



1 **AutoQS v1: Automatic parameterization of QuickSampling** 2 **based on training images analysis**

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9 10 **Highlights**

- 11 • Adaptative calibration as a function of the simulation progression
- 12 • Calibration depends only on the training image
- 13 • Robust parameterization based on a rapid prior analysis of the training image

14 **Abstract.** Multiple-point geostatistics are widely used to simulate complex spatial structures based on a training
15 image. The use of these methods relies on the possibility of finding optimal training images and parametrization
16 of the simulation algorithms. While methods for selecting training images are available, parametrization can be
17 cumbersome. Here, we propose finding an optimal set of parameters using only the training image as input. The
18 difference between this and previous work that used parametrization optimization is that it does not require the
19 definition of an objective function. It is based on the analysis of the errors that occur when filling artificially
20 constructed patterns that have been borrowed from the training image. The main advantage of our approach is to
21 remove the risk of overfitting an objective function, which may result in underestimating the variance or in a
22 verbatim copy of the training image. Since it is not based on optimization, our approach finds a set of acceptable
23 parameters in a predictable manner by using the knowledge and understanding of how the algorithms work. The
24 technique is explored in the context of the recently developed QuickSampling algorithm, but it can be easily
25 adapted to other pixel-based multiple-point statistics algorithms using pattern matching, such as Direct Sampling
26 or Single Normal Equation Simulation (SNESIM).

27 28 **1 Introduction**

29 Geostatistics is extensively used in natural sciences to map spatial variables such as surface properties (e.g., soils,
30 geomorphology, meteorology) and subsurface geological features. Its main applications involve the estimation
31 and simulation of natural phenomena. In this paper, we focus on simulation approaches.

32 Traditional two-point geostatistical simulations preserve the histogram and variogram inferred from point data
33 (Matheron, 1973). However, inherent limitations make the reproduction of complex structures difficult. Multiple-
34 point statistics (MPS), by accounting for more complex relations, enables the reproduction of such complex
35 structures (Guardiano and Srivastava, 1993). However, MPS has its own limitations (Mariethoz and Caers, 2014).

36 To perform satisfactorily, MPS algorithms require analog images (called training images) and appropriate



37 parametrization. Training images can often be provided by expert knowledge. Indeed, the training image is related
38 to the property that is being simulated, and therefore it is common to all MPS algorithms. In addition, several
39 methods have been proposed to automatically select an appropriate training image among a set of candidates (Pérez
40 et al., 2014; Abdollahifard et al., 2019). However, the parametrization of an MPS algorithm depends not only on
41 the chosen training image but also on the specifics of the algorithm. This makes the task of finding good
42 parametrization cumbersome, and therefore, users often have to resort to a trial-and-error approaches (Meerschman
43 et al., 2013).

44 Over the last few years, several studies have addressed the challenge of finding good MPS parameters. These can
45 be categorized into two different philosophies. The first approach is focused on the “simulation grid”, which
46 assumes that a parametrization is related to the simulation grid, the training image and the MPS algorithm.
47 Dagan et al. (2018) proposed using the known hard data from the simulation grid as a reference to compute
48 statistical metrics and then trying to improve the parametrization through a simulated annealing optimization
49 process until the metrics matched as closely as possible. The second approach is focused on the “training image”
50 and assumes that the parametrization is only related to the training image and MPS algorithm. Along these lines,
51 Baninajar et al. (2019) proposed the MPS Automatic Parameter Optimizer (MPS-APO) method based on the cross-
52 validation of the training image (TI) to quantify simulation quality and CPU cost. In this approach, artificially
53 generated gaps in the high gradient areas of the training image are created, and MPS algorithms are used to
54 simulate the gaps. The performance of a particular parameterization is quantified by assessing the correspondence
55 between the filled and original training data. By design, this approach is extremely interesting for gap-filling
56 problems. The authors state that it can be used for the parametrization of unconditional simulations; however, the
57 use of limited gaps cannot guarantee the reproduction of long-range dependencies. Furthermore, due to the design
58 of the framework for generating gaps, each MPS algorithm needs to be able to handle gap-filling problems for the
59 error to be estimated properly.

60 If both approaches show good results, then they are both related to optimization methods, and therefore, the user
61 has no control over the duration of the optimization process. Furthermore, an objective function is needed. Finding
62 this objective function is a challenge in itself because it can change depending on the training image used. Using
63 optimization approaches, many metrics can be accounted for in the objective function, such as histogram,
64 variogram, pattern histogram, connectivity function, Euler characteristic, etc., (Boisvert et al., 2010; Renard and
65 Allard, 2013; Tan et al., 2013) or a weighted combination of these. Similarly, one has to define the meta-
66 parameters linked to the optimization algorithm itself, such as the cooling rate in simulated annealing or
67 maximum number of iterations. As a result, MPS parameter optimization approaches tend to be complex and
68 difficult to use.

69 In this contribution, we propose skipping the complexity of an optimization algorithm and instead simplifying the
70 optimization procedure to a key element: the simulation of a single pixel. The underlying principle of our approach
71 is that a sequence of well-simulated pixels converges to a good simulation overall. Therefore, the goal is to find
72 the optimal parameters to simplify the simulation of a single pixel using the training image as the only reference.
73 Baninajar et al. (2019) showed that computing the prediction error (i.e., the error between the simulation and the
74 reference) is an appropriate metric to find optimal parameters. Following this approach, we propose exhaustively
75 exploring the parameter space by performing pixel predictions over patterns extracted from the training image,



76 and compute the associated prediction error. This results in a prediction error map for each combination of
77 parameters.

78 The remainder of this paper is structured as follows: Section 2 presents the proposed method. Section 3 evaluates
79 the approach in terms of quantitative and qualitative metrics. Section 4 discusses the strengths and weaknesses of
80 the proposed approach and presents the conclusions of this work.

81 **2 Challenges related to inappropriate parameters**

82 The hypothesis behind multiple point simulation is that the neighborhood of a given pixel x (the pattern generated
83 by known or previously simulated pixels) is informative enough to constrain the probability density function of
84 the value $Z(x)$. Therefore, it requires a training image with enough repetition of the pattern (large enough) to allow
85 the computation of such a conditional probability distribution. The Extended Normal Equation Simulation
86 (ENESIM) (Guardiano and Srivastava, 1993) algorithm computes this distribution for the simulation of each pixel,
87 therefore ensuring such a property. To provide a similar guarantee, the SNESIM (Strebelle, 2002) algorithm and
88 the Improved Multiple-Point Parallel Algorithm using a List Approach (IMAPLA) (Straubhaar et al., 2011),
89 include a parameter to define a minimum number of replicates. Direct Sampling (DS) (Mariethoz et al., 2010)
90 adopts a different strategy by allowing for the interrupted exploration of the training image. It includes a distance
91 threshold parameter that defines what is an acceptable match for a neighborhood; however, too small a threshold
92 typically results in a verbatim copy of the training image.

