

We appreciate the reviewer's detailed comments on the manuscript. Those comments has helped improve and clarify the submitted manuscript. Especially, we have added more detailed information on how BUMP NICAS and HDIAG works. Please find our response to each of your comments below.

1. *Page 1, Line 14: "hybrid covariance that combines that used ..." should be "hybrid covariance, which combines that used..."*

Done.

2. *Page 1, Line 19: missing "centers" after numerical weather prediction*

Done.

3. *Page 2, Lines 44-46: What prevents the products of vectors with the univariate spatial correlation matrices from being computed on the native mesh? It is not clear here as to why it is required to compute in a thinned subset of mesh and interpolated to back to full-resolution mesh. Is this related to BUMP NICAS mentioned later?*

Yes, those lines describes the BUMP NICAS procedure mentioned later. If a single (3-d) variable is in a dimension of $[n]$, the univariate spatial correlation matrices would be in a dimension of $[n \times n]$. Construction of $[n \times n]$ matrix and multiplying it with an increment $[n]$ can be too expensive for large systems. Instead, BUMP NICAS constructs the univariate spatial correlation matrix of a subset of mesh $[n_s \times n_s]$ and interpolation from $[n_s]$ to $[n]$, where $n_s \ll n$. This reduces the overall computational cost to multiplying the correlation matrix to an increment. Following reviewer's comment and 10th comment, we have revise the last paragraph of section 3.1 to provide more detailed information on how NICAS works.

4. *Page3, Line 56: "heigh-based" should be "height-based"*

Done.

5. *Page 3, Line 67: "the United Forward Operator" should be "the Unified Forward Operator"*

Done.

6. Page 3, Line 79: “distinguishes” should be “distinguished”

The “distinguishes” looks correct in the originally submitted manuscript.

7. In section 2.3, It is a little difficult to understand the new approach (relative to the previous one used in Liu et al. 2022) as described in the second and third paragraphs. The second paragraph talks about analysis variable change, not increments, however, the third paragraph is all about increments even though the variables being described in the two paragraphs are the same. The most unclear part is from line 92 to line 95. How do p , ρ_d , and θ_d change relative to background forecast even if no observations were assimilated. Why would background forecast change in this case? And how does this new approach contribute to temperature bias reduction in the stratosphere? Assuming the stratospheric temperature bias was an issue in Liu et al. (2022), please provide more context and consider re-write these two paragraphs for more consistency.

Thank you for pointing this out.

[Change from no obs] In the initial implementation of JEDI-MPAS, the major challenge was on how the (3-d) pressure, p , is updated after analyzing surface pressure (p_s). The surface pressure, p_s , is the analysis variable in JEDI-MPAS, but it is a diagnostic variable in MPAS Model, which is extrapolated from p at the two lowest vertical levels. After minimization is done, the p is computed hydrostatically by integrating (analyzed) T , p_s , and q from surface to upper levels, and there happens a non-zero increment for p and consequently for ρ_d and θ_d . This is described as “discretization errors” in the original manuscript, and this non-zero increment is accumulated as the vertical indices increase.

[Benefit of new approach] By using the incremental formula with base state of DA background, if there is no analysis increment from DA, the background p , ρ_d , and θ_d will not change. Also, even with non-zero analysis increment from DA, the error due to the discretization and nonhydrostatic effects will be smaller for p , ρ_d , and θ_d by using the incremental formula. We realized that only the “increments” are mentioned in the third paragraph of section 2.3 and this might make the reader confused.

We have revised the first sentence in the third paragraph of section 2.3 as follows: “**Here, we instead compute increments for p , ρ_d , and θ_d (i.e., δp , $\delta \rho_d$, and $\delta \theta_d$) from the increments δT , δp_s , and δq , by linearizing the corresponding calculations of Liu et al. (2022, steps 3 and 4 of their section 3.3). Then, the full states of p , ρ_d , and θ_d are updated by adding δp , $\delta \rho_d$, and $\delta \theta_d$ to their background fields. If the analysis increment from DA is zero, in this case, the background state will be kept for p , ρ_d , and θ_d .”**

8. *Page 4, Lines 100-101: what is the purpose of mentioning “without a halo region” here? Was it meant to indicate that there are no operations of derivatives or interpolations required for the state(x) and increment (delta x) in JEDI-MPAS as such no halo region is needed? Please explain.*

We’d like to describe technical detail of whether the state and increment objects of JEDI-MPAS contains the halo regions or not. However, the halo exchange should be done when needed, such as horizontal interpolation of state or increment to the observation location, and a linear variation transform from stream function and velocity potential to zonal and meridional winds.

We realized that this might make the readers confusing, so we revised those lines as follows: **“The state and increment objects in JEDI-MPAS only contain their values on own grid point without a halo region. The halo exchange (and its adjoint) is performed when needed, such as horizontal interpolation of state or increment to the observation location and a linear variation transform containing the spatial derivatives.”** in the last paragraph of section 2.3 .

