Letter to the Topical Editor on 'Topography based local spherical Voronoi grid refinement on classical and moist shallow-water finite volume models" for GMD-2021-82

September 24, 2021

We would like to thank the Topical Editor for reviewing and giving constructive comments that helped to improve the paper.

The suggestion for inclusion of variable hyper-diffusion was of most relevance, and we decided to perform new analyses related to filtering requirements of the model. In turn, the new version of manuscript had all experiments re-simulated with a new diffusion mechanism. The results are qualitatevely very similar, and conclusion also, but now it is clearer that the filtering technique is as discreet as possible.

Responses to the reviewers comments were already addressed in the previous version of the manuscript, so we will highlight here only the topical editor's comments.

In what follows we will bring in blue the original comment from the topical editor and reply in normal font bellow, point-by-point.

Response to comments from Topical Editor

- I will mention two minor technical points: 1) In Eq. (5), phi and theta are used for latitude and longitude, while in Eq. (20), phi (different font) and lambda and used for these quantities. Since theta is used for (depth-averaged) temperature, I would use the latter convection for lat/lon throughout.

  Thank you for spotting this. We have adopted phi and lambda for the lat/lon coordinates. We also have fixed other notation mistakes in Sect. 2.2 in this review process.

- 2) My experience with hyperdiffusion is that the coefficient should depend on the local grid resolution, with more hyperdiffusion in coarse regions and less in fine regions. Please see: Guba, O., Taylor, M. A., Ullrich, P. A., Overfelt, J. R., Levy, M. N. (2014). The spectral element method (SEM) on variable-resolution grids: Evaluating grid sensitivity and resolution-aware numerical viscosity. Geoscientific Model Development, 7(6), 2803-2816.

  We agree with the Topical Editor that variable coefficient is more suitable for hyperdiffusion. The coefficient dependence on the cell size is ideal, specially from a physical point of view. Therefore, we have followed a variable fourth-order hyperdiffusion operator proposed by Klemp (2017) and we investigated not only a cell-size dependent coefficient but also a cell-geometry dependent coefficient. We performed new simulations using variable hyperdiffusion, which we give details below, and the results were included in the manuscript.

  In the revised manuscript, we have added a new section *4.1 Hyperdiffusion* where we present the variable diffusion formulation given by Klemp (2017) in Eq. 20. In this new section, we also propose three different hyperdiffusion coefficients: constant coefficient, diameter-based and alignment-based coefficients.

The diameter-based hyperdiffusion follows Zarzycki et al. 2014, where the coefficient depends on the cell diameters and it is reduced 10 times when the cell diameters is divided by 2 (see lines 293-297 in the revised manuscript).

The alignment-based coefficient is defined using the smooth alignment index (see lines 307-313), and it was motivated by the previous versions of this work, where we could observe that the dominant factor for bigger errors is related to the cell geometry.

Both diameter- and alignment-based coefficients depend on a constant $K_{\max}$ that represents the maximum of hyperdiffusion in the coarse cells, for the diameter-based case, and in the ill aligned cells, for the alignment-based case.

We have investigated the error for the test case 2 in Sect. 4.2.1 considering different values of $K_{\max}$ and all the proposed coefficients (see Fig. 7 in the revised manuscript). We could observe that the alignment-based coefficient has a similar behavior to the constant coefficient compared to the diameter-based coefficient. Also, we observed that the error patterns (see Fig. 6) were not related to the grid resolution but to the alignment index. These experiments indicates that, even though the diameter-based hyperdiffusion may be ideal from a physical point of view, it may not be ideal for variable resolution grids. Therefore, we adopted the alignment-based hyperdiffusion for further experiments.

An optimum value of $K_{\max}$ was found for the VR7 grid (see Fig. 8) considering the alignment-based coefficient. This value of $K_{\max}$ was then adopted for all the simulations in Sect. 4.2 and 4.3. We obtained very similar results compared to the previous version of this work, where we considered constant hyperdiffusion, but now we are adding hyperdiffusion only it is necessary.

Klemp, J. B.: Damping Characteristics of Horizontal Laplacian Diffusion Filters, Monthly Weather Review, 145, 4365 – 4379, https://doi.org/10.1175/MWR-D-17-0015.1, 2017.

Zarzycki, C. M., Jablonowski, C., and Taylor, M. A.: Using Variable-Resolution Meshes to Model Tropical Cyclones in the Community 710 Atmosphere Model, Monthly Weather Review, 142, 1221 – 1239, https://doi.org/10.1175/MWR-D-13-00179.1, 2014.

- Also, note that a negative sign is necessary in the inline equation on line 276. The negative sign is necessary to ensure the hyperdiffusion operator is dissipative.

  We fully agree with the Topical Editor. We have added a comment in lines 291-292 pointing out that the negative sign is necessary to ensure the dissipative behavior of the hyperdiffusion operator.