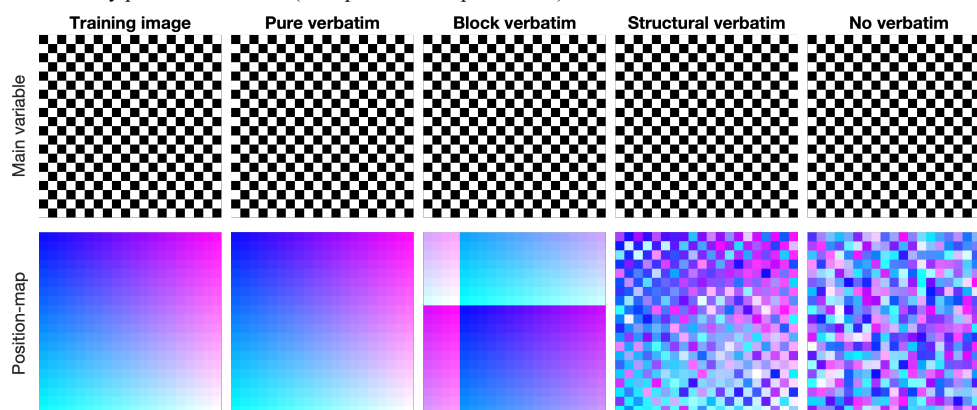
93 To reduce this issue, a maximal fraction of the explored training image is introduced, which is also called the
94 exploration ratio. Since QuickSampling (QS) (Gravey and Mariethoz, 2020) also suffers from the verbatim copy
95 issue with the use of the number of candidates being $k=1$, the authors allow and recommend the use of $k>1$, as k
96 is similar to the number of replicates in SNESIM or IMPALA. A value $k=1.5$ in QS can be visualized as SNESIM
97 with a minimum number of replicates of 1 for 50% of the time and 2 for the remaining 50% of the time.

98 The phenomenon of verbatim copy is the complete pasting of a section from the training image to the simulation
99 (an unintentionally similar process as that in patch-based approaches (Rezaee et al., 2013)). This means that the
100 relative position of the simulated pixels is the same as that in the training image. This occurs when the
101 neighborhood constraints on the simulated pixels are too strong and only the exact same patterns as those in the
102 training image are acceptable. To detect this issue, a common strategy is to create a position map (similar to the
103 index map), which represents the provenance of simulated values by mapping their original coordinates in the
104 training image, as shown in **Figure 1**.

105 Verbatim copy can appear in many forms; **Figure 1** shows the most common ones. The pure verbatim (the most
106 common type of verbatim copy) is a simple copy of the image, with all pixels in the same order inside of the
107 patches. Block verbatim typically appears when there are many replicates of a very specific type of pattern in the
108 training image and few replicates of all other patterns. Therefore, the MPS algorithm is forced to switch at the
109 position of this common pattern. Structural verbatim is an example of pure verbatim over the white pixels.
110 Structural verbatim tends to appear when large-scale structures are unique in the training image, which often
111 allows a visually satisfying image to be quickly obtained. Often, users are ready to sacrifice verbatim on large-
112 scale structures, but this can easily introduce bias, which is one of the hardest types of verbatim to detect. This can
113 typically occur when the maximum neighborhood radius is too large. We then perfectly duplicate the large and



114 initial structure. Finally, no verbatim, which is the expected result of simulations, is when the position pixel does
115 not have any particular relations (their position is unpredictable).



116
117 **Figure 1** Visualization of verbatim copies using a position map. This is an extreme case that highlights that verbatim is
118 not defined by the value simulated but by the source of the value.

119 3 Method

120 The objective of the presented approach is to find an optimal set of parameters using only the training image and
121 knowledge of the mechanics of the simulation algorithm as information. The simulation algorithm is not used in
122 this context; in fact, simulations are not required to obtain a proper calibration. The main target application of the
123 presented approach is the pattern matching simulation algorithm QuickSampling (QS), where the values, at a pixel
124 scale, are directly sampled from the training image. The method is suitable for the simulation of continuous and/or
125 categorical variables. Binary variables are a particular case of continuous and categorical variables.

126 Simulation algorithms, such as QS, can be summarized by Algorithm 1. The key operation occurs at Line 3, which
127 is when the algorithm searches for an optimal match based on the neighboring conditioning data.

128 Algorithm 1 The QS algorithm

129
130 Inputs:
131 T : training images
132 S : simulation grid, including the conditioning data
133 P : simulation path
134 θ : parametrization (including n : number of neighbors)
135
136 1. **For** each unsimulated pixel x following the path P :
137 2. Find the neighborhood $N(x)$ in S composed of the $n(\theta)$ closest neighbors
138 3. Find a candidate in T those matches $N(x)$ using θ
139 4. Assign the value A of the selected candidate to x in S
140 5. **End**

141

142 Here, we propose applying a divide and conquer approach by dividing any pixel-based sequential simulation into
143 its atomic operation: the simulation of a single pixel. We assume that if all pixels are perfectly simulated, then the



144 resulting simulation should also be good. A perfectly simulated pixel is a pixel that respects the conditional
145 probability distribution.

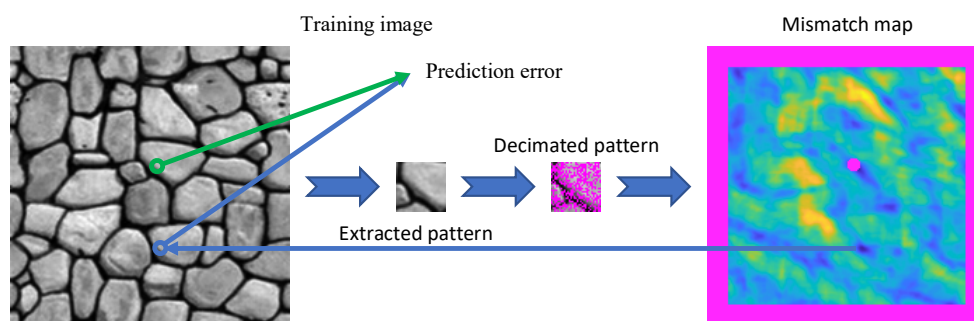
146 Considering the simulation of a pixel, many values can potentially be valid.

$$147 \quad |\{A|P(A|N(x)) > 0\}| \geq 1 \quad (1)$$

148 where $|\cdot|$ represents the cardinality of a set. $P(A|N(x))$ denotes the probability of A (a given value) knowing
149 $N(x)$, the neighborhood. Each possibility will still respect the probability distribution.

150 The proposed approach consists of finding a set of parameters that results in accurate samples for each pattern. An
151 extreme and undesired situation occurs from the simulation of a value that came from the sampling of perfect
152 matches (the neighborhood is available in the training image), which results in a simulation identical to the training
153 image and therefore constitutes verbatim copy.

154 The search for the optimal parametrization is carried out by exhaustive exploration (Algorithm 2). The prediction
155 error is computed, which is the difference between the original value of the pattern and the value of the selected
156 training image pattern (Figure 2).



157

158 **Figure 2** All steps for a single pattern, summarizing Algorithm 2, Lines 2-4.

