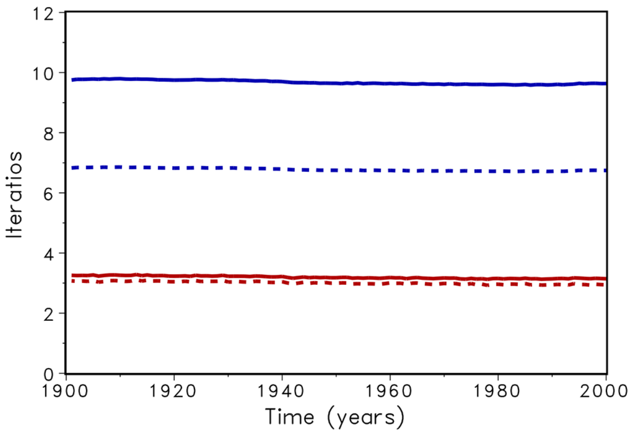

Figure 2. Average number of iteration for bisection (upper lines, blue) and Newton (lower lines, red) for accuracy $x_{\mathrm{acc}} = 0.01$ (dotted) and 0.001 (solid).

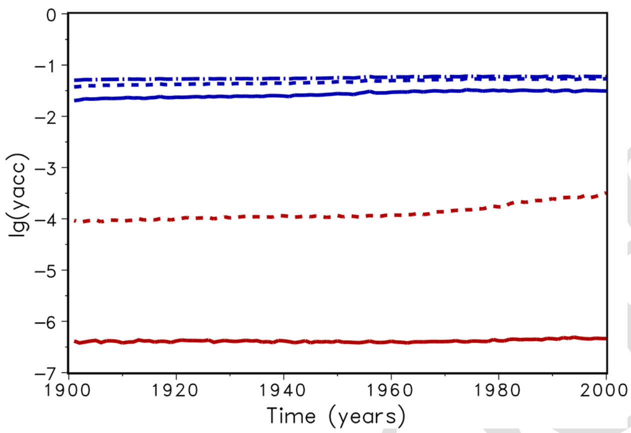

Figure 3. Mean decadic logarithm of the accuracy y_{acc} for bisection (upper lines, blue) and Newton (lower lines, red) for accuracy $x_{\mathrm{acc}} = 0.01$ (dotted) and 0.001 (solid). The dashed–dotted line shows the accuracy of the original version of LPJmL.

In order to check if the implementation of Newton's method is robust for all important model variables, we performed a transient simulation with the LPJmL model starting from the spin-up and covering the years 1901–2000. Model configuration and input data TS13 are as in Schaphoff et al. (2018a). We compared the main diagnostic variables of the published LPJmL4.0 version against the version using Newton's method (see Appendix C). We found that most global diagnostic variables related to fluxes and storage of carbon and water had differences of $< \pm 1.0\,\%$, including total vegetated area. Only marginal changes (+3 gC ~~per mA~~^{2} and month) in net primary productivity (NPP), heterotrophic respiration, and evaporation are seen mainly in Europe and southern as well as southeastern Asia. The reductions in carbon storage in litter and soil are very small and apply only to the boreal zone across the Northern Hemisphere and central Europe (compare spatial maps of carbon and water variables in Appendix C).

The photosynthesis module is also applied to the crop functional types and managed grassland within LPJmL4.0. Therefore, sawing dates, crop productivity, and harvest are among the simulated variables. Comparing both model versions in the model benchmark, we found that global harvest changed for a number of crops. Rainfed and irrigated rice increased by 5 % and 8 %, respectively, mainly in India and southeast Asia. Harvest of rainfed temperate cereals increased by $< 1.0\,\%$, mainly found in central Europe. Harvest of irrigated temperate cereals (incl. wheat) increased by 4.5 %, which mainly applied to India as well. Harvest of irrigated and rainfed soybeans increased by 2.3 % and 1.5 % globally; the differences are mainly found in the US and Brazil. All other crop functional types had marginal to zero changes in global productivity as well as simulated harvest (see Table in Appendix C).

For all global carbon pools (vegetation and soil) and carbon (GPP, heterotrophic respiration, and fire emissions) as well as water fluxes (transpiration and runoff) we found no difference in the temporal changes in the transient simulation over the 20th century. All variables showed similar, if not identical, dynamics (data not shown). Small changes were found in the fractional coverage of plant functional types, i.e. most differences were negligible. The fractional coverage of temperate broadleaved summergreen trees increased by 4.8 % globally, which mainly applies to Europe, the northeastern USA, and parts of China. Increases in temperate C_3 grasses are found in the boreal zone, summing up to 4.8 % globally. Marginal changes of $< 0.5\,\%$ per grid cell are found for all other PFTs, which imply small adjustments in vegetation composition in these vegetation zones (see difference maps in Appendix C). Comparisons using flux tower measurements on carbon and water fluxes as well as discharge data showed no differences so we can conclude that also for these variables the results are robust (data not shown). We can therefore conclude that the LPJmL results were robust before but are now achieved due to improved accuracy of the photosynthesis routine.

