

Letter to the referees on ‘Topography based local spherical Voronoi grid refinement on classical and moist shallow-water finite volume models’ for GMD-2021-82

October 18, 2021

Once again, we would like to thank the referee, Darren Engwirda, for reviewing and giving useful comments. In what follows we will bring in blue the original comment from the referee and reply in normal font.

Response to comments from Reviewer R2

- The conclusion currently takes a rather negative view on variable-resolution runs, arguing that the TRSK discretisation suffers from various deficiencies that are suppressed through use of hyper-viscosity. I’d argue that, rather than being a characteristic of TRSK in particular, almost all GCMs, even those run on quasi-uniform structured grids, require added dissipation terms for numerical stability.

We agree with the referee and have adjusted the text in order to allow a fairer interpretation of the results with respect to TRSK. However, it is important to point out that TRSK was developed to be a mimetic numerical scheme, with relevant conservation properties. In principle, it should not require artificial diffusion on regular grids for stability reasons, except on baroclinic regimes (small equivalent depths). Our results show that, in shallow-water model frameworks, the variable resolution grids require the addition of numerical hyperdiffusion for numerical stability, while the quasi-uniform grids do not. We find it is important to highlight that grid flexibility comes with a price, but we have amended the text (see lines lines 584-587) to correctly highlight that the price is not very high, especially compared to many other GCM.

- The issue, in my view, is mainly related to the discretisation of the advective tendencies (thickness, PV, tracers, etc) with dispersion errors (oscillations, waves) guaranteed to be generated by Arakawa-type schemes that use centred-differences. Adding hyper-viscosity is one popular solution to this problem, with upwind/flux-limited formulations being another.

Thank you for the comment. We agree that centred-differences can be a major source of potential instabilities. In the case of TRSK, these discretizations were carefully tailored to preserve certain numerical properties of the momentum-continuity-PV equations, which are not easily achievable via non-centred approaches (see, for instance, Subich 2018). We found that adaptative (geometry dependent) hyper-viscosity seems to provide a good trade-off between stabilization and mimetic property preservation, but indeed other approaches could be tailored for the purpose.

Subich, C.J., 2018. Higher-order finite volume differential operators with selective upwinding on the icosahedral spherical grid. *Journal of Computational Physics*, 368, pp.21-46.

- In the moist-SWE results in 4.3, it's not clear whether numerical dissipation is added to the tracer equations, or just to the momentum balance?

Thank you for pointing this out. The numerical dissipation was added just to the momentum equation as in section 4.2, aiming to avoid the grid-related errors observed in the variable resolution grids. We have clarified this in lines 480-483.

- I don't believe it should be expected that adding $\text{del}^4(\mathbf{u})$ to the momentum equations will remove the grid-scale oscillations from the rain/cloud tracers, since I expect these effects will be caused by the centred tracer advection scheme used.

As we pointed out previously, the addition of $\text{del}^4(\mathbf{u})$ to the momentum equation aims to mitigate numerical noises triggered by the velocity field and preserve the stability of main flow equations (continuity-momentum equations). So, indeed, we do not expect that adding $\text{del}^4(\mathbf{u})$ will prevent grid-scale oscillations from the tracers, inherent from the centered scheme advection. We believe that, anyhow, these results reveal a relevant aspect of the discrete divergence, also used in other sub-grid scale physical processes. We have added a comment regarding this in lines 483-486.