159 The proposed algorithm explores a discretized parameter space θ (Algorithm 2, Line 1). While this discretization
160 is natural for some parameters, such as n , which is an integer, it can require an explicit discretization for other
161 parameters, such as the kernel in QS (or potentially the th in DS). Furthermore, a key component of our method
162 is the exploration of the parameter space for several representative stages D of the simulation (Algorithm 2, Line
163 1). In the case of a random path, the progression of a simulation is directly related to the density of the
164 neighborhoods, therefore D represents the density of a neighborhood. For each combination D and θ , multiple
165 measures over a set of random locations \mathcal{V} ($500 < |\mathcal{V}| < 10000$) occur at Lines 1-5 in Algorithm 2 and their
166 mathematical expression is shown in Equation 2,

167 **Algorithm 2**



168 Inputs:
 169 List of D , and list of θ
 170 T the training image

171 1. **For** each possible combination of D and θ **do** for all $\nu \in \mathcal{V}$ (\mathcal{V} is randomly generated):
 172 2. Sample a neighborhood $N(\nu)$ from T respecting D
 173 3. Using θ , find a candidate in T that matches $N(\nu)$, excluding for ν itself
 174 4. Compute the error ε between the selected candidate and $Z(\nu)$
 175 5. **End**
 176 6. Analyze the errors ε to determine the best θ for each D .

$$177 \quad \varepsilon(\theta, D, T) = \sqrt{\frac{1}{|\mathcal{V}|} \sum_{\nu \in \mathcal{V}} \left(Z(\nu) - Z\left(\text{Cand}(\theta, N(\nu, D))\right) \right)^2} \quad (2)$$

178
 179 where $\text{Cand}(\theta, N)$ returns a single candidate position for a given neighborhood N and follows the parametrization
 180 θ . $N(\nu, D)$ denotes a decimated neighborhood around ν that respects the condition D . \mathcal{V} represents a random set
 181 of positions in the training image, and $Z(\nu)$ refers to the actual value at position $\nu \in \mathcal{V}$ in the training image.
 182 Finally, for each stage considered, the set of parameters with the minimum associated error ε is considered optimal
 183 (in Algorithm 2, Line 6).

$$184 \quad \varepsilon(\theta_{\text{optimal}}, D, T) = \min_{\theta} \varepsilon(\theta, D, T) \quad (3)$$

185 To avoid over-constrained situations from generating a verbatim copy of the training image, the position ν and its
 186 direct neighbors (in a small radius, usually around 5 pixels, but can be increased depending on the small scale
 187 structure of the training image) are removed from the set of potential candidates. Furthermore, in the case of
 188 equality between several optimal options, we propose the simple rules of taking the cheapest parameter set in terms
 189 of computational cost (e.g., the smallest n).

190 3.1 An efficient implementation

191 In practice, the implementation of Algorithm 2 divides θ into two subsets of parameters: θ_h and θ_s . θ_h contains
 192 all the parameters that affect the computation of the match of a single pattern. This is dependent on the algorithm;
 193 in the case of QS, these are the number of neighbors n , and the kernel (in DS, it would be th , the threshold, and
 194 n). θ_s includes the parameters related to the sampling process of the training image. In the case of QS, these are
 195 the number of matches to retain k , the number of candidates (and in the case of DS, f , which is the fraction of the
 196 training image that is scanned). Interestingly, we can precompute and store all matches for a given
 197 parameterization θ_h . Then, the saved matches of θ_h can be used to quickly measure all possibilities for the
 198 parameters in $\theta = \theta_h + \theta_s$. This two-step approach allows to significantly reduce redundant computations. It is
 199 possible to further accelerate this algorithm by aborting the estimation of ε if the error remains high after having
 200 tested only a small number (at least 500) of samples from \mathcal{V} . To carry out this last operation efficiently, the
 201 algorithm increases \mathcal{V} for the parameter combinations of interest. At each increase step, it checks if more
 202 computations are needed. The following rules proved a good trade-off:



$$\begin{aligned} \varepsilon(\theta, D, T) - \frac{1}{2} \sigma(\theta, D, T) &> \varepsilon(\theta_{min}, D, T) + \frac{1}{2} \sigma(\theta_{min}, D, T) \\ \varepsilon(\theta_{min}, D, T) &= \min_{\theta} \varepsilon(\theta, D, T) \end{aligned}$$

203

$$\sigma(\theta, D, T) = \sqrt{\frac{1}{|\mathcal{V}|} \sum_{v \in \mathcal{V}} \left(\left(Z(v) - Z\left(\text{Cand}_{T \setminus \{v\}}(\theta, N(v, D))\right) \right) - \varepsilon(\theta, D, T) \right)^2} \quad (4)$$

204 With $\varepsilon(\cdot)$ the error, and $\sigma(\cdot)$ the standard deviation. Therefore, a given parametrization is only further explored if
205 the error is a range of a σ .

206 4 Result

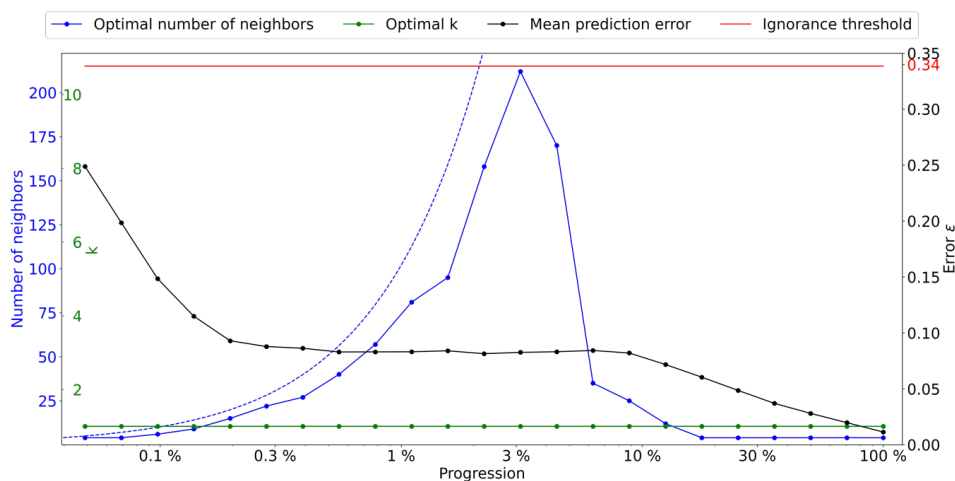
207 All experimental tests in this section are performed using the training image shown in Figure 2. The stages D are
208 distributed following a logarithmic scale. Experimentation shows that the nodes simulated in the initial stages of
209 the path are critical for the overall simulation

210 4.1 Automatic calibration for QS

211 In the case of QS, the method finds optimal values for k the number of candidates, n the number of neighbors and
212 ω the kernel.

213 4.1.1 Automatic calibration for QS with a uniform kernel

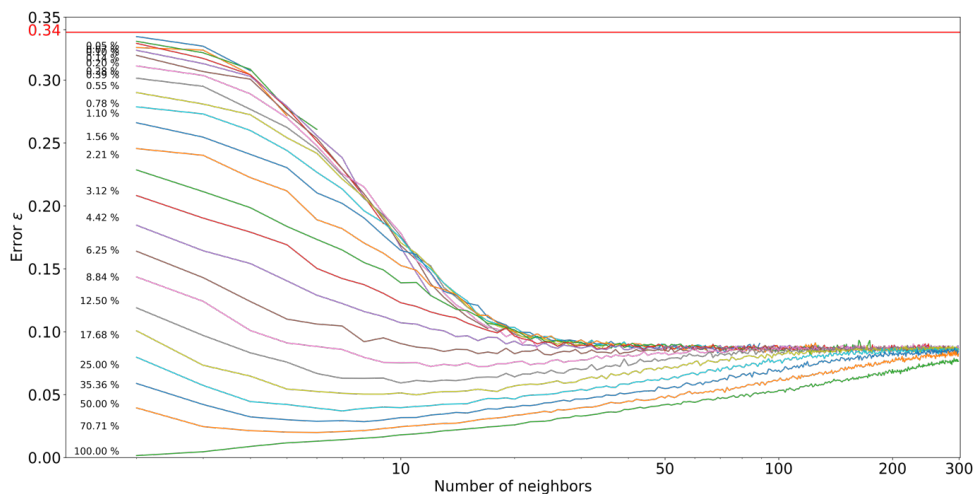
214 In this first test, we use the configuration $\theta_n = \{n\}$ and $\theta_s = \{k\}$. The results are shown in Figure 3. It shows the
215 optimal number of candidates k and number of neighbors as a function of the simulation progress (equivalent to
216 the neighborhood density D). The ignorance threshold is defined by computing the average error between elements
217 of the marginal distribution. It represents the error value beyond which no information is extracted from the
218 neighborhood, stage where the simulated value can then be drawn from the marginal distribution without
219 introducing bias.



