wind also contains vertical velocity information. Could the authors add some discussions regarding the future use of radar radial winds for vertical velocity assimilation?

Response: Thank you for your valuable comments. Based on your suggestion, we have added "In addition, the radial velocity also includes vertical velocity information." in the revised manuscript (Line 322).

2. On page 3, section 2.1, the description of the CMA-MESO 3D-Var system should be more specific.

Response: Thank you for your suggestion. The description of the CMA-MESO 3D-Var system has been revised in our revised manuscript v1: 1) in order to ensure consistency with Eq. (10), we have modified the cost function J to take the form of control variable; 2) the statement "the best analysis $\mathbf{x}$ can be derived from the control variable $\mathbf{c_v}$ (the control variables for CMA-MESO include the zonal and meridional winds, pseudo-relative humidity, temperature, and surface pressure) by minimizing a cost function J of $\mathbf{c_v}$" has been added (Lines 85–89 in the revised manuscript).

3. On page 4, line 106, the insertion of the parameter expression K=cp/R is too abrupt. Please add some description before and after as appropriate.

Response: Thank you for your suggestion. It has been revised to "The parameter κ in Eq. (5) can be expressed as $\kappa = {}^{c_p}/_{R}$." to make it more appropriate (Line 108 in the revised manuscript).

4. The legend for CTRL and DA-W in the upper left corner of Figure 3(a) should be aligned.

Response: Thank you for your suggestion. This figure has been removed in the revised manuscript v1.

5. Figures 4 (a) and 4 (b) can be presented in two figures.

Response: We appreciate your careful attention. Nevertheless, since our revised manuscript already contains 10 figures, we propose merging these two figures (Figures 3 (a) and 3 (b)) into one, taking into account the overall figure count.

6. Figure 4(b) does not show enough differences for the CTRL vs. DA-W group.

Response: Thank you for your suggestion. Based on the previous feedback from other reviewers, we have conducted a new set of batch experiments. Figure 4(b) now is Fig. 3 (b) in the revised manuscript v1. Both the CTRL and DA-W experiments were initialized at 0000 UTC daily from July 1 to July 10, 2020, and run until 1200 UTC each day. The first 3 hours were considered as a "spin-up" period. In the CTRL experiments, observations from aircraft measurements, radiosondes, and other sources (for a comprehensive list, refer to Fig. 3 (b)) were assimilated from 0300 to 0600 UTC with a 1-hour assimilation interval (radial velocity observations are available at each analysis time, while other data sources are only available at 0300 and 0600 UTC). The CTRL-1CY experiment indicates assimilation at 0300 UTC only, while the CTRL-2CY experiment represents assimilation at 0300 and 0400 UTC, and so on (the number preceding the experiment name "CY" represents the assimilation iterations). The DA-W experiments are similar to the CTRL experiments, but include the assimilation of w pseudo-observations (w pseudo-observations are available at each analysis time).

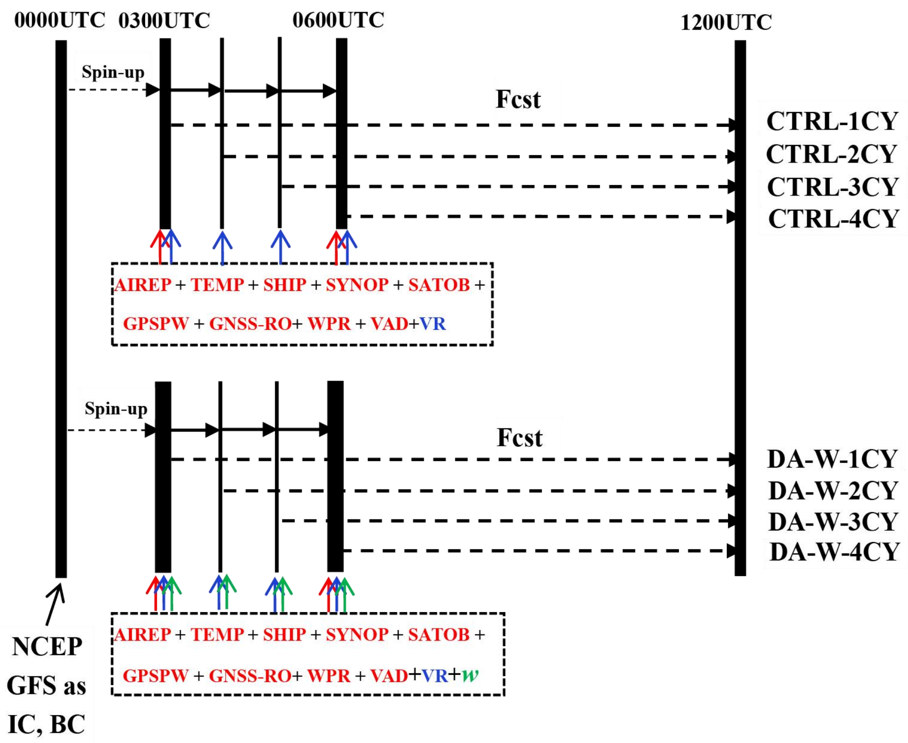

Figure 3 (b) of the revised manuscript v1. Illustration of the numerical experimental scheme for the CTRL and DA-W batch experiments. Both experiments utilize NCEP

GFS data as the initial condition (IC) and boundary condition (BC). The abbreviation "Fcst" represents forecast. The assimilated data comprises conventional observations from aircraft measurements (AIREP), radiosondes (TEMP), ships (SHIP), and ground stations (SYNOP). In addition, cloud-track-wind (SATOB), precipitable water derived from the Global Positioning System (GPSPW), refractivity radio-occultation data from the Global Navigation Satellite System (GNSS-RO), wind profiler radar (WPR), velocity-azimuth display (VAD) wind, and the radar radial velocity (VR) are assimilated. The pseudo-w data is also assimilated for the DA-W experiments.

