



1 Inclusion of the subgrid wake effect between turbines in 2 the wind farm parameterization of WRF

3 Wei Liu^{1,2}, Xuefeng Yang¹, Shengli Chen¹, Shaokun Deng¹, Peining Yu³, Jiuxing Xing¹

4 ¹Institute for Ocean Engineering, Shenzhen International Graduate School, Tsinghua University,
5 Shenzhen, 518055, China

6 ²Mingyang Smart Energy Group Corporation, Zhongshan, 528437, China

7 ³Shenzhen Institute of Information Technology, Shenzhen, 518172, China

8 *Correspondence to:* Shengli Chen (shenglichen@sz.tsinghua.edu.cn), Peining Yu
9 (peining.yu@sziiit.edu.cn)

10

11 **Abstract.** Wind farms, as an important renewable energy source to combat climate change, have had
12 explosive development in recent years. Assessing impacts of wind farms on atmospheric and marine
13 environments requires an accurate parameterization of wind farms in atmospheric models. The current
14 wind farm parameterization scheme (Fitch et al. 2012) in WRF plays an important role in the study of
15 impacts of wind farms. The scheme, however, has some shortfalls, e.g., does not consider the wind wake
16 behind turbines inside a grid cell. In this research, the Fitch scheme in WRF is modified by inclusion of
17 the wake effect of wind turbines. Based on an engineering wake model of a turbine, a wake superposition
18 coefficient and an angle correction coefficient are proposed. A solution model for the inflow wind speed
19 is established to obtain the angle correction coefficient. Other coefficients in the engineering wake model
20 are calculated based on the CFD results. These coefficients are added in the WRF to improve the wind
21 farm parameterization, and sensitivity experiments are conducted. Model results show that the new
22 improved scheme significantly increases wind energy, output power and turbulent kinetic energy in the
23 wind farm area compared with the original scheme. Sensitivity experiments also reveal that, with
24 enlarged model grid size and shortened turbine spacing, the subgrid wake effect becomes more
25 significant, and the new scheme shows more advantages.

26 **Keywords:** wind farm parameterization; wake effect; WRF

27

28

29



30 **1 Introduction**

31 Wind power, as a pollution-free, renewable and widely distributed energy, has gradually become the top
32 priority of international energy transformation. In the process of adjustment of energy structure,
33 promotion of energy production and consumption revolution, the development of wind power industry
34 has played a pivotal role. As a strategic emerging industry, wind power has been guided by a large number
35 of industrial policies and driven deeply by market demand, thus achieving a rapid development period.
36 According to the latest Global Wind Power Industry Report 2022 released by the Global Wind Energy
37 Council (GWEC), by 2021, the quantity of world's new wind power installed capacity is up to 93.6 GW,
38 the cumulative installed capacity has reached 837GW, an increase of 12% over the previous year.

39 With the increase of the scale of wind farms and the size of a single turbine, the impacts of wind farms
40 on the surrounding atmospheric environment is also enlarging. Fitch et al. (2012) found that the wind
41 speed decay in the offshore wind farm could reach 16%, and could extend to the downstream area of 60
42 km. Christiansen et al. (2006) used the SAR radar data to study the impact of two large offshore wind
43 farms in Denmark, and reported that the average wind speed was reduced by 8%~9%, and the wind speed
44 decay zone could extend to 5~20 km along the wind direction. Roy and Traiteur (2010) showed that the
45 vertical mixing is enhanced due to the turbine wake effect during operation. In the stable atmosphere at
46 night, the vertical mixing is enhanced due the turbine wake effect, which leads to the near-surface's
47 warming, while in the unstable atmosphere during the day, the near-surface is cooling. Fiedler et al. (2011)
48 argued that the precipitation in the southeast of and around the wind farms in multiple states is reduced
49 by 1% due to the wind farms, and Vautard et al. (2014) showed that the change of winter precipitation
50 can be 0~5%. Barrie and Kirk-Davidoff (2010) simulated effects of large-scale onshore wind farm and
51 found that the operation of large scale wind farm would change the surface roughness and promote the
52 generation of cyclones, thus causing atmospheric disturbance and significantly affecting atmospheric
53 circulation. In short, the construction of large scale wind farm has a certain impact on climate factors at
54 both global and local scales.

55 At present, numerical model simulations are the mainly method for researching the environmental
56 impact of wind farms. Among these numerical models, Weather Research & Forecasting (WRF),
57 developed in collaboration with the National Center for Atmospheric Research (NCAR) and others, plays
58 an important role in this scope and has been widely accepted by researchers. The grid resolution used in
59 WRF is generally greater than 1 km, and the size of the turbine is only a few hundred meters, numerical



60 models, therefore, cannot resolve wind turbines and the effect of wind farms directly, and it is necessary
61 to use parameterization to characterize the effect of the wind turbine on the atmosphere. The
62 parameterization scheme applied in WRF is proposed by Fitch et al. (2013) which describes wind farms
63 as momentum sink and turbulent source in the environment. Many scholars have employed this scheme
64 to study the environmental impact of wind farms (Boezio and Ortelli, 2019; Jacondino et al., 2021; Pryor
65 et al., 2019). However, the scheme ignores the influence of the wake of the front turbine on the rear
66 turbine, which causes obvious errors. With the rising scale of wind farm in future, the subgrid wake effect
67 will be more prominent. Therefore, it is of great significance to explore the correction method of the
68 subgrid wake effect in WRF wind farm parameterization to improve the accuracy of wind farm
69 representation.

70 For the subgrid wake effect, previous researchers have proposed some solutions. Abkar and Port'e-Agel
71 (2015) tried to average the simulation results of LES and obtained a correction coefficient ξ to correct
72 the error of subgrid wake effect, but there is no universal prediction model or function for the correction
73 coefficient ξ . Pan and Archer et al. (2018) combined the simulation results of LES with the relevant
74 geometric parameters of wind farm layout, and proposed a "hybrid parameterization" scheme, and
75 experimental results show that the hybrid parameterization scheme also has a good effect on the
76 correction of subgrid wake effect. These above schemes use LES technology to achieve the purpose of
77 correcting subgrid wake effect. However, LES requires a large amount of computation, which limits the
78 usage of this method. Even if the LES simulation of wind farm is processed by the actuator model, it will
79 still consume huge computing resources in the face of the tendency of ultra-large wind farm construction
80 and the substantial increase in the number of turbines in a single wind farm. Elshafei et al. (2021) used
81 the spatiotemporal fusion data of deep multi-fidelity Gaussian regression and nonlinear autoregressive
82 algorithms to combine the simulation output of WRF with the field observation data to improve the
83 simulation accuracy. However, the observational data used is difficult to obtain.

84 Although the development of wind farm parameterizations in WRF has undergone several revision
85 iterations, there is still significant room for improvement in the handling of subgrid wake in this model.
86 This study attempts to correct subgrid wake effect errors in a new way, namely, through a simple
87 engineering wake model. Sect. 2 of the paper introduces the parameterization principle of wind farms in
88 WRF and a correction principle of subgrid wake effect based on engineering wake model. Sect. 3 displays
89 the correction and calibration results of the engineering wake model. In Sect. 4, effects of the proposed
90 new parametric scheme are analyzed and a series of sensitivity experiments are carried out.



