Then, the real numbers $u_{m,j}$ and v_j, with $j \in \{1,\ldots,s\}$ and $m \in \{1,\ldots,r+1\}$, are calculated as a solution of the nonlinear system

$$\begin{cases} \vdots \\ \displaystyle\sum_{m=1}^{r+1} u_{m,j_1} \cdots u_{m,j_\xi} {v_m}^{2-\xi} = \mu_{j_1,\ldots,j_\xi} \\ \vdots \end{cases} \tag{1}$$

with one equation for each $\xi \in \{0,\ldots,h\}$ and for each $j_1,\ldots,j_\xi \in \{1,\ldots,s\}$, for a total of $\left(s^{h+1}-1\right)/(s-1)$ equations. The system represents the matching between the statistical moments of the ensemble and the normalized uncertainty pdf. In fact, its first equation, i.e.,

$$\sum_{m=1}^{r+1} {v_m}^2 = 1,$$

ensures that the weights will sum to 1; the following block of s equations, i.e.,

$$\begin{cases} \displaystyle\sum_{m=1}^{r+1} u_{m,1} v_m = 0 \\ \vdots \\ \displaystyle\sum_{m=1}^{r+1} u_{m,s} v_m = 0, \end{cases}$$

ensures that the ensemble has 0-mean; the third block of s^2 equations, i.e.,

$$\begin{cases} \displaystyle\sum_{m=1}^{r+1} u_{m,1} u_{m,1} = \delta_{1,1} = 1 \\ \vdots \\ \displaystyle\sum_{m=1}^{r+1} u_{m,1} u_{m,s} = \delta_{1,s} = 0 \\ \\ \vdots \\ \\ \displaystyle\sum_{m=1}^{r+1} u_{m,s} u_{m,1} = \delta_{s,1} = 0 \\ \vdots \\ \displaystyle\sum_{m=1}^{r+1} u_{m,s} u_{m,s} = \delta_{s,s} = 1, \end{cases}$$

forces the ensemble covariance matrix to be equal to the identity matrix, and so on and so forth. The general equation reported in system (1) represents all the moment-matching equations up to order h, where $\mu_{j_1,\ldots,j_\xi}$ are the statistical moments of a s-dimensional probability distribution that approximates the assumed error distribution shape. Such probability distribution must be uncorrelated, normalized and have 0 mean, since the ensemble mean and covariance will be later adjusted by the sampling algorithm, which will also rescale the other moments accordingly (see equation (6)).