Review of 'MPAS-Seaice (v1.0.0): Sea-ice dynamics on unstructured Voronoi meshes' by A. K. Turner et al.

I can only recommend this manuscript for the journal. It describes the MPAS-Seaice model which is based on the B-grid discretization for meshes obtained by the Spherical Centroidal Voronoi Tesselation. The sea ice velocity is placed at the corners of mesh polygons (generally hexagons). This leads to a discretization that is similar to that using sea ice velocities placed on triangle centers on triangular grids, which have been proposed earlier. However, MPAS-Seaice uses different methods (variational and weak) to compute the strain rates and the divergence of stresses. The description of these methods presents the most interesting part of the manuscript (in my opinion). The incremental remapping scheme used for the advection of scalars follows essentially the earlier paper, but even in this case there are details that might be of interest to the community.

The difficulty of B-grid discretization on quasi-hexagonal meshes lies in the existence of a numerical mode in velocities. This mode is related to the fact that the two corners of hexagons sharing a common edge (or the centers of triangles on a dual mesh) are geometrically different. One can see this in a transparent way by looking at triangles of dual mesh: The orientation of the pattern of neighbors is different for two triangles sharing a dual edge. This leads to some difference in discrete differential operators for these two triangles, hence the mode in velocities. This mode can be filtered by the discretization, which is the case for the weak method, and this mode can be emphasized by the discretization, which is the case for the variational method. While filtering may lead to better accuracy for strain rates, it leaves a non-trivial kernel in strain rates, i.e., there are non-trivial discrete sea ice velocities, which lead to zero stress divergence and thus may contaminate solutions. The variational method might show less accurate strain rates because of the the contribution of spurious velocity mode, but the mode will be suppressed in dynamics, and no null space will be created. The manuscript does not consider these details, and I do not think it should. However, they might be helpful in interpreting the results on the accuracy of strain rate and stress divergence computations. I think that the availability of variational and weak methods is an advantage of MPAS-Seaice, and future practice will show which one is optimal. I tried the weak method in FESOM (slightly modified for median-dual control volumes of FESOM2), but was satisfied with its performance only when I added stabilization that removes the null-space. So my guess is that the variational method will be more reliable.

In summary, this is an excellent contribution which should be published.

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Minor comments

line 26 'parallelization' does not require the EVP, but the EVP may improve scalability.

28-29 Why metric terms are specially mentioned? They must be present.

71 'twelve pentagons' are misleading if we think about the ocean mesh which does not cover the entire sphere.

85 'we rotate u and v' – the coordinate system is rotated, not u and v.

113-114 'Here, the directions ...' This has already been said above.

115 Say that you ignore boundary effects for simplicity, they simply lead to additional terms on the rhs of (3).

119 → 'contributions'

Formula (4) Already here the readers should be informed that the values of stresses are considered as parameters in D. It would be better to mention this explicitly in the argument list. The formula is misleading otherwise, for sigma is also a function of velocities. A still better approach is to take a test function instead of $\mathbf{u}$, as one will do in the finite element method.

179 'methods'→ basis functions

186 Does it imply that $2n_v^2$ coefficients have to be stored per scalar cell?

210 In FESOM sea ice implementation we define metric cosine at triangles, compute the areas of triangles, and then compute the areas of median-dual control volumes by summing 1/3 of the contributions from triangles. This is the same as taking the mass matrix and lumping it. In this way our scalar areas are always consistent with the areas of elements where the cosine of latitude is estimated. Your (34) is the lumped mass matrix too. In reality, if one thinks about your discretization as an example of finite-element method, the question about the control volumes associated to velocity points does not occur (which means that they appear as the result of the method). Let us write $\partial_t \mathbf{u} = \nabla \cdot \boldsymbol{\sigma}$. Multiply this with a test function $\mathbf{w}$, integrate, and apply the integration by parts in the rhs:

$$\partial_t \int \mathbf{w} \cdot \mathbf{u} dA = -\int \nabla \mathbf{w} : \boldsymbol{\sigma} dA,$$

where the boundary terms are omitted. The rhs here is the same as in (3) of the manuscript if one takes into account that $\boldsymbol{\sigma}$ is symmetric tensor. Now take $\mathbf{w} = \mathbf{w}_j W_j$, $\mathbf{u} = \mathbf{u}_j W_j$ where summation is implied over the repeating indices. The result is

$$\mathbf{w}_i M_{ij} \partial_t \mathbf{u}_j = -\int \nabla(\mathbf{w}_i W_i) \boldsymbol{\sigma} dA,$$

where $M_{ij} = \int W_i W_j dA$ is the mass matrix. The requirement that the equation above holds for arbitrary $\mathbf{w}_j$ is equivalent (for the rhs) to differencing of D over u_i and v_i (take $\mathbf{w}_i = (1, 0)$ or (0,1)). Thus we obtain the equations which correspond to your (32). Lumping the mass matrix leads to your (34). Thus, (34) gives the only consistent definition for the area of dual cells. Other definitions are not allowed.

228 The variational method is also a 'weak' method, differential operators are obtained by projecting on some functions. Your 'weak' method is actually

a finite-volume method.

Equation (38): one can start from it from the very beginning. Since the metric terms are added independently, it is sufficient to know how to express a gradient of scalar u and v, which is just the generalized divergence theorem.

Equation (46): Here, a part of metric terms is taken twice. Look at (45). If one takes a vector quantity instead of the tensor sigma, (45) will be an exact finite volume divergence. No metric terms are needed, they all are taken into account through l_i. In the case of second rank tensor, $\boldsymbol{\sigma}_i \cdot \mathbf{n}_i$ is a vector, and unit vectors are different at different i. Bringing them to a single location is equivalent to the account of the contributions from differentiating unit vectors. The result will be $(-\sigma_{12}, \sigma_{11}) \tan\theta / r$ in (46). Once again, the reason lies in using (45) for the primed quantities. Had you used finite differences to compute the divergence, the metric terms in (46) would be correct.

444 Why do you need a citation here? This is so by definition.

446 I doubt that it is of interest to look at the behavior of gradients at a point, because only weighted combination of gradients appears in the code. Averaged gradients make much more sense.

475 Indeed, the operator of gradient or divergence on triangle is only first order accurate.

A general comment here: If one starts from a smooth f sampled at the corners of primary mesh, there might be a grid-scale noise in the divergence of stresses for the variational method. The weak method may show cleaner results because of using a larger stencil for strain rate computations. However, conclusions can be different for sea ice dynamics evolving with time. The noise will likely be suppressed by dynamics in the variational method. In contrast, for the weak discretization, if some grid-scale noise will be added to the velocity, it may stay longer if it does not affect the discrete stress divergence. I think the behavior in 3.1.1 and 3.1.2 can be partly explained through the presence of mode. The transient behavior can be of interest as well (but not for this paper).

705 '400 nodes' Do you mean cores? Also it would be of interest to present the results shown in Fig. 22 in terms of the mean number of mesh cells per core. This should make the results for global and polar meshes similar, and the statement on the scaling will be more general. In FESOM (see www.geosci-model-dev.net/12/3991/2019/) scaling of sea ice is always sublinear, but continues to about 200-300 vertices (equivalent to cells on the Voronoi mesh) per core, which is similar to your result.