91 2 Principle of subgrid wake effect correction

92 2.1 Wind farm parameterization in WRF

93 The WRF model is a completely compressible and non-hydrostatic multi-layer forecasting model for
94 small and medium scale weather systems, developed by the National Oceanic and Atmospheric
95 Administration, the National Center for Atmospheric Research and other agencies. The horizontal
96 resolution of the grid in WRF model generally ranges from 1 to 10 km, which is larger than the feature
97 scale of some motion elements. In order to better describe physical processes of these subgrid scale
98 motions, it is necessary to use parametrization methods for representing the interaction between the
99 solvable scale and the unsolvable scale. WRF model utilizes parameterization schemes of physical
100 processes including short-wave radiation and atmospheric long-wave radiation, microphysical processes,
101 boundary layer, cumulus convection, etc., to improve simulation accuracy (Skamarock et al, 2008).
102 Since the height of wind farms is in the order of 100 meters which are located in the atmospheric
103 boundary layer, it is necessary to supplement the boundary layer parameterization scheme when
104 exploring the impact of wind farms on the environment with WRF. According to the "momentum sink -
105 turbulent source" theory proposed by Fitch et al. (2013), parameterization of wind farms is realized by
106 adding a momentum trend term Eq. (1) to the momentum equation and a turbulent energy generation
107 term Eq. (2) to the equations of turbulent energy. In addition, a power generation term Eq. (3) is
108 introduced to calculate the power output of the entire wind farm.

$$109 \frac{\partial V_{ijk}}{\partial t} = -\frac{1}{2} \frac{N_t^{ijk} C_T(V_{ijk}) V_{ijk}^2 A_{ijk}}{(z_{k+1} - z_k)}, \quad (1)$$

$$110 \frac{\partial TKE_{ijk}}{\partial t} = \frac{1}{2} \frac{N_t^{ijk} C_{TKE}(V_{ijk}) V_{ijk}^3 A_{ijk}}{(z_{k+1} - z_k)}, \quad (2)$$

$$111 \frac{\partial P_{ijk}}{\partial t} = \frac{1}{2} \frac{N_t^{ijk} C_P(V_{ijk}) V_{ijk}^3 A_{ijk}}{(z_{k+1} - z_k)}, \quad (3)$$

112 where V_{ijk} is the wind vector at the grid (i, j, k) ; TKE_{ijk} is the turbulent kinetic energy at the grid $(i,$
113 $j, k)$; P_{ijk} for the power output at the grid (i, j, k) ; $C_T(V_{ijk})$, $C_{TKE}(V_{ijk})$, $C_P(V_{ijk})$ are turbine thrust
114 coefficient, turbulent kinetic energy generated coefficient and power factor, respectively, which are
115 functions of velocity; A_{ijk} is turbine swept area; N_t^{ijk} is the number of the turbines. These equations
116 show that the inflow wind speeds of all turbines are the same within one grid in this original
117 parameterization. At the same time, it can be seen that the core variable is the inflow wind speed of the



118 turbine which determines the accuracy of the parameterized scheme. Therefore, this study starts with the
119 correction of the inflow wind speed of the turbine by which the error of the subgrid wake effect based on
120 the engineering wake model can be reduced, so as to improve the accuracy of the simulation of wind
121 farm effect in WRF.

122 In this study, a wake superposition coefficient and angle correction coefficient will be used to correct the
123 inflow wind speed of the turbines, and the specific relationship is shown in Eq. (4). The wake
124 superposition coefficient considers the wind speed change due to the wake superposition in front of each
125 turbine when the inflow wind speed is perpendicular to the wind farm, and the angle correction
126 coefficient is further corrected for any wind direction. In fact, there is a wind direction angle between
127 inflow wind and wind farm layout. The wake superposition coefficient corrects the inflow wind speed
128 under the condition of 0° wind direction angle, while an angle correction coefficient is to correct the
129 inflow wind speed under any θ° wind direction.

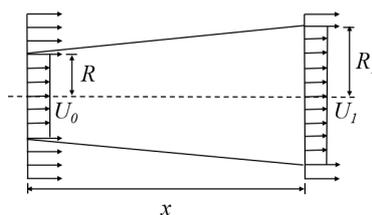
$$130 \quad u = C_a \cdot C_b \cdot u_0, \quad (4)$$

131 where C_a is the wake superposition coefficient, C_b indicates the angle correction coefficient, and u_0
132 denotes the original wind speed.

133 2.2 Wake superposition coefficient

134 The wake superposition coefficient is proposed based on the wake analytic model and the wake
135 superposition model. The analytical model of a single turbine wake is a mathematical expression of the
136 distribution of turbine wake velocity. Currently, there are many expressions of wake analytic models,
137 among which the Jensen model (Jensen et al., 1984) appears earlier and is widely used.

138



139

140 Figure 1: Schematic diagram of the wake analytic model.

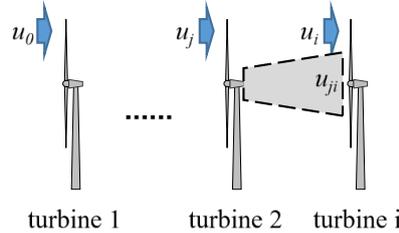
141 This model assumes that the expansion of wake width behind the turbine is linear along the flow direction



142 (Fig. 1), and that the loss of wake speed is related to C_T . According to the one-dimensional momentum
 143 theorem and the mass conservation theorem, the relation between the wind speed u_i and the inflow speed
 144 u_0 can be given as (Jensen et al., 1984):

$$145 \quad u_i = u_0 \left[1 - \left(1 - \sqrt{1 - C_T} \right) \left(\frac{R}{R + kx} \right)^2 \right], \quad (5)$$

146 where u_0 represents the inflow wind speed of turbine i , u_i represents the wake speed of the turbine i at the
 147 downstream position x ; C_T is the thrust coefficient of the turbine; R is the radius of the turbine's sweeping
 148 area; and k is the wake expansion coefficient, which is related to the roughness coefficient of the ground.
 149 Eq. (5) only describes the speed distribution of the wake behind a single turbine. In practice, the
 150 downstream turbine is often partly blocked by the upstream turbine, therefore it is necessary to introduce
 151 a wake superposition model. The wake superposition models proposed mainly include the quadratic sum
 152 superposition model, the linear superposition model and the energy conservation superposition model.



153

154 Figure 2: Schematic diagram of quadratic sum wake superposition model.

155 Among them, the more widely used is the quadratic sum superposition model proposed by Katic (1986)
 156 (Fig. 2), and is expressed as:

$$157 \quad \left(1 - \frac{u_i}{u_0} \right)^2 = \sum_{j=1}^n \left(1 - \frac{u_{ji}}{u_j} \right)^2, \quad (6)$$

158 where u_i represents the inflow wind speed of turbine i ; u_0 is the initial wind speed before superposition;
 159 n is the total number of superposition turbines in front of turbine i ; u_j denotes the inflow wind speed of
 160 turbine j ; and u_{ji} represents the wake wind speed of turbine j at turbine i . According to Eq. (2.5), the
 161 expression of u_{ji} is:

$$162 \quad u_{ji} = u_j \left[1 - \left(1 - \sqrt{1 - C_T} \right) \left(\frac{R}{R + kx_j} \right)^2 \right], \quad (7)$$

163 where x_j represents the distance between turbine j and turbine i . Substituting Eq. (7) into Eq. (6):



$$164 \quad u_i = u_0 \left[1 - \sqrt{\sum_{j=1}^n [(1 - \sqrt{1 - C_{Tj}}) (\frac{R}{R+kx_j})^2]^2} \right], \quad (8)$$

165 By contrast with Eq. (4), the expression of wake superposition coefficient can be given by:

$$166 \quad C_a = \left[1 - \sqrt{\sum_{j=1}^n [(1 - \sqrt{1 - C_{Tj}}) (\frac{R}{R+kx_j})^2]^2} \right], \quad (9)$$

167 2.3 Angle correction coefficient

168 The same as the wake superposition coefficient, because the turbine effect in the grid is directly
 169 superimposed, it is only necessary to correct the total effect of the turbine wake superposition effect
 170 under the condition of θ° wind direction angle. The total effect of the turbine's wake superposition is
 171 averaged for each turbine, and the angle correction coefficient is used to express the effect of the averaged
 172 wind speed of a single turbine on the wake superposition. That is to say, instead of correcting the specified
 173 effect of a single turbine, only the overall deviation of the wind farm relative to the 0° wind direction is
 174 evaluated. For additive terms Eqs. (1-3), the velocity variables have different powers, so the angle
 175 correction coefficient needs to be divided into quadratic and cubic according to the power number of the
 176 additive terms.