9. *Page 4, Line 106: “of” is missing after “independent”*

Done.

10. *Page 4, Lines 112-113: Please explain in more details on how BUMP models the univariate spatial correlations differently from the recursive filters, which was used in GSI? Perhaps try to make efforts to connect this paragraph with those on the last paragraph (Lines 165-170) of Page 6.*

We have modified the text as **“... except in our use of BUMP-Normalized Interpolated Convolution from on Adaptive Subgrid (NICAS; Ménétrier, 2020), rather than recursive filters, to model the univariate spatial correlations (see further description at the end of this section).”** and provided more information in the last paragraph of section 3.1 as **“... The spatial correlation matrix is pre-computed from the given correlation lengths with BUMP-NICAS. Similar to the GSI recursive filters, NICAS works in the grid-point space. However, it applies the convolution function explicitly, instead of recursively for GSI. Thus, the choice of the convolution function in NICAS is free, as long as it is positive-definite. We choose a widely-used fifth-order piecewise function of Gaspari and Cohn (1999), which resembles the Gaussian function but is compactly supported. To make the explicit convolution affordable for high-dimensional systems, it is actually performed on a low-resolution unstructured**

mesh. A linear interpolation is required from the unstructured mesh to the full model grid. Finally, an exact normalization factor is pre-computed and applied to ensure that the whole NICAS correlation operator is normalized (i.e. diagonal elements of the equivalent correlation matrix are "1"). Thus, the NICAS correlation matrix can be written as: $C=NS\tilde{C}S^T N^T$, where \tilde{C} is the convolution operator on the low-resolution mesh, S is the interpolation from the mesh to the full model grid, and N is the diagonal normalization operator. The low-resolution mesh density can be locally adjusted depending on the diagnosed correlation lengths (or provided by the user)." .

11. Section 3.2: Another concern I have is regarding the parameters of the multivariate background error covariance being diagnosed from NCEP GFS forecasts, which is a different model from MPAS. The authors mentioned in their future work that they plan to use an ensemble from JEDI-MPAS to train the covariance model, but why not use the MPAS forecasts to diagnose these parameters in the first place? Isn't the MPAS forecast a natural and more straightforward choice that can realistically represent the multivariate structure of the forecast errors of MPAS compared to NCEP GFS? In addition, knowing that the covariance is meant to be used for 6-hourly cycling DA (which is typical), why would the author follow the traditional approach and train the covariance model using data from 24 hour forecast differences and then apply a re-scaling factor to address the gap? This begs the question whether the re-scaling could be avoided if the covariance was trained from 6-h differences of MPAS forecasts. In addition, what is the spatial resolution of the NCEP GFS samples?

Thank you for your comment. As you mentioned, it is natural to use MPAS Model's own forecast samples to diagnose the B parameters to consider the MPAS Model's own characteristics. In this initial development and validation work, however, we have wanted to use the pre-existing forecast samples from external model. After submission of this manuscript, we did use the MPAS Model's own forecast samples (still with NMC-type perturbations) to diagnose the B parameters, but with a recent version of JEDI-MPAS source code (early June 2023). To summarize, the overall structures of B parameters (such as regression coefficients, vertical profiles of horizontal- and vertical correlation lengths) diagnosed from MPAS-based samples was similar to that from GFS-based samples. The largest difference was in the error standard deviation parameters, which were in larger values for MPAS-based samples, especially in the upper levels. In one-month cycling experiment, this lead to a reduction in temperature and wind RMSEs in 6 hour forecasts fields in the upper levels. In future efforts of further refinement on the JEDI-MPAS static B, we will definitely use the MPAS Model's own forecast samples (either NMC type or ensemble samples).

To generate the sample error perturbations, the NMC method uses two forecast fields with different forecast lead time, verified at the same valid time. The 24 hour different lead time is typical used in defining NMC samples because we want to remove the effect of diurnal

cycle in the perturbations. If we try the ensemble-based samples to diagnose the B statistics in future, it would be natural to use the 6 hour forecast samples, and the rescaling of error standard deviation parameters might be avoided as the reviewer commented.

The resolution of GFS model itself is approximately 13 km (horizontal) with 64 vertical levels. This 13 km GFS model produces the forecast dataset with 0.25 degree lat/lon grids on the standard pressure levels. For GFS-based samples in this study, the 0.25 degree GFS dataset is interpolated into MPAS 60 km grid (also with a vertical interpolation).

We have added: **“Here, the 24-hr forecast lead time difference is chosen to remove the effect of diurnal cycle in the perturbation samples. The GFS forecast files of 0.25 degree resolution on the pressure levels are interpolated to 60 km MPAS mesh with 55 vertical levels for following training procedures.”** in the first paragraph of section 3.2 .

12. Page 8, Lines 211-219: Does this modification suggest that the assumed correlation function (the fifth-order, compactly supported function from Gaspari and Cohn 1999) is not optimal for stream function and velocity potential, but okay for other variables such as temperature, specific humidity and surface pressure?

