

Thanks for the careful reading and insightful comments. We have carefully revised the manuscript accordingly.

The study introduces the concept of constrained EnKF that is used to improve estimation of CO<sub>2</sub> in the GEOS-Chem model by conserving the mass of CO<sub>2</sub> in analysis updates. The study is important to this field, and it is believed that authors spent lots of efforts on preparation of data, implementation, and interpretation, however, as the reviewer pointed out, the writing skills are very disappointing, there are too many typos, grammar errors, wrong choices of wording and incomprehensible phrases. There are much more than the reviewer already listed. Reviewers are primarily supposed to provide scientific evaluation of the work. Therefore, I strongly suggest that authors do careful cross proof-reading in the revision.

Response: Thanks for the suggestions. We have carefully revised the manuscript. And it has been polished by a native speaker from AJE (America Journal Expert).

There are some other comments from my side:

1. There are too many abbreviations. They are very disturbing while reading. Use abbreviations only if they are necessary. For example, it is unnecessary to use "AW" for "assimilation window".

Response: Thanks for the suggestion. We have deleted some abbreviations and summarized the abbreviations to table A3.

2. It would be much easier to understand the mathematical expressions if authors can use thin, bold and bold capital to differentiate scalar, vector and matrix.

Response: Thanks for the suggestion. We have revised the equations.

3. Throughout the paper, it is not natural to use the word of "priori" instead of "background" while using "analysis" (not posterior).

Response: Thanks for point out this. We replace the word of 'prior' with 'a priori' in the main text to reduce misunderstandings. In our study and most of the CO<sub>2</sub> inversion studies, 'a priori' SCF inventory is used to regularize the ill-posed problem. And we use the 'a priori' to perturb the ensembles in the inflation step. 'A priori' is different from the background (or first guess) used in state-oriented DA (weather/ocean). So, in this paper, we used 'a priori' together with background (first guess) and analysis.

4. Can authors provide a flow chart for the algorithm of LETKF+CEnKF? It would be helpful to understand how the algorithm works.

Response: Thanks for the suggestion. We added a flow chart for the overall algorithm (Fig. 1).

5. Can authors illustrate differences between assimilation window, observation window and overall window for the run-in-place method?

Response: Thanks for the suggestion. We have added a flowchart to explain the windows (Fig. 1) and added more illustration on this (Line 135).

6. Can authors explain why the RTPS can maintain mass conservation? I am not sure about this.

Response: Thanks for the comment. First, RTPS can reduce but can not solve the mass conservation issue. Because the original error sources come from the flux. The RTPS for CO<sub>2</sub> could maintain the error structure developed by the flux ensemble. While additive inflation for CO<sub>2</sub> will destroy the structure and leading to larger mass loss/gain.

7. Can authors explain more clearly how the initial ensemble is created? Is it a time-lag ensemble?

Response: Thanks for the comment. We have revised the explanation (line 213). Yes, it is a time-lag ensemble.

8. Since OSSE is done in this study, I assume that the observations are created by adding the noise to truth. But it seems that real observations are used. Can authors make this more clear in the text?

Response: Thanks for the comment. We rephrase the illustration of how the pseudo-observations are created (line 221). We use the real observation network (time, location, representative error of the observation) instead of the real observation values to create the observations in OSSE.

9. If I understand correctly, authors use the mass of background ensemble mean as the proxy for true value. However, this is not the ideal choice, for example, due to forecast error. Can authors provide some discussion on this?

Response: Thanks for the comment. An important rule for CO<sub>2</sub> inversion is strict mass conservation. We can also use the global mass of analysis CO<sub>2</sub> as a proxy to constrain the SCF instead of CO<sub>2</sub>. It should reduce the mass imbalance but can not strictly conserve the mass. From the causal relationship point of view, it is the SCF that drives the accumulation/absorption of CO<sub>2</sub> mass. So, there is no SCF that drives the additional analysis CO<sub>2</sub> mass. And this is the reason why we apply the CEnKF to the CO<sub>2</sub> state. The forecast error of the CO<sub>2</sub> distribution could be large, but the forecast error of the global mass may be small if the assimilation window is short enough.

10. Authors show the importance of mass conservation constraint within data assimilation. Does the constraint have some feedback effects on dynamical components of the model? Have authors also considered the impacts on the longterm forecasts? Is it important?

Response: Thanks for the comment. The dynamic component is the CO<sub>2</sub> transport. And it has some direct feedback effects on the short-term forecast (Fig. 12b, c). But for long-term forecasts, the effects could be diluted or smoothed out (Fig. A1).

11. Line 73-74: Zeng and Janjic 2016 showed the LETKF can violate the conservation properties (e.g., total energy and enstrophy), and Zeng et al. 2017 introduced a new algorithm which can conserve non-linear properties. However, their studies have not showed the imbalanced dynamics. For imbalance, it is more appropriate to cite other papers, e.g., Greybush et al. 2011, Bick et al. 2016 or Zeng et al. 2021a,b.

Response: Thanks for the suggestion. We have added some of the references (Line 78).