177 According to Eq. (1), after correcting using the wake superposition coefficient, that is, when the wind
 178 direction is 0° , the total effect of the turbine in one grid for the momentum trend term is:

$$179 \quad \frac{\partial V_{ijk}}{\partial t} = -\frac{1}{2} \frac{A_{ijk} \sum_1^n C_T (v_{n0}) v_{n0}^2}{(z_{k+1} - z_k)}, \quad (10)$$

180 where v_{n0} is the inflow wind speed of the turbine in 0° wind direction. Introducing the quadratic angle
 181 correction coefficient when the wind direction is θ° , the total effect of the turbine in the momentum trend
 182 term is:

$$183 \quad \frac{\partial V_{ijk}}{\partial t} = -\frac{1}{2} \frac{A_{ijk} \sum_1^n C_T (C_{b2} \cdot v_{n0}) (C_{b2} \cdot v_{n0})^2}{(z_{k+1} - z_k)} = -\frac{1}{2} \frac{A_{ijk} \sum_1^n C_T (v_{n\theta}) v_{n\theta}^2}{(z_{k+1} - z_k)}, \quad (11)$$

184 where $v_{n\theta}$ is inflow wind speed of the turbine in θ° wind direction. Therefore, the quadratic angle
 185 correction coefficient C_{b2} is expressed as:

$$186 \quad C_{b2} = \sqrt{\sum_1^n v_{n\theta}^2 / \sum_1^n v_{n0}^2}, \quad (12)$$

187 Similarly, from the equation of turbulent kinetic energy Eq. (2), the cubic angle correction coefficient
 188 C_{b3} is given by:

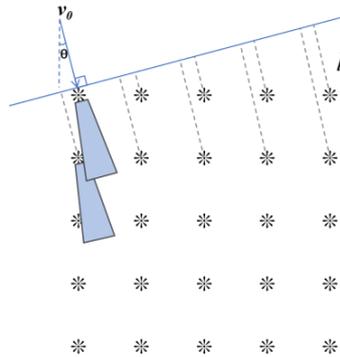


$$C_{b3} = \sqrt[3]{\frac{\sum_1^n v_{n\theta}^3}{\sum_1^n v_{n0}^3}}, \quad (13)$$

190 The reason why two correction coefficients are computed in two steps, instead of directly calculating the
 191 inflow wind speed of a single turbine in one step, is that the usage of two correction coefficients will
 192 reduce much computing consumption. As the scale of wind farm tends to be larger in future, the
 193 consumption of computing resources in the simulation process will increase greatly, therefore it is of
 194 great application significance to simplify the calculation.

195 The calculation of $v_{n\theta}$ and v_{n0} in Eq. (12) and (13) is carried out through the wind farm modeling. As
 196 shown in Fig. 3, in the modeling process, for a wind farm composed of n turbines, l_n is the windward
 197 distance of each turbine along the wind direction angle of θ° , and n turbines are numbered according to
 198 the order of l_n from small to large: $1\dots i, j\dots n$, where turbine i is upstream of turbine j . The distance
 199 between turbine i and turbine j along the wind direction is denoted $l(i,j)$, and the wake wind speed of
 200 turbine i at turbine j is denoted $v(i,j)$, then $l(i,j), v(i,j)$ can form an $n \times n$ upper triangular matrix, denoted
 201 as the distance matrix \mathbf{L} and the wind speed matrix \mathbf{V} (Eq. 14). The element $v(i,i)$ on the diagonal of \mathbf{V}
 202 represents the inflow wind speed of turbine i at a wind direction angle of θ° , i.e. $v_{n\theta}$.

$$V = \begin{bmatrix} v(1,1) & \dots & v(1,i) & \dots & \dots & v(1,n) \\ & \dots & \dots & \dots & \dots & \dots \\ & & v(i,i) & v(i,j) & \dots & \dots \\ & & & v(j,j) & \dots & \dots \\ & & & & \dots & \dots \\ & & & & & v(n,n) \end{bmatrix} \quad (14)$$



203
 204

Figure 3: Schematic diagram of wind farm modeling principle.

205 The wind velocity matrix \mathbf{V} is calculated in the row sequence. According to Jensen wake analytic model
 206 Eq. (5), elements $v(i,j)$ in row i of the wind speed matrix \mathbf{V} can be obtained as:



207
$$v(i, j) = v(i, i) \left[1 - \left(1 - \sqrt{1 - C_T} \right) \left(\frac{R}{R + k \cdot l(i, j)} \right)^2 \right], \quad (15)$$

208 Due to the superposition of upstream turbines, the calculation of downstream turbine inflow wind speed
 209 needs to consider the superposition effect of the wake. According to the quadratic sum superposition
 210 model Eq. (6), the $v(i+1, i+1)$ can be calculated as :

211
$$v(i+1, i+1) = v(1, 1) \left[1 - \sqrt{\sum_{n=1}^i \gamma(n, i+1) \left(1 - \frac{v(n, i+1)}{v(1, 1)} \right)^2} \right], \quad (16)$$

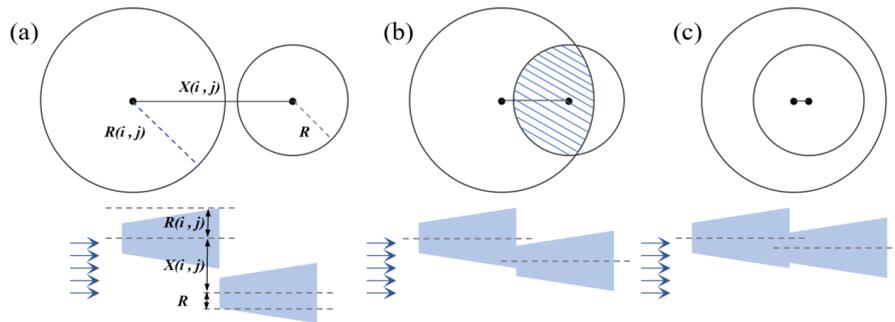
212 Given the inflow wind farm wind speed $v(1, 1)$, combining the Eq. (15) and Eq. (16), all the elements
 213 in the matrix \mathbf{V} can be solved, namely the inflow wind speed $v_{n\theta}$ of all wind turbines in the wind farm
 214 in θ° wind direction angle.

