

## [Response to Reviewer's Comments]

We are very grateful to the reviewer for her/his careful reviews and kindly giving us valuable and constructive comments and suggestions that we have generally accepted. Here, we provide our point-by-point responses to each of the comments. The revisions are highlighted by red in the revised manuscript. Supplemental PDF file would be useful to check revisions and their corresponding comments.

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### [Reviewer RC1]

(1) I think this is an interesting article, which shows the potential of particle filters (and particularly two flavours of localised particle filters) in large scale atmospheric applications. The authors build on an existing LETKF computational framework to implement and experiment with these localised particle filters. The implementation is very nicely detailed with schematics and direct comparisons of the algorithms, including step-by-step contrasts, and quantifying the computational costs. I have some minor corrections and questions I would like to have answered before I can recommend this article for publication.

Response: Thank you very much for your encouraging comment. We have revised the manuscript accordingly following the suggestions.

(2) Short Summary: I would eliminate 'millions'.

Response: Revised (P1L33)

(3) Cite van Leeuwen 2021 (a consistent interpretation of the stochastic EnKF) when discussing perturbed observations.

Response: Revised (P2L39)

(4) Cite Wang et al 2004 next to Bishop 2001. After all, it is the symmetric transform variant which is used in practice.

Response: Revised (P2L50)

(5) Line 62. Even PFs with proposal densities collapse, although at a slower rate.

Response: Revised (P2L64)

(6) Line 69. How do local PFs ensure continuity between particles in different regions? Maybe mention something brief on this regard.

Response: We added discussion on this point (P5L122, P16L489)

(7) Lines 96-97. The sentence 'Transform matrix...' could be rephrased to avoid word repetition.

Response: Revised (P4L98).

(8) Line 147. Why write  $H(x)-y$  in the Gaussian likelihood. You had used  $y-H(x)$  when describing the EnKF. It is better to keep consistency in the paper.

Response: Revised as suggested (Eqs. 12 and 15)

(9) Line 205. Can there be a similar 'adaptive inflation' as Miyoshi's (and previously Anderson's) for the weights of the LPF?

Response: We added discussion on this point in section 4.3.1 (P16L472)

(10) Line 225. Does the LPFGM reduce to the LPF when  $\gamma \rightarrow 0$ ?

Response: Revised (P9L235).

(11) Figure 2 is really nice an illustrative of the differences between the LPF and the LPFGM.

Response: Thank you for your comment.

(12) Line 298. There is an incomplete sentence starting with 'Although... '

Response: We are sorry the original manuscript contained this typo. This sentence was deleted in the revised manuscript (P11L318).

(13) Line 305. Are these identical twin or fraternal twin experiments? I.e., is there model error?

Response: They are identical twin experiments. We added a description (P12L324)

(14) Line 325. Why was this specific variable chosen to tune the localisation scales?

Response: We added descriptions (P11L341).

(15) Line 330. Why was  $T$  chosen? I would think that, if you want to exploit the PF advantage, you would choose a variable like relative humidity.

Response: We added explanations on this point (P13L351).

(16) Line 380. In the regular PF, the number of particles to avoid filter collapse has to be of the order  $\exp(N_{\text{eff}}^2)$ , where  $N_{\text{eff}}$  is the effective size of the problem, according to Snyder et al 2008. Then van Leeuwen and Ades (2013) showed that  $N_{\text{eff}}$  is proportional to the number of independent observations. I therefore thought that filter collapse would happen with the sparse observation network. It is clear that filter collapse and filter divergence are different then. Could you explain the difference and the mechanisms leading to them?

Response: We added discussion on this point (P14L404).

(17) Line 390. Why does frequent resampling lead to filter divergence? Does this have to do with the space-wise continuity of the fields not being ensured?

Response: Thank you very much for the valuable comment. While we conducted further investigations, we could not find a clear reason for the filter divergence. We added related discussions in the revised manuscript (P15L422 and section 4.3.2).