We sincerely appreciate the reviewer for her/his constructive comments on the manuscript. Our responses are listed as follows in blue. Text is revised accordingly.

Review from Referee #2

The authors present a new numerical method to calculate sea spray induced heat fluxes given a sea spray generation function. Specifically, the authors propose the Gaussian Quadrature (GQ) method as a computationally efficient alternative to the spectrum integral method, and more accurate method than simplifications of the integral method. The results show improved agreement of the GQ method with the spectrum integral in comparison to the simplified parameterization. Impact of such difference in the context of world's oceans are presented using a coupled ocean-atmospheric-wave model.
Overall, the manuscript is interesting and the modelling community may benefit significantly from the proposed GQ-method. However, the authors are not clear in the type of advancement and implications their work has on current and future works. The method does not improve the physics of sea spray induced heat fluxes, instead it improves the numerical accuracy of solving the involved equations. While I nevertheless believe this is important work worthy to be published, it is an important distinction that is currently not appropriately phrased nor discussed. I therefore recommend a major revision.

Major concerns

As per the above, I believe that the authors need to be more explicit in what their method is improving. I think that the authors haven't necessarily overstated their conclusions, but the interpretations of some statements are right now too ambiguous or sometimes incorrect. This largely involves the word 'improved' throughout the manuscript, including the title and abstract, where it reads as if the parameterizations are improved. This is not really true, it is actually the numerical method used to solve the parameterizations that has been improved (at least against the simplified parameterization discussed in the manuscript). Thus, when these different methods are compared when implemented in the applied coupled model, the authors are not improving the bulk parameterizations, but simply presenting the numerical error of the methods discussed in the context of the coupled model. Importantly, it is the numerical error assuming that the sea spray generation function is correct.
Response: As suggested, we revise the title, the abstract and the text to focus on the improvement of the numerical method. The title is revised to "Accelerated Estimation of Sea Spray-Mediated Heat Flux Using Gaussian Quadrature: Case Studies with a Coupled CFSv2.0-WW3 System". For the result comparison in the context of the coupled system, we also clarify that we are not improving the bulk parameterization, but presenting the numerical errors of the methods.

A related point is the absence of a clear discussion on the interpretation of the results. As mentioned by the authors, the sea spray generation function has an uncertainty of several orders of magnitude. Thus, if the RMSE of the proposed GQ method reduces the numerical error by say 1-10 W/m^2 (e.g., figs 3-5), how relevant is such an improvement in the broader context of the physics and model *uncertainty? Such uncertainty in the physics could perhaps get into the 100s of W/m^2. I'm not suggesting the GQ method proposed here is therefore irrelevant, but it does change the interpretation and application of the model/results in practice, for now and the future. This also brings up some other questions regarding the interpretation of the improved SST as observed in Figs 6 and 7. The uncertainty in the physics of sea spray is considerable larger than the improvement in approximating the spectral integral using the GQ method. Thus, any improvements in the modelled SST cannot be reliably be assigned to the usage of the GQ method.*

Response: We apologize for the unclear writing. Although the sea spray generation function (SSGF) has an uncertainty of several orders of magnitude, the sea spray-mediated heat fluxes in A92 have been tuned by non-negative constants based on observations and the COARE algorithm to reduce the uncertainties (Andreas and Decosmo, 2002; Andreas et al., 2008; Andreas et al., 2015; Andreas, 2003). In this study, we use the constants (Eqn. A7-A8 in Appendix A) for the SSGF (Fairall et al., 1994) to get a mean bias of 3.70 and 0.095 W/m^2 for latent and sensible heat flux respectively in A92 compared to observations (Andreas et al., 2015). Therefore, a few W/m^2 improvements of numerical errors in this study are relevant. Even though, we agree that the improved SST and other variables cannot be reliably assigned to the usage of the GQ method, due to the uncertainties of the coupled model itself and SSGF. A discussion about the uncertainty as suggested is added to clarify.