215 Eq. (6) applies to the situation when the wake is completely overlapping, i.e., the downstream turbine is
 216 completely in the wake of the upstream turbine. In practice, due to the existence of wind direction angle,
 217 the downstream turbine is not completely in the wake of the upstream turbines, hence we introduce a
 218 shielding factor $\gamma(i, j)$ of turbine i to turbine j as in Eq. (16). The shielding factor, representing the degree
 219 to which the upstream turbine's wake affects the downstream turbine, is the proportion of the overlap
 220 area between the upstream turbine's wake and the downstream turbine's disk surface to the swept area of
 221 the downstream turbine's impeller. The distance between turbine i and turbine j perpendicular to the wind
 222 direction is $X(i, j)$, and the wake radius of turbine i at turbine j is $R(i, j)$, then

223
$$X(i, j) = l(i, j) \cdot \tan\theta, \quad (17)$$

224
$$R(i, j) = k \cdot l(i, j) + R, \quad (18)$$

225 where θ is the wind direction angle, k is the coefficient of wake expansion, R is the radius of the impeller's
 226 sweeping area, and $l(i, j)$ is the distance between turbine i and turbine j in the windward direction.



227
 228 Figure 4: Schematic diagram of the shielding factor calculation(a) no shielding; (b) partial shielding; (c) complete
 229 shielding.



230 As shown in Fig. 4, the shielding factor has three types of situation:

231 (1) when $X(i,j) > R(i,j) + R$ (Figure. 2.4a), i.e., the downstream turbine impeller is completely not in the
 232 wake of the upstream turbine, $\gamma(i,j)=0$;

233 (2) when $R(i,j) - R < X(i,j) < R(i,j) + R$ (Fig. 2.4b), i.e., part of the downstream turbine impeller is in the
 234 wake of the upstream turbine, $\gamma(i,j) = S_{sd} / S_{tb}$;

$$235 \quad S_{sd} = R(i,j)^2 \arccos \frac{R(i,j)^2 + X(i,j)^2 - R^2}{2R(i,j)X(i,j)} + R^2 \arccos \frac{R^2 + X(i,j)^2 - R(i,j)^2}{2RX(i,j)} -$$

$$236 \quad X(i,j)R(i,j) \sin(\arccos \frac{R(i,j)^2 + X(i,j)^2 - R^2}{2R(i,j)X(i,j)}), \quad (19)$$

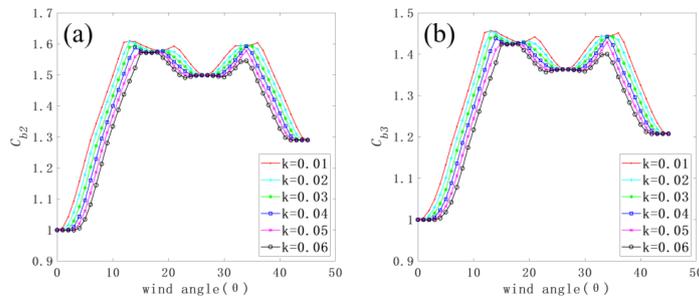
$$237 \quad S_{tb} = \pi \cdot R^2, \quad (20)$$

238 (3) when $X(i,j) < R(i,j) - R$ (Fig. 2.4c), i.e., the rotating surface of the downstream turbine impeller is
 239 completely in the wake of the upstream turbine, $\gamma(i,j)=1$.

240 3 Correction of engineering wake model

241 3.1 CFD experiments of wake

242 When calculating the inflow wind speed of the turbine at different wind angles, the wake expansion
 243 coefficient plays an important role. When the total number (n) of turbines in a wind farm is 25, the
 244 distance between turbines is 5 times the turbine diameter, and the inflow wind speed $v_0=10$ m/s, the
 245 dependence of the quadratic and cubic angle correction coefficients on the wake expansion coefficient
 246 can be seen in Fig. 5.



247
 248 Figure 5: Relation between the angle correction coefficient with the wake expansion coefficient (k) (a) the quadratic
 249 angle correction coefficient; (b) the cubic angle correction coefficient.

250 With the rising of wake expansion coefficient, both angle correction coefficients diminish. Because the
 251 average inflow wind speed of the turbine at 0° wind angle is the same, and with the increase of expansion



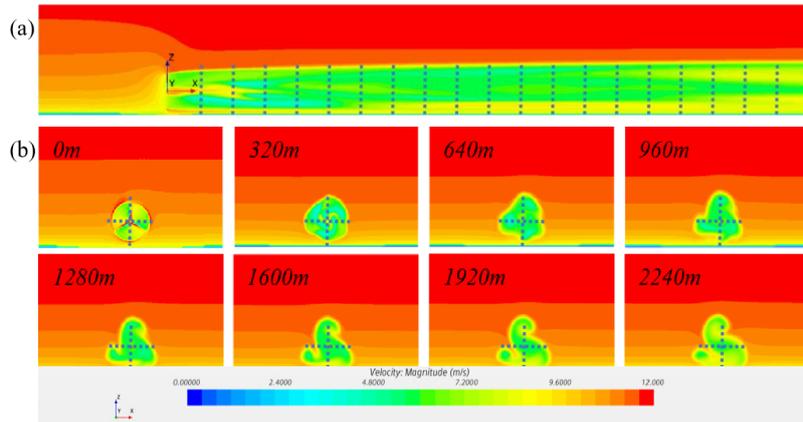
252 coefficient, the distribution of the wake along the downstream gradually becomes divergent. For the case
253 of θ° wind angle, the downstream turbine is affected more by the upstream turbine, and the inflow wind
254 speed becomes smaller, hence the two angle correction coefficients are reduced due to Eqs. (12) and (13).
255 It is concluded that the change of wake expansion coefficient has a significant influence on the angle
256 correction coefficient.

257 The quadratic sum superposition model (Eq. 6) is used in the model for solving the angle correction
258 coefficient. In the use of quadratic sum superposition model, it is assumed that the inflow wind speed of
259 the turbine is evenly distributed. However, in the actual situation, the inflow wind speed of the turbine is
260 affected by ground friction in the form of wind profile distribution. In short, when the wake analytic
261 model and wake superposition model are used in this study, certain corrections need to be made according
262 to the CFD simulation results of the turbine's wake.

263 3.2 Calibration of the wake expansion coefficient

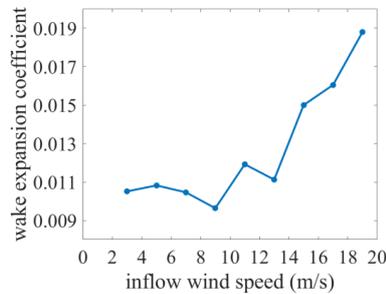
264 The expansion coefficient is examined by the CFD modelling of a single turbine under different inflow
265 wind speed conditions. In the single turbine experiment, the incoming wind speed of the turbine is in the
266 form of wind profile. There are 9 groups of experiments are set according to the wind speed at the hub
267 height, with the wind speed at the hub height ranging from 3 to 19 m/s (an interval of 2 m/s). As shown
268 in Fig. 6, speed monitoring surfaces are set $1D$ apart in the downstream direction, and two speed
269 monitoring lines are set in the horizontal and vertical directions on each speed monitoring surface, and
270 100 monitoring points are set on each monitoring line to monitor the speed amplitude.

271 After obtaining the wind speed amplitude at the monitoring line in the wake, it is necessary to determine
272 the boundary of the wake in the horizontal and the vertical direction according to distribution of the speed
273 amplitude. Then the relationship between wake radius and wake distance can be obtained. With a linear
274 fit to the change of the wake radius at each inflow wind speed, the slope of the fitted line is the wake
275 expansion coefficient k . After calculation, the relationship between wake expansion coefficient and
276 inflow wind speed is shown in Fig. 7. The wake expansion coefficient remains relatively stable for the
277 low wind speed (3-9 m/s), but has a significant linear growth for the high wind speed (9-19 m/s), reaching
278 to a value of 0.019 for 19 m/s. These wake expansion coefficient is applied to the wake analytical model
279 (Eq. 9).