220

221 **Figure 3** Optimal parameters for QS (k in green and number of neighbors in blue) as a function of the progression,
 222 with the associated prediction error (in black). The red line represents the ignorance threshold. The dashed blue line
 223 indicates the average density for the neighborhood considered.

224 The optimal k remains small throughout the simulation because the training image is not sufficiently repetitive
 225 (large). At early stages of the simulation, it seems important to use many neighbors. The number of neighbors
 226 increases until approximately 3% of the simulation, followed by a subsequent drastic reduction. This tends to
 227 indicate that once the large structures are informed, only the few direct neighbors are important. We also note that
 228 even if the parametrization is logical, it is generally difficult to predict. This indicates that the use of a single
 229 parametrization for the entire MPS simulation is generally suboptimal. Figure 3 also shows that the first few
 230 simulated pixels are hardly predictable (close to the ignorance threshold).



231

232 **Figure 4** Error as a function of the number of neighbors, with $k=1$,
 233 where each curve represents the associated density of the neighborhood D (which is equivalent to the fraction of the
 234 simulation path).

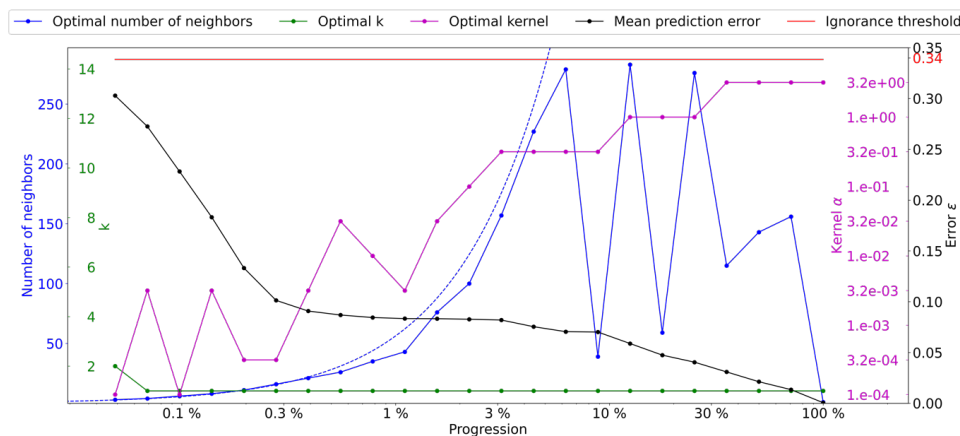
235 Figure 4 shows the evolution of ϵ as a function of the number of neighbors in the simulation stage. This indicates
 236 that two regimes exist in the simulation. In the first percentages of the simulation, an optimal prediction can be



237 obtained even with a small number of neighbors. However, as the neighborhoods become denser, the importance
 238 of spatial continuity takes over. This two-step process is expected, as random large-scale features are generated
 239 first; then, in a second step, the MPS algorithm fills the image with a consistent fine-scale structures. Furthermore,
 240 it shows that using a large number of neighbors at the end of the simulation generates suboptimal results, which
 241 could explain the small-scale noise that is sometimes visible in some MPS simulations.
 242

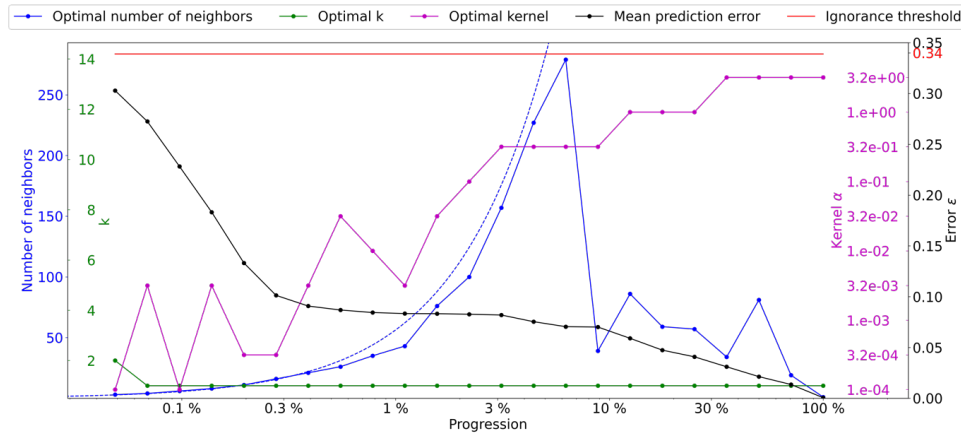
243 4.1.2 Automatic calibration of QS considering different kernels

244 Here, we use the following configuration $\theta_n = \{n, \omega\}$ and $\theta_s = \{k\}$. We consider kernels as having a radial
 245 exponential shape, $w_i = e^{-\alpha \cdot d_i}$.



246 **Figure 5** Optimal parameters for QS (k in green, number of neighbors in blue, and best kernel in magenta) as a function
 247 of the progression, with the associated prediction error (in black). The dashed blue line indicates the average density
 248 for the neighborhood considered.
 249

250 Figure 5 shows the evolution of the QS parameters, where interferences between the number of neighbors and
 251 skewed kernels (high α) are visible. This interaction can be explained by the fact that the last neighbors will receive
 252 negligible weights with large α values, and thus n becomes insensitive. In that case, the differences between
 253 possible configurations are negligible, with random noise in the metric. As expressed in the methodology section,
 254 in cases of a similar error, the cheapest solution is considered. In the case of QS, having a large number of
 255 neighbors can marginally increase the computational time; therefore, we introduce a small tolerance that results in
 256 favoring small n values. This tolerance is introduced as a small extra cost for each extra neighbor, for example
 257 adding $5e-5$ for each extra neighbor. When the gain during simulations was limited, up to a 10% computational
 258 gain was observed using Equation 4.
 259

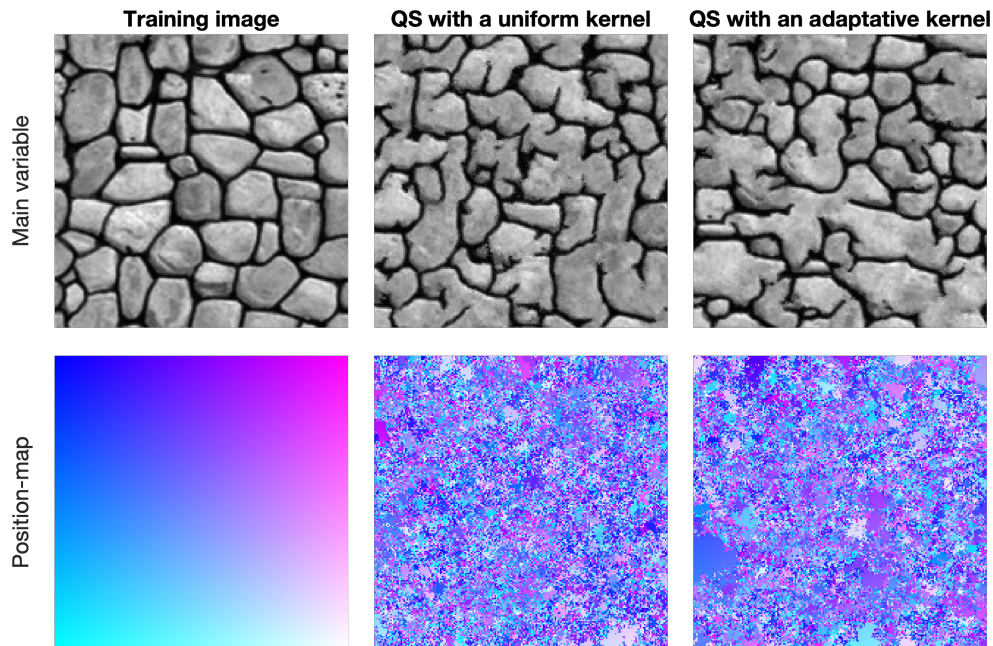


260

261 **Figure 6 Optimal parameters for QS (k in green and number of neighbors in blue, and the best kernel in magenta)**
 262 **as a function of the progression, with the associated prediction error (in black). The dashed blue line is the average**
 263 **density for the neighborhood considered.**

264 Figure 6 shows similar quality (ϵ curves) as Figure 5. However, the number of neighbors required during the
 265 simulation drastically decreases as advanced simulation stages, and the wild oscillations are avoided.