7. On page 4, section 2.3 Accuracy check, the presentation of the judgment logic of the adjoint test under double precision could be made clearer, as is your account of verification of gradient correctness in the second half of the section.

Response: Thank you for your suggestion. According to your comments, the authors have made revisions to these two sections for clarity, as shown below (Lines 123–141 in the revised manuscript):

After completion of the w observation operator, the correctness of the adjoint operator should be checked (adjoint check). For the tangent linear $\mathbf{H}$ and its adjoint $\mathbf{H}^{\mathrm{T}}$ of an observation operator, the following formula is always satisfied:

$$< \mathbf{H}(\delta x), \mathbf{H}(\delta x) >=< \mathbf{H}^{\mathrm{T}}\big(\mathbf{H}(\delta x)\big), \delta x >, \tag{9}$$

where δx represents a small perturbation and $< >$ stands for the inner product of the vectors. The difference between the left-hand side and the right-hand side of Eq. (9) is expected to approach zero, typically with at least 13 significant digits. The test results show that term $< \mathbf{H}(\delta x), \mathbf{H}(\delta x) >$ is equal to 0.100159014620902D-17 (D: double precision), and term $< \mathbf{H}^{\mathrm{T}}\big(\mathbf{H}(\delta x)\big), \delta x >$ is equal to 0.100159014620902D-17. The difference between the two terms is 0.577778983316171D-33, which is achieved with 16 digits of accuracy. As a result, the adjoint check has successfully passed under double precision.

For a tangent linear operator, it is also necessary to verify the correctness of the gradient (gradient check) using the following standard:

$$\Phi(\alpha) = \frac{J(\mathbf{c_v}+\boldsymbol{\alpha})-J(\mathbf{c_v})}{\boldsymbol{\alpha}\nabla J(\mathbf{c_v})}, \tag{10}$$

$$\lim_{\boldsymbol{\alpha}\to 0} \Phi(\alpha)=1, \tag{11}$$

where ∇J is the gradient of J and the symbol α indicates a small scalar value. For values of α that are near but not too close to the machine zero, the value of $\Phi(\alpha)$ is expected to be close to 1. The results of the gradient check are presented in Table 1, showing a satisfactory approximation of the gradient with 8 digits of accuracy achieved (α=10^{-7}). This suggests that the tangent linear operator is accurate within the rounding error of the computer.

Table 1. Verification of gradient correctness: values of $\Phi(\alpha)$ for different α values (symbols defined in Eq. (10)).

α	$\Phi(\alpha)$
10^{-4}	1.00000684582308
10^{-5}	1.00000068454433
10^{-6}	1.00000006939151
10^{-7}	1.00000000569911
10^{-8}	1.00000003421803
10^{-9}	1.00000055706492
10^{-10}	1.00000626084813
10^{-11}	1.00001576715311
10^{-12}	1.00053861393468
10^{-13}	0.998162037654562

8. On page 9, section 4.1.2 Results, it can be seen that when the precipitation threshold is 20mm h-1, the enhancement effect of the experimental group on the forecast is no longer obvious compared to the precipitation threshold of 1mm h-1 and 5mm h-1, and the experiments in the section 4.2 The batch experiment also show a similar situation. Therefore, this phenomenon can be elaborated.

Response: Thank you for your suggestion. Based on the previous feedback from other reviewers, we have rerun the batch experiments (considering more assimilation cycles). Additionally, to ensure consistency, the individual case is derived from the batch experiments. The phenomenon is no longer apparent from Figures 4-6 in the revised manuscript v1.

9. The case of heavy precipitation analysis selected in the section 4 Validation could be explained a bit more. That is, why was this case chosen, what is its particularity, and what are its impacts? In particular, what are the characteristics of the vertical velocity during the evolution of this case?

Response: Thank you very much for your comments and professional advice. Your concern about the individual case has been raised by another reviewer previously, suggesting that we use an individual case from the batch experiments for consistency. Therefore, we have replaced the individual test with a case in the batch experiments, specifically the case initialized on July 9, 2020 in our revised manuscript v1.

10. The structure of the paper needs to be organized more logically, e.g. on page 6, the text between the section 4 Validation and the section 4.1 The heavy rainfall case study would be better suited to a separate section. Alternatively, there needs to be a separate section on the experimental setup, where the details of section 4.1 and 4.2 related to the setup are described together, which would be more conducive to enhancing the readability of the paper.

Response: Thank you for your suggestion. The text between the Section 4 Validation and the Section 4.1 (which is now the text from Lines 177–194 in the revised manuscript) has been separated into Section 4.1 as suggested. Since in the revised manuscript, the individual case is derived from the batch experiments, we have also adjusted the order of the batch experiments and individual case study, with batch experiments numbered as Section 4.2 and case study as Section 4.3, thus enhancing the readability of the article.

We would like to express our sincere gratitude to you for your invaluable feedback and insightful comments, which have greatly contributed to improving this manuscript.