280

281 Figure 6: Schematic diagram of monitoring surface setup in the turbine wake (a) Parallel wake profile; (b) Vertical
 282 wake profile.



283

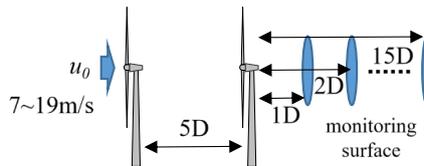
284 Figure 7: Dependence of wake expansion coefficient k on the inflow wind speed.

285 3.3 Correction of the wake superposition model

286 The wake superposition model is corrected based on the results of the CFD two-turbine wake experiment.

287 The experiment settings are shown in Fig. 8. The distance between turbines is $5D$, and 7 experiments are

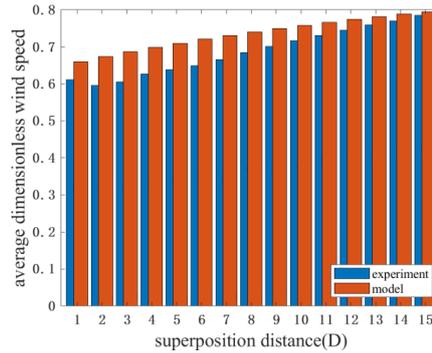
288 carried out within the working wind speed ranging from 7 to 19 m/s with an interval of 2 m/s.



289

290

Figure 8: The experiment of two-turbine wake superposition.



291

292 Figure 9: Comparison of the average dimensionless wind speed behind the second turbine between the
293 superposition experiment and the superposition model. The dimensionless wind speed is obtained by being
294 dividing by the inflow wind speed.

295

296 It can be seen from the comparison results (Fig. 9) that there are significant differences between results
297 of the CFD experiment and the engineering model. Results calculated using the engineering
298 superposition model are slightly higher than that simulated using the CFD experiment, and their
299 difference gradually diminishes with the increase of the distance. Such differences are mainly due to the
300 assumption that the inflow wind speed is evenly distributed at different altitudes in the calculation of
301 wake superposition model, while the CFD simulation adopt the inflow wind speed with a wind profile
302 closer to the actual situation. This assumption is one of the error source of the wake superposition model
303 in this study, so it is necessary to correct the wake superposition model according to the CFD results to
304 improve the accuracy of the wind farm model, further improve the accuracy of the angle correction
305 coefficient, and finally make the correction of the subgrid wake effect more accurate. Thus a correction
306 factor (C_{over}) is proposed as in Eq. 21. The dimensionless velocity of the experiment and the model is
307 averaged over the wake distance, and the ratio between the experiment simulation (\bar{v}_{sim}) and engineering
308 model (\bar{v}_{mod}) is 0.93. By applying this coefficient to Eq. (9), the correction of the wake superposition
309 model can be completed.

310
$$C_{over} = \bar{v}_{sim} / \bar{v}_{mod}. \quad (21)$$

311 **4. Implement of new parameterization and sensitivity experiments**

312 **4.1 Implement of new parametrization**



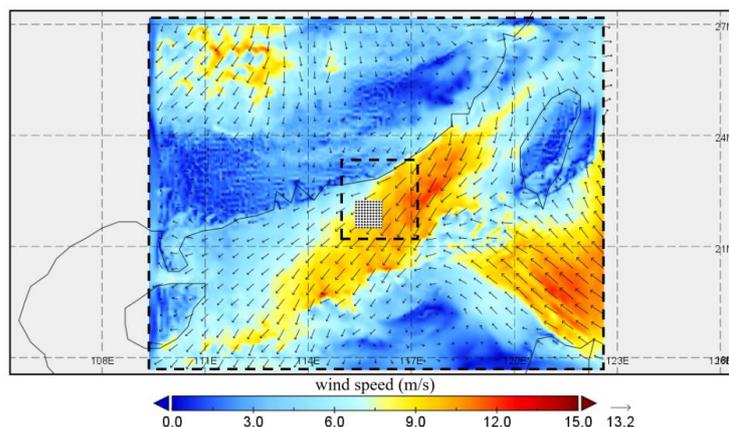
313 The wake superposition coefficient (C_a), the quadratic angle correction coefficient (C_{b2}) and the cubic
314 angle correction coefficient (C_{b3}) are used to modify the original parameterization scheme of wind farm
315 in WRF. The differences between the original and new schemes are compared, and the correction effect
316 of the subgrid wake effect is verified and discussed.

317 In the experiments, the model domain is set from 17.68° N to 27.16° N, from 109.38° E to 122.62° E
318 (Fig. 10). The wind farm is located in an area between 21.70° N~ 22.50° N, 115.14° E~ 116.00° E . The
319 type of turbine used in the WRF simulation is the same as that used in the CFD simulation. The turbines
320 are arranged parallel to the longitude and latitude lines with an equal spacing of $5D$. The total number of
321 turbines is 25600, and the output power of a single turbine is 3 MW.

322 The experimental area nests two layers of inner and external meshes with grid resolutions of 2.8 km and
323 8.4 km, respectively, and 30 layers are used in the vertical direction. The wind farm is located in the inner
324 area of the nested model. The inner grid spacing is equal to 5 times the turbine layout spacing, i.e., there
325 are 25 turbines in the one grid cell.

326 Physical and dynamic schemes used in the simulations are identical to what was performed in the
327 previous studies (Hong et al.,2006; Mlawer et al.,1997; Fouquart et al.,1991; Grell et al.,2002;
328 Nakanishim et al.,2009). For the land surface process, the parameterization scheme of heat diffusion is
329 adopted. Goddard parameterization scheme is used for short-wave radiation process, while RRTM
330 parameterization scheme is used for atmospheric long-wave radiation process. For microphysical process,
331 WSM6 parameterization scheme is selected to improve the accuracy of vertical profile and reduce the
332 influence of time step on physical parameterization scheme. The Grell-Freitas scheme is employed to
333 parameterize cumulus cloud. MYNN-2.5 is selected for the parameterization scheme of the planetary
334 boundary layer, which contains various physical processes in detail, and can simulate the influence of
335 wind farm on atmospheric boundary layer more accurately. The WRF model is initialized at 0000 UTC
336 on 4 January 2022, using the data from the National Centers for Environmental Prediction Global
337 Forecast System (GFS). All the simulations are run for about 3 days.

338 It can be seen that there is a relatively stable northeast wind in the wind farm area at this time (Fig. 10)
339 and the wind speed is moderate, about 10 m/s. The kind of speed belongs to the critical wind speed of
340 the turbine to reach the rated power, which is conducive to observe and compare the difference
341 phenomenon at this moment.