266 **4.2 Sequential simulation using automatic calibration**



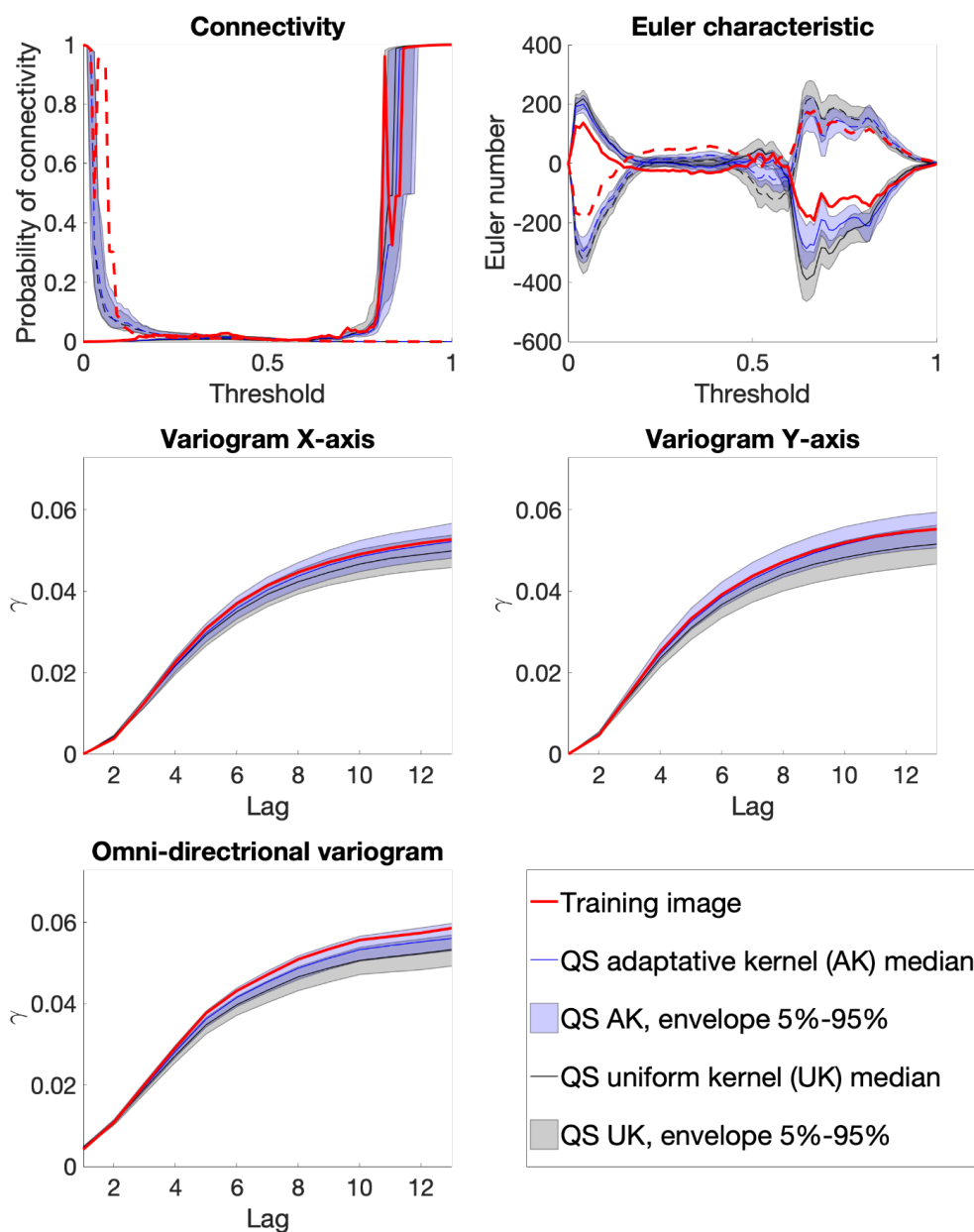
267

268 **Figure 7 Simulation using QS with parameters generated by the automatic calibration.**

269 Figure 7 shows qualitative results using the evolutive parametrization resulting of the proposed autocalibration.
 270 QS with kernel refers to the use of different values of alpha for the kernel. In this case, the results are similar to
 271 state-of-the-art simulations using a manual calibration. Tests using QS with a uniform kernel fail to reproduce



272 some structures; in particular, the size of the objects is incorrect. Each verbatim map shows few homogenous areas;
273 therefore, realizations are produced with a low rate of verbatim copy.
274 From a quantitative point of view, Figure 8 illustrates different metrics across a set of 100 simulations. The
275 automatic calibration method proposed here allows for obtaining better quality simulations than in the original QS
276 article.
277 Figure 9 shows that variogram and connectivity metrics are well reproduced, although they have not been directly
278 constrained in the calibration process. Indeed, the parameter optimization only considers the simulation of single
279 pixels and never computes global metrics over an entire grid.
280



281
 282 **Figure 8 Benchmark between QS with an adaptive kernel (Figure 6) and a uniform (without) kernel (Figure 3) over 100**
 283 **simulations for 5 different metrics.**

284 **5 Discussion**

285 The proposed method allows for the automatic calibration of QS and potentially similar pixel-based MPS
 286 approaches, reaching a similar quality as that of manual parameterization from both quantitative and qualitative



287 points of view. The metrics confirm the good reproduction of training patterns because the method finds a
288 calibration that avoids verbatim copy.

289 The major advantage of our approach is the absence of a complex objective function, which often itself requires
290 calibration. The method runs in a predictable maximum time, which depends of the number of patterns tested, that
291 relate on the expected quality of the calibration σ . The calibration can even be refined based on previous results,
292 without running all the processes again, by adding steps, kernels, or by increasing $|\mathcal{V}|$.

293 Our approach cannot be used to determine an optimal simulation path because it focuses on the simulation of a
294 single pixel. Furthermore, the method does not take into consideration the computational cost required for a
295 simulation.

296 The computation time of the optimal parameters depends on the expected quality. For example, sometimes a kernel
297 provides only a small improvement but requires many computations. A full exploration of a 250x250 image takes
298 approximately 30 min. However, a result using a degraded parameter space can provide close results in less than
299 10 min. This degraded space can be constructed, for example, by subsampling the number of neighbors following
300 a squared function or using some external/expert knowledge.

301 The method was explored for multivariable images, resulting in a larger parametrization space than with a single
302 variable. The method provides good results independent of the task. Unfortunately, when performing this
303 approach, each new parameter increases the computation time. This can lead to impractical scenarios, especially
304 in the case of 4 or more variables.