We have found the largest discrepancy between assumed and modelled correlation function for stream function and velocity potential, thus we have aimed to tune the horizontal correlation lengths for these two variables. As described in Lines 215-219, reducing the horizontal correlation lengths for stream function and velocity potential has increased the error variances for zonal and meridional wind analysis variables. This has also greatly improved the fit-to-obs for single zonal wind assimilation test. This does not necessarily mean that the use of modelled correlation is okay for other variables, and further efforts are needed to improve the diagnostics of better B parameters.

13. The authors didn't show analysis increments of specific humidity (q) from the two single observation tests. Is it because there is no correlation between q and other analysis variables (meaning q is univariate) so there is zero q increment? In addition, there is no single observation test for observation of q. Is it due to the same reason?

Yes, there are no analysis increments from two single observation tests because there is no correlation between specific humidity (q) and other variables. If we perform a single observation test from a single q observation, the increments will be shown only for q fields (i.e., univariate).

This might be the simplest way of handling the moisture analysis variable in the data assimilation. More sophisticated moisture analysis variable and moisture B variables, for

example, pseudo relative humidity (Dee and da Silva, 2003) or normalized relative humidity (Hólm et al, 2002) will be an important future research topic in JEDI-MPAS. We have added this remark at the last paragraph of section 6 as follows: **“We will also explore more sophisticated moisture variables, such as pseudo relative humidity (Dee and da Silva, 2003) or normalized relative humidity (Hólm et al., 2002).”** .

14. Page 9, Lines 253-257: As stated here, a one-month “cycling” experiments were performed, but at “each” cycle, a 20-member ensemble of 6-hour MPAS forecast was performed using IC from GEFS. Are the experiments fully cycled or partial cycled? The latter one means each cycle always cold-start from GEFS. I would think the experiments are fully cycled, but it is not clear to me which approach was really taken by the authors.

Yes, the experiments are fully cycled for the *deterministic* analysis. However, the 6-hour MPAS ensemble forecasts were performed as a cold-start mode, initialized from GEFS analysis ensemble. Then, the 6-hour MPAS ensemble forecasts have provided the ensemble background covariance at the valid time.

We have added a text “... **to provide the ensemble background error covariance.**”

15. As stated in the conclusions, the formulation of the JEDI-MPAS static B generally follows Wu et al. (2002), but with the novel use of BUMP for multiple elements of the covariance model. Although the BUMP package including VBAL and VAR are introduced to perform variable transforms, these transformations generally follow Wu et al. (2002). The most novel parts of BUMP are perhaps the NICAS and HDIAG drivers that are used to model the univariate correlation in place of recursive filters. The NICAS and HDIAG drivers allow one to specify spatial correlation functions and compute convolutions on (semi-) native mesh, which should be considered an enhancement over recursive filters used in GSI. However, the subsequent modification which halves the diagnosed horizontal correlation length for stream function and velocity potential makes this enhancement less promising. If the specified correlation function (i.e., the fifth-order compactly supported function from GC1999 chosen in this study) leads to results that are far from the sample statistics, then why not make efforts to find a more appropriate correlation function that may be more suitable for certain variables? It gives the impression that the novelty that BUMP brought about was not fully exploited.

One biggest benefit of BUMP NICAS/HDIAG is that it can be applied on an unstructured grid, while other spatial correlation operators, for example, recursive filters (in GSI or WRF DA) or spectral transforms in (SSI or UKMO), require a structured grid, such as (reduced) gaussian grid or orthogonal grid with two directions. The GC1999 function models the compactly supported Gaussian shape with a *single* parameter (i.e., separation distance

from the origin), so it is very efficient both when we diagnose the correlation lengths and when we calculate the convolution.

While choosing the other functions can be an interesting research topic, now NICAS has a more advanced capability to diagnose and to apply the multiple components of GC functions. By superposing the multiple GC functions with different correlation lengths, this capability is expected to fit the raw sample correlation better. This has not yet been tested in a large dataset, and we'd like to test this with JEDI-MPAS in near future and will see if it resolves the issue of additional tuning. This was briefly mention at the last sentence of section 6 in the original manuscript.

16. Figure 1 Caption: regression coefficients of (a) delta T, (b) delta phi, and (c) delta kai are the nonzero elements at this mesh cell of the submatrices should correspond to M, L, and N, respectively..., not L, M and N, according to Equation (4).

Thank you for finding this. It is fixed.

17. Figure 6.: "using the length scale the gives the best fit" should be "using the length scale that gives the best fit"

Done.

18. Figure 7: Please draw the reduced horizontal correlation length in a different color to show the effect of the additional modifications made to the raw statistics.

Done.

19. Figures 7-9: Please include variable units.

Variable units are added in the color bars and figure captions.

Reference

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Gaspari, G. and Cohn, S. E.: Construction of correlation functions in two and three dimensions, *Quarterly Journal of the Royal Meteorological Society*, 125, 723–757, <https://doi.org/https://doi.org/10.1002/qj.49712555417>, 1999.

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