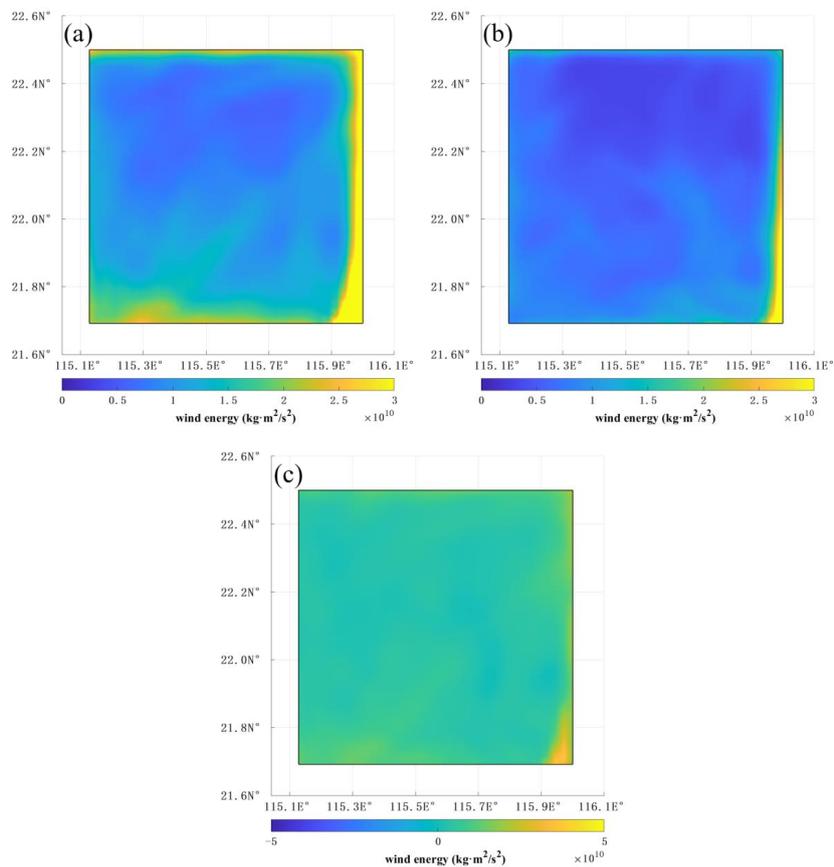


342

343

344

Figure 10: Snapshot of wind vectors and wind speed on 4 January 2022. The dash black lines denote the inner and external model domains, respectively. The white region with black dots represent the wind farm.



345

346

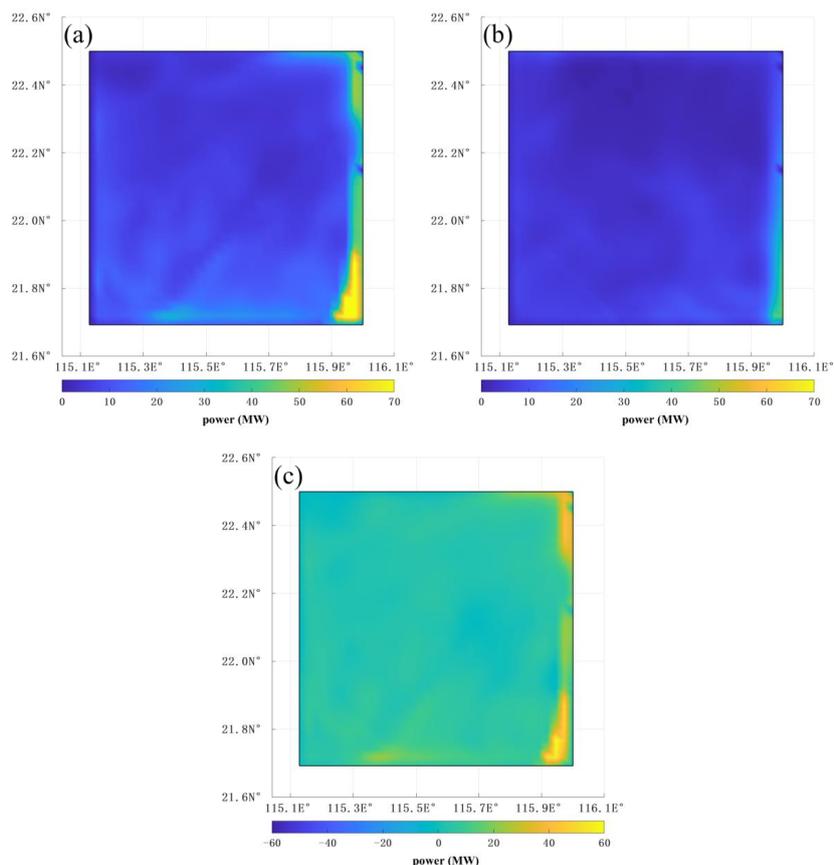
Figure 11: Comparison of wind energy distribution for (a) the new parameterization scheme; (b) the original



347 parameterization scheme; (c) their difference.

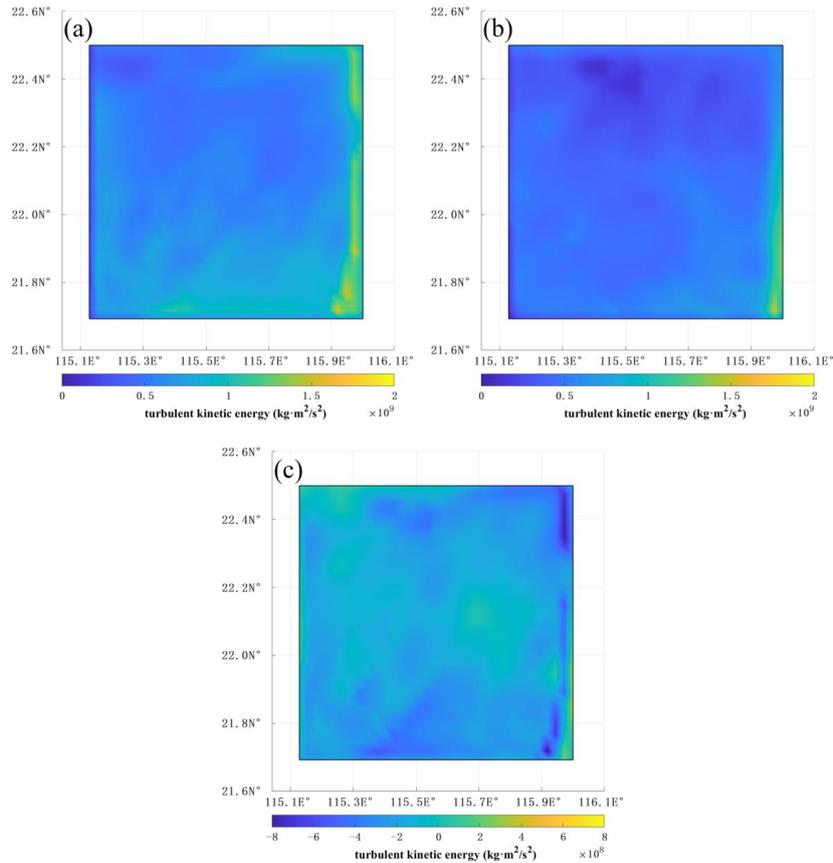
348 As shown in Fig. 11, the wind energy inside the wind farm region using the new scheme, which is
349 $1.44 \times 10^{13} \text{ kg} \cdot \text{m}^2/\text{s}^2$ in total, is higher than that of the original scheme ($8.54 \times 10^{12} \text{ kg} \cdot \text{m}^2/\text{s}^2$), increasing
350 by 68%. When the subgrid wake correction is added to the new parameterization scheme of wind farm,
351 the inflow wind speed of the rear turbine is reduced. Therefore, the absorption of wind energy in the wind
352 farm region is reduced. At the same time, the total wind energy becomes greater than that in the original
353 scheme, reducing the error of overestimation of wind speed attenuation in the original scheme. The wind
354 energy in the upstream edge region of the wind farm is significantly enhanced. This is because in the
355 new parameterization scheme, the kinetic energy absorption of the grid in the upstream region is reduced
356 more, so that the wind energy is more fully developed downstream and resulting in greater wind energy
357 in the edge region of the wind farm.

358



359
360 Figure 12: Comparison of power output for (a) the new parameterization scheme; (b) the original parameterization
361 scheme; (c) their differences.