305 In the context of testing the generality of the proposed approach, calibration was computed on multiple training
306 images (found in the Appendix B, C). Unexpectedly, the calibration pattern with two regimes (n large, then n
307 smaller) seems to be universal, at least for single variable simulations. While the position of the abrupt transition
308 between each regime seems to vary greatly (between 0.5% and 20% of the path), the overall shape remains the
309 same. Therefore, the approach proposed by Baninajar et al. (2019), in which long ranges are not considered, can
310 be extended by using large n values in the early stages of the simulation.

311 **6 Conclusion**

312 The proposed approach allows for the automatic calibration of pixel-based MPS algorithms. Furthermore, it
313 demonstrates that for optimal results, the parametrization cannot remain constant during the simulation and instead
314 needs to evolve with the simulation progression. A visually appealing result of complex features without verbatim
315 copy is difficult to simulate, especially when using a uniform kernel.

316 The proposed method allows for the calibration of a parametric kernel. However, in future work one can envision
317 the optimization of a nonparametric kernel where the weight of each individual neighbor w_i is considered a
318 variable to optimize using ε as an objective function (e.g., using machine learning frameworks).

319 The study of the evolution of parameters shows a smooth behavior of the average error. Therefore, the use of a
320 multivariate fitting approaches to estimate the error surface with fewer evaluations could be an interesting solution
321 to speed up the parametrization by capitalizing on neighbors (in parameter space). The use of machine learning to
322 take advantage of transfer learning between training images also has a high potential. These two solutions will
323 allow for the interpolation between parameters such as α of the kernel.



324

325 **Code availability**

326 The source code of the autoQS algorithm is available as part of the G2S package at: [https://github.com/GAIA-](https://github.com/GAIA-UNIL/G2S)
327 UNIL/G2S (last access: 15
328 Sept 2022) under the GPLv3 license. Platform:
329 Linux/macOS/Windows 10+. Language: C/C++. Interfacing functions in MATLAB, Python3, and R.

330 **Author contributions.**

331 MG proposed the idea, implemented and optimized the autoQS approach and wrote the article. GM provided
332 supervision, methodological insights and contributed to the writing of the article.

333 **Competing interests**

334 The authors declare that they have no conflict of interest.

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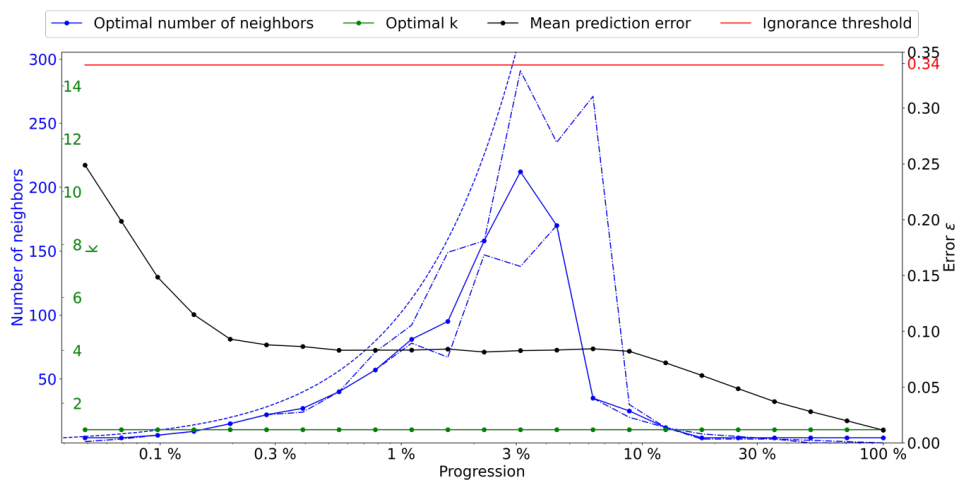
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376
377



378 Appendix

379 This supplementary material contains a similar calibration for other training images.

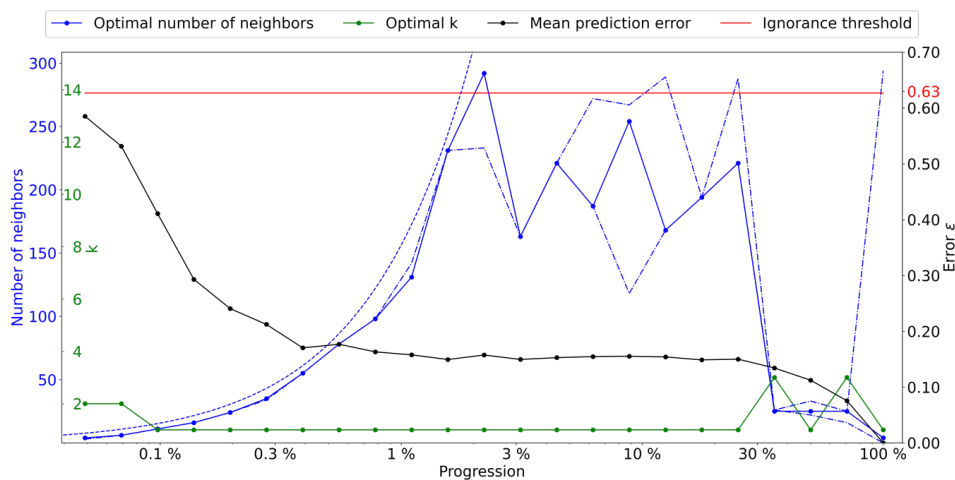
380 A. Stone



381

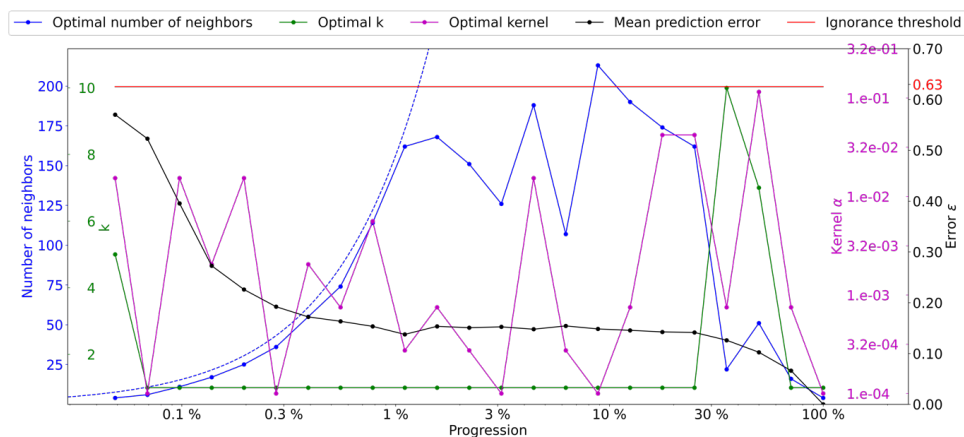
382 **Figure A.1** Optimal parameters for QS (k in green and number of neighbors in blue) as a function of the progression,
383 with the associated prediction error (in red). The red line represents the ignorance threshold. The dashed blue line is
384 the average density for the neighborhood considered. The dot-dashed line represents the variability in 1% of the error.