Minor comments:

Line 66: 'huge amount' sounds a bit vague. Especially later on the authors actually provide a number, so they can be more accurate here.

Response: To be more accurate, we calculate the runtime of CFSv2.0-WW3 global experiments for 7-day forecast with different parameterizations (Table R1). The text is revised as suggested.

Table R1. The runtime of CFSv2.0-WW3 global experiments for 7-day forecast with different parameterizations.

7-day Forecast	Runtime (h)
SPRAY-A92	126.94
SPRAY-A15	7.60
SPRAY-GQ	7.67

Line 70: 'apt to produce significant bias', more a numerical error. However, as per one of my major concerns, is this bias/error of significance in the context of the existing uncertainty in sea spray parameterizations?

Response: Considered that the A92 has a mean bias of 3.70 and 0.095 W/m^2 for latent and sensible heat flux compared to observations (Andreas et al., 2015), the numerical error here is significant. Text is revised to clarify.

Line 86: 'thus provided reliable hourly estimates', is very vague. Either provide a reference about its reliability, or simply state that this dataset is used.

Response: As suggested, the text is revised to state that this dataset is used.

Line 100: This sentence reads a bit odd. 'based on eddy correlation observations' refers to the

turbulent heat fluxes in cited papers, not the sea spray induced heat flux.
Response: We agree. The text is revised to "Based on observations of turbulent heat fluxes and the COARE algorithm".

Line 108: 'requires huge amount', just say is computationally expensive.
Response: Corrected. Thanks.

Line 123-124: somewhat confusing, please rephrase sentence.
Response: The text is revised to "Since the sea spray-mediated heat flux is not sensitive to salinity (Fig. 1e&f) and only monthly observation data is available, the ESA monthly salinity is applied".

166: '36 times' this is an interesting and useful statistic. Perhaps consider to provide what this means in terms of the fully coupled model run, i.e., is it still saving much time percentage wise?
Response: As suggested, we test the runtime of the fully coupled experiments for 7-day forecast (Table R1). The runtime of SPRAY-GQ experiment is about 17 times less than the runtime of SPRAY-A92 experiment. Text is revised accordingly.

Line 201-202: confusing sentence, please rephrase.
Response: The text is revised to "The increased (decreased) SSTs north (south) of 50°S in SPRAY-GQ compared to those in SPRAY-A15 (Fig. 6b) reduce the RMSE of SST in SPRAY-GQ".

Line 204: 'fig 12g' best practice is to keep order of figures intact.
Response: Corrected. Thanks.

Line 233: 'significant improvements', I disagree. Fig. 8c and 9c seem to show very similar RMSE. Their variability after day 18 is very similar.
Response: The sentence is revised. The differences of WSP10 RMSEs between SPRAY-GQ (black) and SPRAY-A15 (red) are very small in the first two weeks. Afterwards, the mean RMSE in SPRAY-GQ is lower than that in SPRAY-A15 significantly at 95% confidence level in both boreal winter (Fig. 8c) and boreal summer (Fig. 9c).

Line 260: This manuscript would greatly benefit from a separate and in-depth discussion on the interpretation and implications of the results.
Response: Thanks. The following discussion is added in the text.
When wind speed is larger than 10 m/s, spray-mediated heat flux can become as important as the interfacial heat flux (Andreas and Decosmo, 1999, 2002). Particularly, even in the absence of air-sea temperature difference, the spray-mediated sensible heat flux is still present (Andreas et al., 2008). As indicated by previous studies (e.g., Garg et al., 2018; Song et al. 2022), it is necessary to superimpose the spray-mediated heat flux on the bulk formula to complete the physics of turbulent heat transfer for coupled simulation. Since the full microphysical parameterization (A92) is computationally expensive, an efficient algorithm that captures the main features of A92 can be beneficial to large-scale climate systems or operational storm models. The GQ method proposed in the study can efficiently calculate the spray-mediated heat flux, and agree better with A92 than A15. Thus, the GQ method has a great potential to be applied in large-scale climate systems and

operational storm models.

References

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