362 The power output in the new scheme is higher than that in the original scheme (Fig. 12). The total regional
363 power output of the wind farm in the new scheme is calculated and the result is 11639.56MW, while that
364 in the original scheme is 5703.2MW, with an increase of 104%. This is because after the new scheme
365 corrects the excessive kinetic energy absorption error of the turbine, the wind speed in the wind farm
366 area increases, thus increasing the instantaneous output power of all turbines. Therefore, the output power
367 generated by the wind farm increases.



368

369 Figure 13: Comparison of turbulent kinetic energy for (a) the new parameterization scheme; (b) the original
370 parameterization scheme; (c) their differences.

371 As shown in Fig. 13, the turbulent kinetic energy of the wind farm region in the new scheme is also
372 enlarged. The total regional turbulent kinetic energy under the new scheme is $7.00 \times 10^{11} \text{ kg} \cdot (\text{m}^2/\text{s}^2)$, while
373 that in the original scheme is $4.96 \times 10^{11} \text{ kg} \cdot (\text{m}^2/\text{s}^2)$, with a rise of 41%. It can be seen from the expressions
374 of the power output and turbulent source terms in the parameterization principle of wind farm Eqs. (2
375 and 3) that power output and turbulent kinetic energy have the same tendency. Therefore, the growth of
376 power output is bound to be accompanied by the enhanced turbulent kinetic energy generation.

377 In summary, compared with the original parameterization scheme, the simulation results of the new
378 scheme have increased significantly for the simulation results of each parameter quantity, and the
379 differences between the two schemes also conform to the relevant law. Therefore, the rationality and



380 feasibility of the new scheme for the correction of subgrid wake effect can be proved. It forms a
381 foundation for exploring the further optimization of the new parameterization scheme and its sensitivity
382 to wind farm parameters.

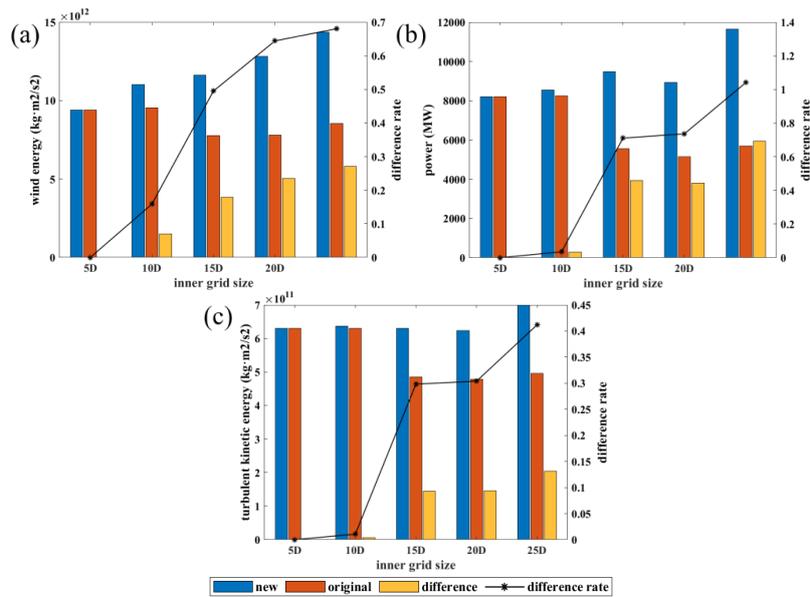
383 4.2 Sensitivity experiments

384 A series of model experiments is conducted to investigate the sensitivity of the new scheme to wind farm
385 parameters. Experiments are divided into two groups: the Grid experiment and the Space experiment. In
386 all experiments, the simulation area, simulation time and other parameterization schemes are the same
387 as in the last section. Details of the settings in each group of experiments are shown in Table 1.

388 Table 1: Setting for sensitivity experiments.

	Internal grid size	Spacing	Number	Number in one grid
Grid experiment	5 <i>D</i>	5 <i>D</i>	25600	1*1
	10 <i>D</i>	5 <i>D</i>	25600	2*2
	15 <i>D</i>	5 <i>D</i>	25600	3*3
	20 <i>D</i>	5 <i>D</i>	25600	4*4
	25 <i>D</i>	5 <i>D</i>	25600	5*5
Space experiment	10 <i>D</i>	2 <i>D</i>	25600	5*5
	15 <i>D</i>	3 <i>D</i>	25600	5*5
	20 <i>D</i>	4 <i>D</i>	25600	5*5
	25 <i>D</i>	5 <i>D</i>	25600	5*5

389
390 In the Grid experiment, the size of the inner grid size is changed with an interval of 5*D*, so that the
391 number of turbines in one grid varies. As for the first experiment, the inner grid size is 5*D*, equaling to
392 the turbine spacing, so there is only one turbine in one grid and has no subgrid wake effect in the original
393 and new parameterization schemes. With the number of turbines in the grid changing, it is easily to
394 observe the sensitivity of the subgrid wake effect to the grid size. In the Space experiment, the scale of
395 the wind farm is kept unchanged and only the turbine spacing is adjusted. In order to keep the same
396 number of grid turbines, i.e., to ensure the consistency of the subgrid wake effect on the number
397 superposition, the simulated grid size is changed to adapt to the change of turbine spacing. The simulation
398 results of the two groups of experiments are processed and shown in Figure 15 and 16.



399

400 Figure 14: Comparison of simulation results between the original and new schemes for the Grid experiments for
401 (a) the wind energy; (b) the power output; (c) the turbulent kinetic energy. The difference rate is calculated as the
402 difference between results of two schemes being divided by the old scheme results.

403 As shown in Fig. 14, the simulation results indicate that compared with the inner grid size of 5D, when
404 there is no subgrid wake effect, the wind energy, the output power and the turbulent kinetic energy are
405 all significantly reduced with the grid size, which confirms the influence of subgrid wake effect on the
406 simulation. In these Grid experiments, the size of the grid is gradually enlarged, while the spacing and
407 the total number of turbines remain unchanged, so the range of the wind farm remains unchanged.
408 Theoretically, the simulation results of the wind farm should also remain unchanged, however, the
409 simulation varies due to the various intensity of the subgrid wake effect associated with different grid
410 size. It can be found that the difference rate increases gradually with the increase of grid size. This is
411 because the larger the grid size, the more turbines contained in the same grid, correspondingly, the more
412 significant the subgrid wake effect in the original parameterization scheme. Under the same conditions,
413 when the mesh size is larger, the subgrid wake effect is more significant, and it is more necessary to
414 employ the new wind farm parameterization scheme. As the number of turbines in the grid diminishes,
415 the difference between the original and new parameterization schemes is gradually reduced. When there
416 is only one turbine in the grid, i.e., there is no subgrid wake effect, results of the original and new
417 parameterization schemes are the same, proving that the new parameterization scheme can be compatible



436 parameterization scheme of wind farms. The accuracy of the engineering model is improved using the
437 CFD simulation of the turbine wake. The verification and sensitivity analysis of the new parameterization
438 scheme are carried out.

439 In the existing Fitch scheme of WRF, the inflow wind speed of all turbines in one grid are the same,
440 which ignores the wake effect between turbines. The wake superposition coefficient corrects the subgrid
441 wake effect under the condition of 0° inflow wind angle, and the angle correction coefficient further
442 corrects the condition under any inflow wind angle. An engineering wake model is calibrated and
443 modified based on the CFD simulation results. The wake expansion coefficient in the wake analytical
444 model is calibrated by the change of wake radius of single turbine under different inflow wind speed
445 conditions. At the same time, the velocity calibration of the wake superposition model is carried out by
446 the wake superposition of two turbines under different inflow wind speed conditions. The above
447 correction coefficients are applied to WRF to present a new parameterization scheme of wind farms.