After improving the computational efficiency and numerical precision, we can now test the parameter uncertainties following Walker et al. (2021), who tested the sensitivity of θ, α_{C_3}, b_{C_3}, k_{c25}, and K_{o25} on their impacts on global GPP. The LPJmL model computes V_{m} as follows (Schaphoff et al., 2018a, Eq. 35) TS15 : TS16

$$V_{\mathrm{m}} = \frac{1}{b_{C_3}} \cdot \frac{c_1}{c_2} \cdot ((2\theta - 1) \times s - (2\theta \times s - c_2) \times \sigma) \cdot \mathrm{APAR}. \quad (19)$$

Therefore, the sensitivity of V_{cmax} results from varying b_{C_3} indirectly since the reciprocal of b_{C_3} is used to calculate V_{cmax} in a linear equation. Varying b_{C_3} is therefore the adequate sensitivity test which relates to V_{cmax}. We varied each parameter by 10 % independently and find that θ (α_{C_3}, b_{C_3}, k_{c25}, K_{o25}) increases global annual GPP (AGPP, hereafter)

Table C1. Global sums of actual vegetation, including land-use, comparing Newton approach (benchmark run) against bisection approach (run). Tece is temperate cereals. TS20

Parameter	Lit. estimates	Run	Benchmark fun	Diff. abs.	Diff [%]
Vegetation carbon [GtC]	460–660[a,b,c]	595.9	596.2	0.231	0.039
Total soil carbon density [GtC]	2376–2456[d], 1567 TS21	1862	1862	−0.08	−0.004
Litter carbon [GtC]	NA TS22	151.3	151.4	0.116	0.077
Fire carbon emission [GtC yr^{-1}]	2.14 (1.6 Nat. Fire)[g,h,i,j] TS23	3.108	3.109	0.001	0.036
Establishment flux [GtC yr^{-1}]	NA	0.161	0.161	0	−0.002
Area all natural vegetation [M ha TS24]	NA	7767	7767	−0.119	−0.002
Area tropical broadleaved evergreen tree [M ha]	NA	1180	1179	−0.237	−0.02
Area tropical broadleaved raingreen tree [M ha]	NA	1280	1280	0.448	0.035
Area temperate needle-leaved evergreen tree [M ha]	NA	364	360.8	−3.166	−0.87
Area Temperate broadleaved evergreen tree [M ha]	NA	322	321.5	−0.467	−0.145
Area Temperate broadleaved summergreen tree [M ha]	NA	136	142.5	6.517	4.792
Area boreal needle-leaved evergreen tree [M ha]	NA	429.2	426.8	−2.393	−0.558
Area boreal broadleaved summergreen tree [M ha]	NA	916.8	919.6	2.814	0.307
Area boreal needle-leaved summergreen tree [M ha]	NA	378.3	380.7	2.398	0.634
Area tropical C_4 grass [M ha]	NA	893.2	890.6	−2.573	−0.288
Area temperate C_3 grass [M ha]	NA	535.7	545.2	9.472	1.768
Area polar C_3 grass [M	NA	1332	1320	−12.93	−0.971
NPP [GtC yr^{-1}]	~~66.05~~[j], 62.6[b], 49.52–59.74[l]	62.81	62.87	0.064	0.102
Heterotrophic respiration [GtC yr^{-1}]	NA	50.78	50.83	0.044	0.086
Evaporation [~~10.,~~ TS25 km^3 yr^{-1} TS26]	NA	9.644	9.661	0.017	0.173
Transpiration [~~10.,~~ km^3 yr^{-1}]	NA	47.83	47.82	−0.011	−0.024
Interception [~~10.,~~ km^3 yr^{-1}]	NA	7.914	7.912	−0.002	−0.024
Runoff [~~10.,~~ km^3 yr^{-1}]	NA	54.3	54.23	−0.064	−0.118
Harvested carbon rainfed tece [Mt DM/year TS27]	524.08[m]	458.5	462.6	4.106	0.895
Harvested carbon rainfed rice [Mt DM/year]	492.66[m]	125.2	131.5	6.304	5.035
Harvested carbon rainfed maize [Mt DM/year]	498.33[m]	434.9	434.8	−0.07	−0.016