

Dear Editor and Reviewers,

We are very grateful for your comments and suggestions, which have helped to improve our manuscript significantly. We have revised the manuscript accordingly, and the changes can be found in the track-changes file. The following is a point to point response to your comments and suggestions. Corresponding changes in the manuscript are also made available below at the appropriate places, if applicable.

Sincerely,

Benjamin Zanger and Jia Chen on behalf of all co-authors.

Editor and Reviewer #2:

On line 298, in the sentence "Even though, for both domains ...", the comma is not needed.

Thank you for the remark, we changed that in the text.

Throughout the text, the word "relative" has been abbreviated as a rel. I think it should be spelled out (the typesetter might do that anyway before the paper appears in print).

We changed that throughout the text, and in the axes labels of the figures. The changes in the figures are not found in the track changes.

In the abstract, the authors state, "A common top-down approach for solving this problem is Bayesian inversion that uses a given Gaussian distributed prior emission field. However, such an approach has drawbacks when the assumed spatial emission distribution is incorrect." I worry that this statement could be misinterpreted by readers. Technically speaking, traditional Bayesian inverse models can be used to correct the spatial (and/or temporal) distribution of emissions. Rather, this approach typically assumes that the correction follows a multivariate normal distribution, and this assumption may not be appropriate for all inverse problems, like the urban applications discussed in this paper.

Thank you for this remark. We adapted the sentence in the following way:

Lines 3 ff:	<i>A common top-down approach for solving this problem is Bayesian inversion that—uses with the assumption of a given multivariate Gaussian distributed distribution for the prior emission field. However,</i>
-------------	--

	such an approach <i>assumption</i> has drawbacks when the assumed spatial emissions distribution—is are incorrect or not <i>Gaussian distributed</i> .
--	--

Line 26: Numerous inverse modeling studies also enforce temporal correlation.

Thank you for this remark. We added this to the text.

Lines 26 ff:	<i>Instead, sectors (Jones et al., 2021), spatial correlations (Wesloh et al., 2020), and/or temporal correlations (Jones et al., 2021; Wesloh et al., 2020) are used to construct alternative parameterizations of emission fields to prevent overfitting.</i>
--------------	---

Line 41: The description of Hase et al. (2017) could be confusing to some readers. E.g., it's not clear what "limitations" are being referred to or what "multiple parameterizations" refers to. Hase et al. (2017) use a dictionary that includes different types of spatial patterns or basis functions. As a result, they can represent a diversity of spatial patterns in the emissions field. (I'm not sure if my description is any less confusing, but I think it's another way to describe the important features of their work.)

Thank you for this remark.

We modified the text to make it clear that this approach increases the sparsity of the emission field and the representation of the field is not unique:

Lines 26 ff:	<i>Hase et al. (2017) demonstrated sparse reconstruction with enforced positive emissions estimates of anthropogenic CH₄ emissions from synthetic observations in the US. To overcome the limitation of sparse emission field estimations To increase the sparsity of the emission field, Hase et al. (2017) used a redundant dictionary representation, where multiple parameterizations are possible for the same emission field the representation of the emission field is not unique.</i>
--------------	---

Lines 86-89: This description of maximum likelihood could be confusing to some readers. E.g., "From this posteriori distribution a parameter estimation can be made using a maximum likelihood (ML) detector on the a posteriori." It's not clear what parameters this sentence is referring to. The word "detector" here also feels a bit confusing to me (i.e., it implies that ML is being used to detect something). Here's how I think of this process: inverse modelers estimate emissions by maximizing a likelihood function or equivalently, minimizing the negative log of this function. The

resulting estimate is often referred to as the ML estimate of the emissions. This likelihood function is based on Bayes Theorem (which leads into Eq. 2).

Thank you for this remark. We have the same understanding of this process, where the likelihood function is the probability (or probability density) of the parameters derived by Bayes' theorem from the prior probability distribution and the measurement probability distribution.

In engineering this is sometimes referred to as maximum a posteriori (MAP) detector, or maximum likelihood (ML) detector on the posteriori.

But a more common expression in statistics is maximum likelihood (ML) estimation on the posteriori or maximum a posteriori (MAP) estimation.

As parameters, we understand the coefficients describing the emission field.

We adapt our text to be compliant with the literature of statistics and inverse modeling:

Lines 26 ff:	<i>From this posteriori distribution an estimation of the parameters, and therefore the emission field, estimation can be made using a maximum likelihood (ML) detector estimator on the a posteriori. Since the ML detector estimator acts on the a posteriori, this is commonly referred to as Maximum a posteriori (MAP) detector estimator.</i>
--------------	--