

### **General response:**

We thank the reviewers for going through this manuscript and providing valuable comments. Before we go into point-by-point responses, we would like to suggest four questions to facilitate the reviewing process.

- 1) What hypothesis is proposed for quantifying the causal contributions?
- 2) Why is this causal method so important for understanding complex Earth system?
- 3) Is our hypothesis reasonably tested? Are the assessment framework and criteria reasonable?
- 4) What is the potential theoretical explanation behind our hypothesis?

From reviewers' comments, we felt that our answer to question 1 was effectively delivered. In our revision, we further explain the different terms in the modified normalizing factor, since it seems that this was unclear. For clarity, we also split certain equations, such as those defining our hypothesis, as well as definitions of normalized and unnormalized causal sensitivities. We have also placed our discussion of the "sign" into a separate sub-section, and included further explanation of the use of the original sign of IF and nIF for a "second sign-adjustment".

It is also clear to us that Question 2 was not sufficiently clearly addressed in the previous version of our manuscript. The term "linear model" may have caused misunderstanding that this causal method has limited relevance to complex non-linear Earth systems. We have clarified that "nonlinearity" exists in the varying causal sensitivity and the noise-contributions. We elaborated this using the methane-climate feedback as an example. Related to this confusion over the term "linear model", we have also reframed the use of such terms as "second order and first order", by replacing it with "state-dependent and rate-dependent", which better explains the scenario to which it should be applied.

In regard to point 3 above, we are sorry that the contents and figures in our previous version proved too information-rich and difficult to understand. In our revision, we have added a sub-section on "assessment criteria". In brief, criterion 1 focuses on whether the causal "quantifier (e.g. nIF)" can represent the nCS in the causal function, and criterion 2 highlights whether the factor  $\alpha$  can represent the maximal  $|CS|$  of the causal function. Criterion 1 is supported by Figs 2,3,4. Criterion 2 is supported by Table 1, and Fig. 2k-l as an example for the 1:2:3 ratio (the extended assessment criterion 2). Figures for all 1D tests are revised, with the focus now on the comparison of the designed and estimated  $\partial Y_1(X_1)/\partial t$  and  $\partial X_1(Y_1)/\partial t$  in the main text. The designed and estimated trends are placed together in same sub-figures for easier comparison. The confusion over our use of the 1:2:3 ratio has been addressed with a more extensive explanation. Since it is just an extended assessment criterion, its comparison is only shown in two sub-figures in the main text, preventing overloaded information in the Figures. Comprehensive results are now moved to the Supplementary Information for detailed comparison.

In regard to the fourth question, question 4 above, a more detailed theoretical development of our approach would certainly make our hypothesis more convincing, but the key message from this paper is "empirical assessment". Nevertheless, in this revision, we have better explained why both "normalized causal sensitivity" and "normalized information flow" appear to be applicable over various times, locations, and with different causes. The main reason is because both of them are normalized over the "causal function of the cause variable on the effect variable". Based on this, we have also provided a more detailed explanation of the different normalizing factors  $mdZ$ , as well as the associated terms  $nIF_{(non-X \rightarrow Y)}$ ,  $nIF_{(Y \rightarrow Y)}$ , and  $|mdZ_2 - mdZ_1|/mdZ_3$ . In the conclusion, we have also further illustrated the limitations of our method.

*In addition, we consider the data processing methods as secondary, although clearly a discussion of them is necessary for interested readers to repeat our work. We therefore chose to park those details in Supplementary Information. One of data-processing methods concerns preliminary removal of the state-dependent and noise contributions for estimating the rate-dependent contribution. We have also corrected some mistakes in the earlier manuscript (and we thank reviewer 2 for pointing them out). We have corrected those and included a Supplementary Figure to better explain this aspect of data processing.*

*Lastly, we understand that both reviewers have commented on the notation. We would like to highlight that the foundation of this causal method is still the normalized information flow between only TWO time series. Complicated matrix notation may, we feel, distract away from such focus without improving clarity. We therefore have not changed the notation. Nevertheless, to improve readability, we have aligned better between equations and Figures of 1D tests. Previously,  $\partial X(Y)/\partial t$  was on the left column of Figures (default priority of reading and testing sequence) while the equations in the paper are mainly based on  $\partial Y(X)/\partial t$ . We have revised our Figures, Matlab codes and Supplementary Table, so that  $\partial Y(X)/\partial t$  now appear in the left column.*

*We would be grateful if any further or subsequent comments could first consider the issues that we describe around the four questions posed above, followed by any other detailed comments.*