385 B. Strebelle



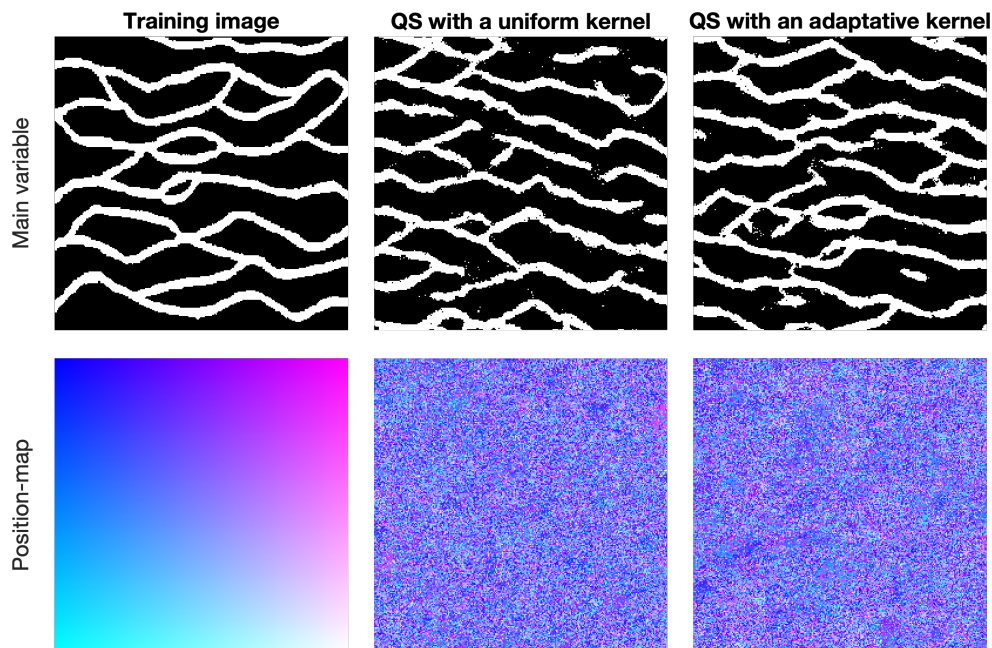
386

387 **Figure B.1** Optimal parameters for QS (k in green and number of neighbors in blue) as a function of the progression,
388 with the associated prediction error (in red). The red line represents the ignorance threshold. The dashed blue line is
389 the average density for the neighborhood considered. The dot-dashed line represents the variability in 1% of the error.



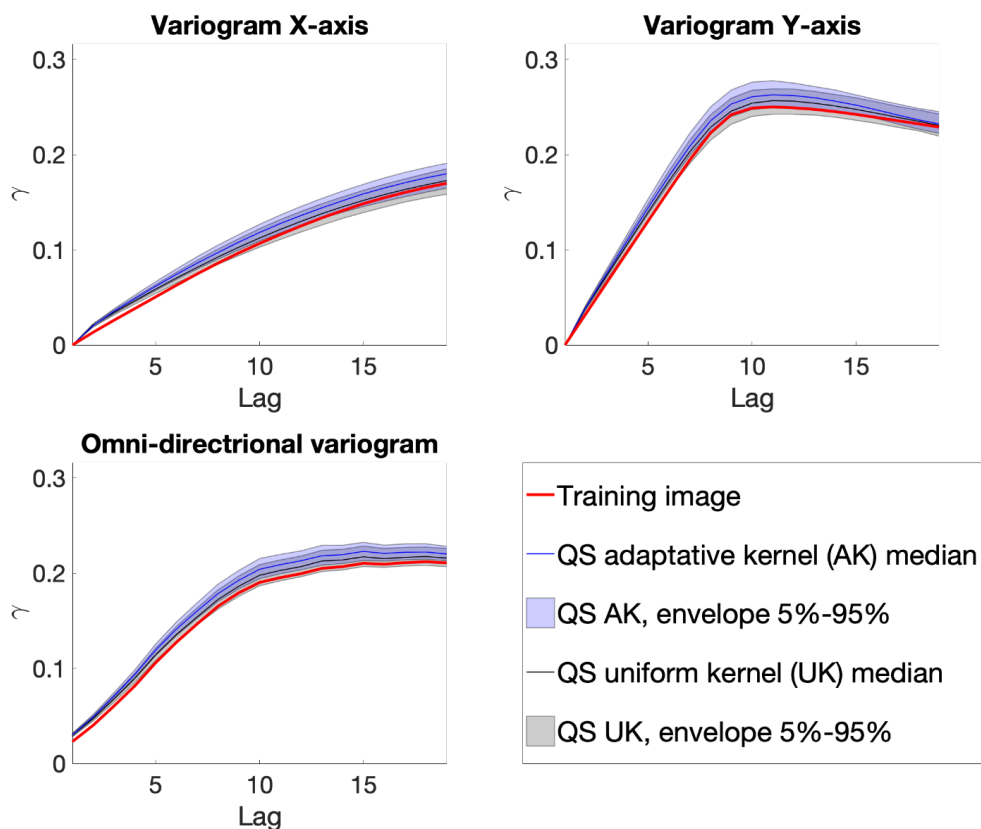
390

391 **Figure B.2** Optimal parameters for QS (k in green and number of neighbors in blue, and the best kernel in magenta)
 392 as a function of the progression, with the associated prediction error (in red). The dashed blue line is the average density
 393 for the neighborhood considered.



394

395 **Figure B.3** Simulation using QS using parameters generated by the automatic calibration.



396

397

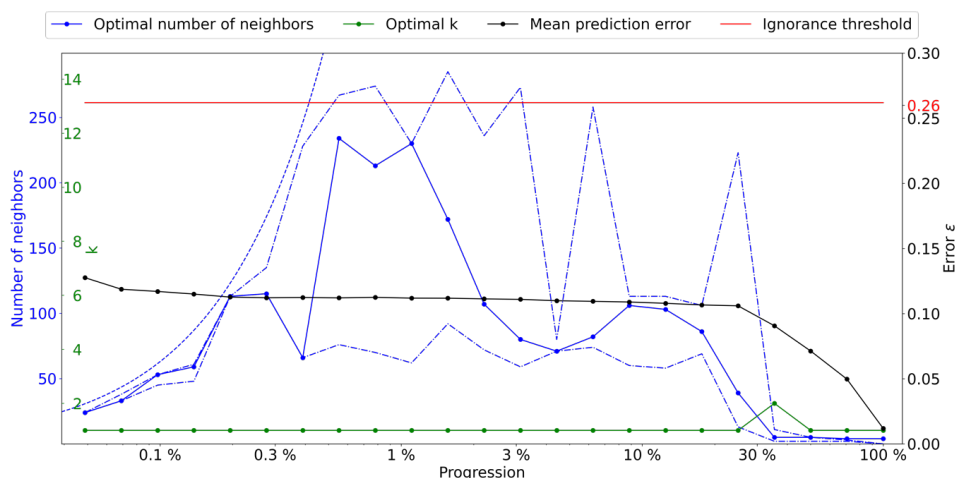
398

Figure B.4 Benchmark between QS with adaptative kernel (Figure B.2) and uniform (without) kernel (Figure B.1) over 100 simulations for 5 different metrics.

