Review of Pandy et al., gmd-2021-339

General comments

This manuscript presents a new method that employs a parallel operation of 4DVAR for the application of atmospheric methane inversions. The original development is done for CO_2 application, but this study takes care of atmospheric lifetime in addition. The basic idea is to split the simulation time and run that split blocks in parallel to reduce computational time. This concept itself is not new, and becoming increasingly popular today as required simulation years lengthens. Such development is also important for near real-time understanding of greenhouse gas budgets. There are various ways to deal with the error in the initial condition of each block, and the presented method is scientifically sounding and applicable in other 4DVAR models. The work suites the scope of the journal, and is generally well presented. However, I would like to have some additional explanations and clarifications before publication.

- Study design: This study not only examines the computational performance, but also the accuracy they archived from the new method. Such accuracy would be more precisely examined using perturbed observations from the sequential simulation, i.e. "truth" you aimed to archive at. Was there any specific reason why you did not do so, but used real-life mole fraction data? When using real-life data, exactly the same set of observations should be assimilated, but that information was unclear from the manuscript.

- Calculation of uncertainty: PPVI introduces additional uncertainty to the model estimates, but also probably reduces computational time to calculate flux uncertainties. Please add how uncertainty will be calculated from PPVI properly taking care of the uncertainty in the correction factors. If that is impossible or still under development, please at least add discussion on this point. Would you in future inform uncertainty from those estimates e.g. for all simulation years?

- Consequence to the point above, the study examined only for cases where initial concentration fields are at equal grid resolution to the actual runs. However, the application is meant to run for higher resolution inversion as well. I would not ask to do additional inversion, but can you speculate from e.g. previous studies how it affects the flux errors? For example, you used a single correction factor (n^i_k) for each iteration/block globally and annually, but is it a good assumption?

- Introduction: I would like to have more background information on the block-based method. The authors present Chevallier (2013) for the bases of this study, but is it the only study that employs the block-based methods? How do the atmospheric methane inversions generally generate/correct initial conditions? This information would give further insight into what is new in this method.

- As I understood, the method is not to improve the calculation accuracy, but the reduce computational time yet achieving some accuracy at annual/regional levels. Please add in the Conclusion how widely you recommend this method to be used. The authors mention that CAMS simulations will be replaced by this method, but do you see other applications? Can we use those for a more detailed analysis of temporal/spatial distributions? Is this method also applicable in other inversion methods than 4DVAR?

- Additional figures/tables: I would like to see more details about the performances. Please consider including additional information.

- More details on computational time. How much clock time did each step take? As I understood, Table 1 presents the total clock time, but would like to see that for each step, and not only the totals.
- Mole fraction differences in a map. Figure 3 presents the representative sites from the SH and NH, but I would like to see the spatial distribution in more detail.

Specific comments
Please ensure that all terms are precisely defined, and not used interchangeably. The term "correction factors" is the one to pay special attention to. You have several correction factors, and were sometimes confusing.

P4 L115: Where the "emission to mole fraction conversion factor f = 0.361 ppb/Tg" came from? Please give information on how it was driven.

P5 L147: Please specify which of those steps can be done in parallel (i.e. independent of each other), and which of those steps are needed to re-do (update) for any new inversion runs with changes in inputs, chemistry or years.

P6 L172-174: Did you do any preprocessing of the data? Please also see general comment on point about the observations used.

P6 L181: How do you decide on "9 months"? Do I understand correct that you have 9 months of overlap, i.e. 4.5 months before and after the year-in-question?

P8 L 230: "a hypothetical 35-year inversions using the TM5-4DVAR setup."
What is this?

Technical comments
Please check some minor English language errors and technical typos.

Equations and notations therein. Please ensure that the vectors and matrices are in the bold fonts, and add vector/matrix sizes in the explanations. Please also check Figure 1 on this point.

Section 3.1: Please add coordinates of the sites.

Units: Please make sure that units are properly presented in text (e.g. Section 3.1 "The observation-prior mismatch is -6.7 ± 6," → The observation-prior mismatch is -6.7 ± 6 ppb?).

Figure 2: Please present the latitude/longitude units in N/E/S/W, i.e. 60°N instead of +60°.

Figure 4: Please add units to the x-axis.

Figure5: I assume this is regional total emissions, averaged over 1999-2010. Please consider rephrasing the caption.