448 Verification and sensitivity experiments of the new parameterization scheme are carried out, compared
449 with the original parameterization scheme under different simulation conditions. The experimental
450 results show that the simulation results of wind energy, power output and turbulent kinetic energy of the
451 new parameterization scheme are significantly higher than those of the original scheme. The differences
452 between them are analyzed to be caused by the overestimation of the wind energy absorbed by the
453 turbines in the grid in the original scheme. Sensitivity experiments show that in the experimental grid
454 size range ($5D\sim 25D$), with the increase of grid size, the difference rate between the original and new
455 schemes grows gradually. In experiments of different turbine spacing ($2D\sim 5D$), with the shortened
456 turbine spacing, the different rate between the new and the original schemes is increased gradually. Due
457 to the limitations engineering practice, there are still some shortcomings in the improved scheme. The
458 method of solving the angle correction coefficient should be optimized. In the process of solving the
459 angle correction coefficient, an average method is used to deal with the inflow angle, which cannot
460 accurately represent the inflow wind speed in front of a specific turbine to a certain extent. It is hoped
461 that other solving methods can be explored in future to compare the differences with the solution in this
462 paper and improve the accuracy of the solution. The new and original parameterization schemes should
463 be used to systematically carry out the experiments of wind farm's influence on various weather and
464 climate systems, so as to investigate the application performance of the new parameterization schemes
465 more systematically.



466 In future, after the new parameterization scheme is verified systematically, it is hoped that the new
467 scheme can be promoted and integrated into the WRF parameterization scheme for wind farms, so as to
468 make the simulation of wind farms in WRF more accurate, and provide a better tool to estimate the wind
469 power and study the environmental impact of wind farms.

470

471 *Code availability.* The source code modifying the wind farm module in the WRF can be accessed
472 through this link (<https://doi.org/10.5281/zenodo.8253825>).

473

474 *Author contributions.* Methodology and experiment: WL, SC, JX, SD, PY. Funding acquisition: JX,
475 SC. Original draft preparation: WL, SC. Manuscript review and editing: SC, JX, XY, PY.

476

477 *Competing interests.* The authors declare that they have no conflict of interest.

478

479 **Acknowledgements**

480 This study is supported by funds from Shenzhen Science and Technology Innovation Committee
481 (WDZC20200819105831001), the Guangdong Basic and Applied Basic Research Foundation
482 (2022B1515130006). SC is also supported by the Scientific Research Start-up Fund (QD2021021C).

483

484 **References**

485 Abkar, M., & Porté-Agel, F. A new wind-farm parameterization for large-scale atmo
486 spheric models. *J. Renew. Sustain. Energy*, 7(1), 013121, 2015. DOI:10.1063/1.49076
487 00

488 Archer, C. L., Mirzaeisefat, S., & Lee, S. Quantifying the sensitivity of wind farm
489 performance to array layout options using Large-Eddy Simulation. *Geophys. Res. Lett*
490 *t.*, 40(18), 4963-4970, 2013. DOI:10.1002/grl.50911

491 Barrie, D. B., Kirk-Davidoff, D. B. Weather response to a large wind turbine array.
492 *Atmos. Chem. Phys.*, 10(2), 769-775, 2010. DOI:10.5194/acp-10-769-2010

493 Boezio, G. C., & Ortelli, S. Use of the WRF-DA 3D-var data assimilation system t
494 o obtain wind speed estimates in regular grids from measurements at wind farms in
495 Uruguay. *Data*, 4(4), 142, 2019. DOI:10.3390/DATA4040142

496 Christiansen, M. B., & Hasager, C. B. Using airborne and satellite SAR for wake
497 mapping offshore. *Wind Energ.*, 9(5), 437-455, 2006. DOI:10.1002/we.196



498 Jacondino, W. D., Nascimento, A., Calvetti, L., et al. Hourly day-ahead wind power
499 forecasting at two wind farms in northeast Brazil using WRF model. *Energy*, 230,
500 120841, 2021. DOI:10.1016/j.energy.2021.120841

501 Elshafei, B., Pena, A., Xu, D., et al. A hybrid solution for offshore wind resource
502 assessment from limited onshore measurements. *Appl. Energy*, 298, 117245, 2021. D
503 OI:10.1016/j.apenergy.2021.117245

504 Fiedle, B. H., & Bukovsky, M. S. The effect of a giant wind farm on precipitation
505 in a regional climate model. *Environ. Res. Lett.*, 6(4), 45101, 2011. DOI: 10.1088/
506 1748-9326/6/4/045101

507 Fitch, A. C., Olson, J. B., & Lundquist, J. K. Parameterization of wind farms in cl
508 imate models. *J. Climate*, 26(17), 6439-6458, 2013. DOI:10.1175/JCLI-D-12-00376.1

509 Fouquart, Y., & Bonnel, B. Intercomparing shortwave radiation codes for climate stu
510 dies. *J. Geophys. Res. Atmos.*, 96(D5), 8955-8968, 1991. DOI:10.1029/90JD00290

511 Grell, G. A., et al. A generalized approach to parameterizing convection combining
512 ensemble and data assimilation techniques. *Geophys. Res. Lett.*, 29(14), 38-42, 2002.
513 DOI:10.1029/2002GL015311

514 Hong, S. Y., & Noh, Y. A new vertical diffusion package with an explicit treatmen
515 t of entrainment processes. *Mon. Wea. Rev.*, 134(9), 2318-2341, 2006. DOI:10.1175/
516 MWR3199.1

517 Jensen, N. O. A note on wind generator interaction. Risø National Laboratory, Rosk
518 ilde, 1984:15-20.

519 Katic, I., Højstrup, J., & Jensen, N. O. A simple model for cluster efficiency. In E
520 uropean Wind Energy Association Conference and Exhibition (pp. 407-410). Rome:
521 EWEA, 1986.

522 Mlawer, E. J., Taubman, S. J., Brown, P. D., et al. Radiative transfer for inhomoge
523 neous atmospheres: RRTM, a validated correlated-k model for the longwave. *J. Geo
524 phys. Res. Atmos.*, 102(D14), 16663-16682, 1997. DOI:10.1029/97jd00237

525 Nakanishi, M., et al. Development of an improved turbulence closure model for the
526 atmospheric boundary layer. *J. Meteor. Soc. Japan*, 87(5), 895-912, 2009. DOI:10.21
527 51/jmsj.87.895



528 Pan, Y., & Archer, C. L. A hybrid wind-farm parametrization for mesoscale and cli
529 mate models. *Bound-Layer Meteorol.*, 168(3), 469-495, 2018. DOI:10.1007/s10546-01
530 7-0259-9

531 Pryor, S. C., Shepherd, T. J., Barthelmie, R. J., et al. Wind Farm Wakes Simulated
532 Using WRF. *J. Phys. Conf. Ser.*, 1256(1), 12025, 2019, DOI:10.1088/1742-6596/125
533 6/1/012025.

534 Roy, S. B., Traiteur, Impacts of wind farms on surface air temperatures. *PNAS*, 10
535 7(42), 17899-904, 2010. DOI:10.1073/pnas.1000493107

536 Skamarock, W. C., Klemp, J. B., Dudhia, J., et al. A Description of the Advanced
537 Research WRF Version 3. NCAR Technical Note, 2008. DOI:10.13140/RG.2.1.2310.6
538 645

539 Vautard, R., Thais, F., Tobin, I., et al. Regional climate model simulations indicate
540 limited climatic impacts by operational and planned European wind farms. *Nat. Com
541 mun.*, 5(1), 3196, 2014, DOI:10.1038/ncomms4196.
542