Response to Anonymous Referee #2 (RC2)

In this manuscript, the authors present spyro, a full waveform inversion finite element solver based on Firedrake. The solver features wavespeed-adapted triangular/tetrahedral meshes, a fully-explicit time stepping scheme based on a mass-lumping technique, and conforming elements of variable orders (up to degree 5 in 2D, and 3 in 3D).

The paper is well-written and fits the scope of this journal. The numerical results also look promising and the proposed solver seems to have several advantages over other FWI methods.

R: We thank the referee for these comments and for the time he/she spent evaluating our work.

I only have a few remarks/questions:

-p3, line 61: spectral triangular elements are known at least up to degree 9 (see, e.g., Mulder 2013: New triangular mass-lumped finite elements of degree six for wave propagation, Cui et al. 2017: High order mass-lumping finite elements on simplexes, and Liu et al. 2017: Higher-order triangular spectral element method with optimized cubature points for seismic wavefield modeling).

R: The referee is right, and we thank him/her for calling our attention to that. We have updated the sentence in the paper and added references to Cui et al. 2017 and Liu et al. 2017, which presented 9th order mass lumped triangular elements.

However, our numerical results already gave diminishing returns at high order, so increasing from degree 5 is unlikely to help our application in practice.

-p6, line 161: \sigma_i is not defined here. I suggest to give the definition of \sigma_i and \Psi_i here and explain that p_i and w only need to be computed in the boundary layer.

R: We have modified the text to “... and Ψ_{i} are the damping matrices. These damping matrices are calculated using damping functions, which are referred to as σ_{i}. Note that Ψ_{i}, p and ω only need to be calculated in the PML.”

-Section 2.2: is there a reference for the derivation of the adjoint equations? Especially the boundary conditions given on page 8, line 207 need some explanation. The adjoint problem has 2 boundary conditions while the original problem had only 1.

R: The original problem has actually an extra boundary condition, namely “(n.p)=0”. This condition was implemented through making an integration by parts on the term “-\nabla.p” on equation (1) in the FEM formulation and taking the remainder boundary term to be zero. This was done since no clear boundary condition on “p” was originally mentioned in the paper by

Kaltenbacher et al 2013, and this one did stabilise the scheme. Also, the adjoint equations, together with their boundary conditions were derived by us in this work, and no reference on them was found. A derivation of the adjoint is now added in Appendix A1.

-p9, line 230: definition of F is missing.

R: Indeed, this definition was missing. It has been corrected in the new version.

-Section 3.1: is the stiffness matrix assembled and stored or are the computations matrix-free?
R: In the UFL firerake, the “right-hand-side” vectors of a linear system where the stiffness matrices would appear (terms related to matrices A_n in equation (26)), are computed at every iteration without the knowledge of an a priori stored stiffness matrix. So the computations are matrix-free.

-p12, top line: In 3D, alpha is computed taking the cube root?
R: Indeed. This has now been clarified in the new version.

-p12, definitions A_{n+1}, A_n, A_{n-1}: please double check these definitions.
--In A_n top-right: should be M_{omega, 1} instead of M_{omega}.
--In A_n bottom-right: should be 0.
--In A_{n-1} second row: second and third term should be swapped.
--In A_{n-1} bottom-right: should have a minus sign.
R: Thank you for pointing out those definition errors. They all have been corrected.

-p14, equation 36: the definition of G is not really clear. What is its continuous counterpart? Also, the right-hand-side should be a vector. Please give a more precise definition.
R: Indeed, the way the equation is written, the rhs is a scalar, but, if evaluated for every test function "delta c", makes a vector. We made a change in the text to clarify this.

-p16, line 396: 32 nodes instead of 50.
R: The referee is correct. This has been corrected.

-p17, Algorithm 1: step 10 is done via L_BFGS and step 11 via ROL? Or are both used for both steps?
R: Indeed, the text was misleading and was not clearly related to the algorithm. It has been clarified now. Both the descent direction and the step lengths are part of the l-BFGS algorithm and are implemented in ROL.

-p17, line 441: equation (36) instead of (19)?
R: The referee was correct. We have modified the cross-reference.

-p18, Figure 5: This figure is rather unclear. Does gradient.py do steps 8+9 of Algorithm 1? This figure also contains several equations, whereas I would expect steps of an algorithm.
R: We have modified the figure. gradient.py does both 8 and 9 of Algorithm 1.

-Section 5: it seems that the z-coordinate is always given first. Please explicitly mention this somewhere.

R: Okay this is now mentioned.

-p22, line 548-550 and Figure 8: for a fixed p, the relation between C and G is actually linear. In Figure 8, straight lines are only expected when using a log-log plot. I would suggest to use a log-scaling for the horizontal axis in Figure 8 or remove the linear fits.
R: We have removed the linear fits.

-p26, final paragraph: please double-check the domain sizes.
R: They have been double checked.

-p32, Table 3: it would be convenient to also have the final J here as an additional column.
R: Thank you for the suggestion, this is now annotated.

-Section 6: with 2 full pages, the conclusions seem to be overly long. Please try to make it shorter and more concise. A summary of the results and corresponding conclusions should be the main focus. Ideally, this section has one or just a few clear takeaway messages.

R: We have made the conclusions significantly shorter and more concise, it is now less than half of its original length.

0-p41, Appendix: The definition of superscripts x_k/x_l are missing. Also, does there need to be a summation over l in the last two equations?

R: Thank you for pointing that out. This was corrected in the new version.