399

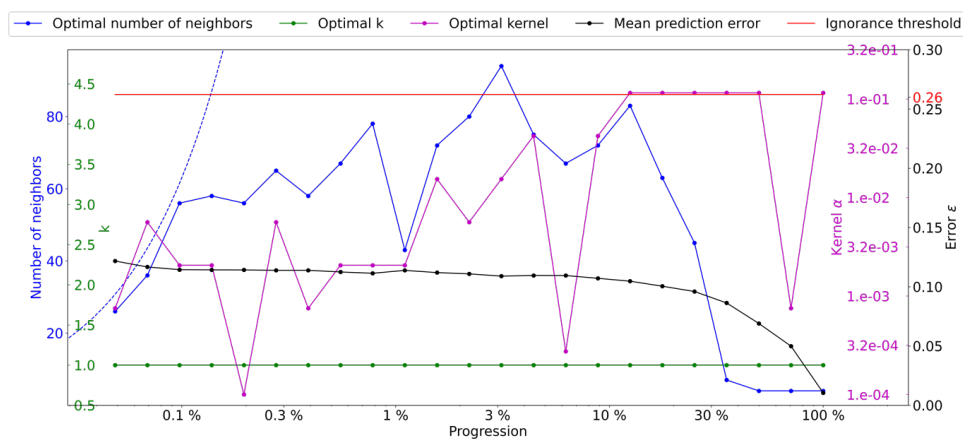


400 **C. Delta Lena**



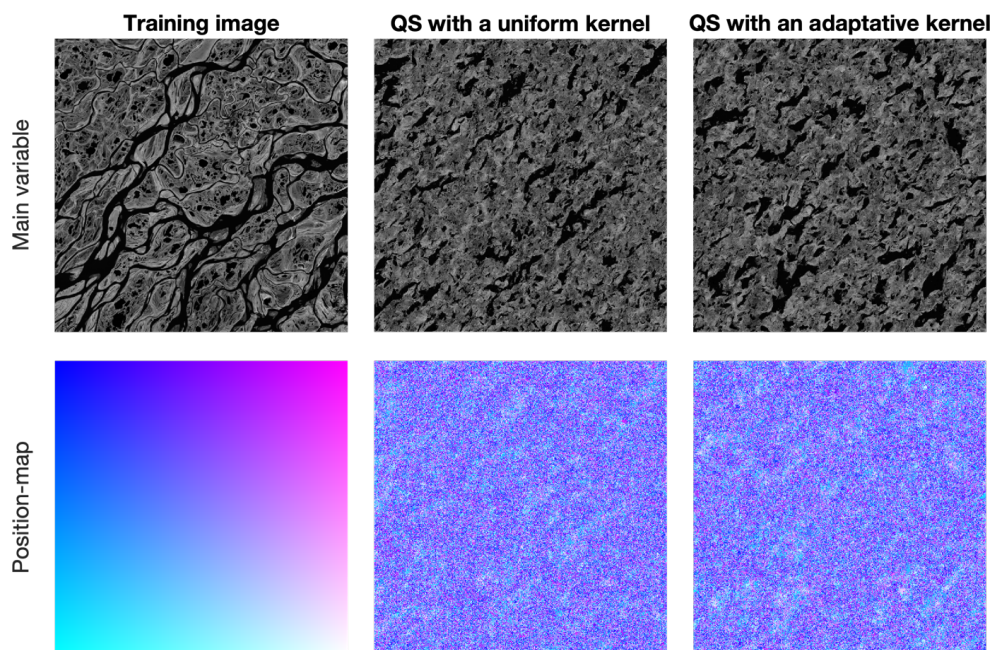
401

402 **Figure C.1** Optimal parameters for QS (k in green and number of neighbors in blue) as a function of the progression,
 403 with the associated prediction error (in red). The red line represents the ignorance threshold. The dashed blue line is
 404 the average density for the neighborhood considered. The dot-dashed line represents the variability in 1% of the error.



405

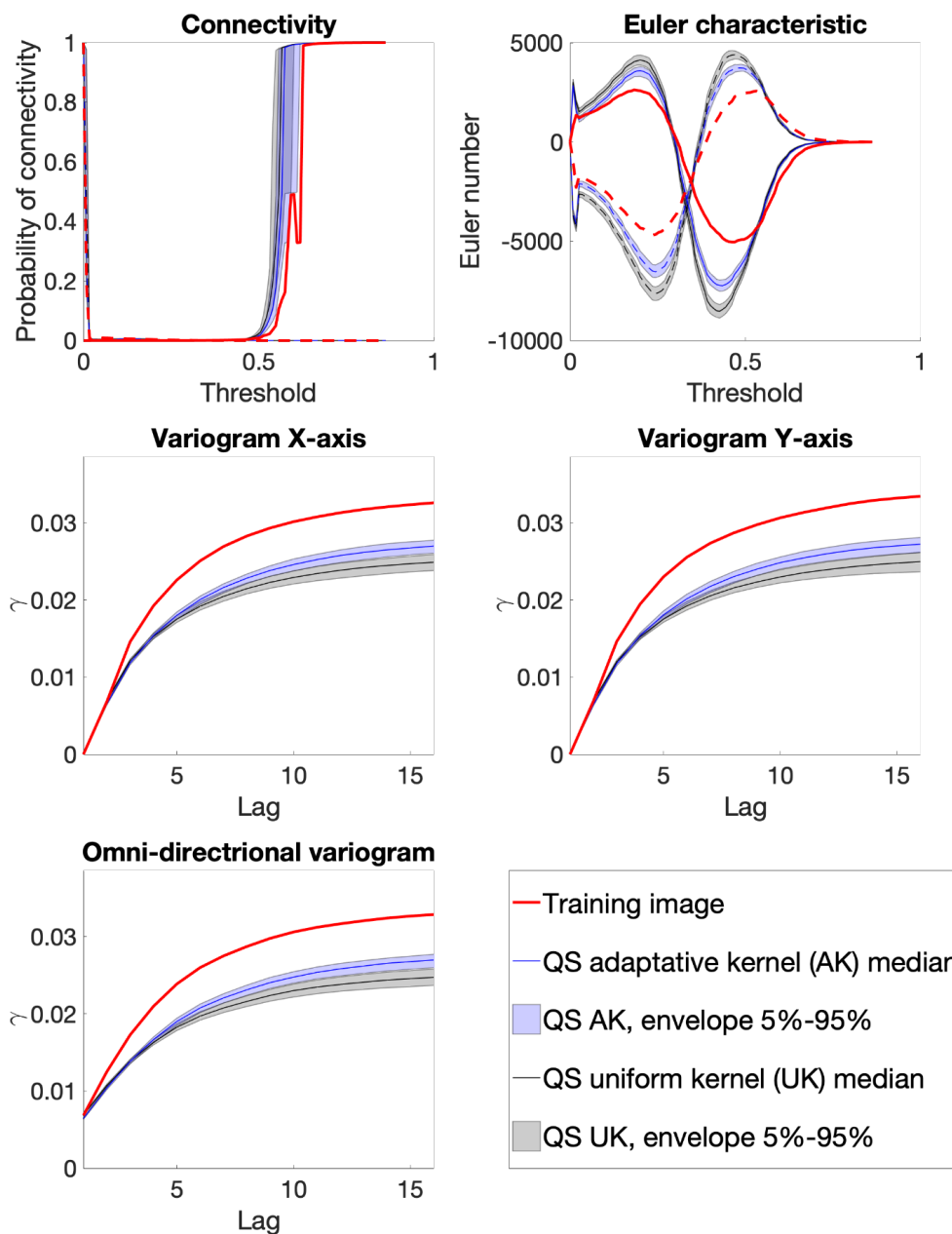
406 **Figure C.2** Optimal parameters for QS (k in green and number of neighbors in blue, and the best kernel in magenta)
 407 as a function of the progression, with the associated prediction error (in red). The dashed blue line is the average density
 408 for the neighborhood considered.



409

410

Figure C.3 Simulation using QS using parameters generated by the automatic calibration.



411
 412 **Figure C.4** Benchmark between QS with adaptive kernel (Figure C.2) and uniform (without) kernel (Figure C.1)
 413 over 100 simulations for 5 different metrics.
 414