Review of Metrics for Intercomparison of Remapping Algorithms (MIRA) applied to Earth System Models

by Mahadevan et al, January 2022

This paper presents a methodology for evaluating regridding algorithms and applies it to compare four different libraries: ESMF Regrid, TempestRemap, Generalized Moving-Least-Squares (GMLS), and WLS-ENOR. The methodology proposes a set of metrics to measure the quality of the regridding based on different criteria: sensitivity, consistency, conservation, monotonicity, dissipation.

The paper starts with an in-depth review of the mathematical basis of current regridding algorithms used in climate modelling. Then it defines metrics to evaluate the criteria listed above and presents the workflow to calculate the metrics for various meshes at different resolutions for different fields. Three quasi-uniform (Cubed-Sphere, Voronoi, and Regular Latitude-Longitude) meshes at five different resolutions and two regionally refined (Cubed-Sphere RRM, Voronoi RRM) meshes at three different resolutions are used. The regridding of two analytical fields and three real fields (Total Precipitable Water, Cloud Fraction, Global Topography) are studied.

To analyse the regridding consistency, the convergence rates with respect to the mesh refinement for quasi-uniform grids are calculated for the different libraries. The 2nd order regridding for ESMF Regrid and TempestRemap show degraded convergence rates in some cases while GMLS (and GMLS-CAAS i.e. applying a post-processing filter) and WLS-ENOR (that do not require computation of mesh intersection) show high-order consistency. Regarding global conservation, ESMF Regrid and TempestRemap are conservative by construction while GMLS is not; however, GMLS-CAAS and WLS-ENOR show good global conservation. The meshless and hybrid remap schemes, GMLS-CAAS and WLS-ENOR, preserve the global extrema conservation (monotonicity) while mesh-based algorithms implemented in ESMF Regrid and TempestRemap do not. The paper also provides a detailed analysis of the local extrema preservation for the four libraries and on the differences brought by using regionally-refined meshes.

The paper is well written and structured. As such, it provides an interesting contribution to the theory and practical implementation of code coupling for Earth System Modeling even if the precise impact of the conclusions for real-world coupled systems are not straight forward to infer. Overall, I recommend that the paper be accepted for publication after taking into account the comments I list here below.

Important comments

1. The notion of grid resolutions and convergence rate (as a function of the grid refinement) are central to the paper but are not well defined:
   - p.16, L.11 : in « uniform mesh refinement », are you talking about source or target mesh, or both ? Or do you include the “cross-resolutions”, e.g. the calculations done for CS(0) - MPAS(4)?

- p.24, L.6: what are exactly the 5 (uniform) and 3 (refined) resolutions used for the calculations? Please describe them more precisely as this is important for the calculation of the convergence rate.
- p.25, L.8: How you calculate the convergence rate (as a function of h the mesh refinement)? Please give more detail, this is central to the paper.
- p.25, L.9: Please better justify (or refer precisely to numbers in Table 1) why you state "ESMF first-order conservative scheme yields expected rates"? What rates do you expect? At the bottom of p.24, you provide some details on the expected convergence rates but the definition involves h and I don't think that h has been defined precisely although it is clearly linked to the grid resolution (see my comment above).
- p.29, L.1: again, please detail what are the "theoretically expected rates"; how do you evaluate them? Please illustrate why you state that "theoretically expected rates are observed" by referring precisely to numbers on Table 4.
- p.37, L.4 and p.38, Table 6. Be more precise on the resolution used for each grid; what does "the finest CS-MPAS RRM mesh combination" mean precisely?
- Figure 7-8-9-10-11: Define more precisely CS(0), CS(4), MPAS(0), MPAS(4) (see also my comment above for p.24, L.6.
- p.39, Fig. 13: Define more precisely CS-RRM(0), MPAS-RRM(0), CS-RRM(2), MPAS-RRM(2). In the caption, put "for a) coarse to fine, and b) similar refinements …"

2. For all figures, the x and y axes should be redrawn with bigger and clearer fonts.
3. Figure 6:
   - I don't understand the y axis. How can you have negative values for those error norms? Given equations (7) and (9), I don't think this is possible.
   - I suppose that each curve is for one specific pair of grids a specific resolution. For example, the left-most plot is for the grid pair CS-MPAS with a specific resolution of CS (among the 5 possible) and a specific resolution of MPAS (among the 5 possible). If I am right, please indicate which is the resolution for CS and which is it for MPAS.
4. p.32, L8-9 and Figures 8 and 9:
   - I think L_{min} should be G_{min} .and L_{max} should be G_{max} and refer to equations (12) and (13) as you are describing here global extrema and not local extrema.
   - Fig. 9 a): How can G_{min} be negative (for TempestRemap)?
   - Fig 9 b): Please specify in the captions where is the TempestRemap curve?
5. p.37, L.5: You write "The global errors with respect to all error norms are considerably smaller in WLS-ENOR and TempestRemap." This is particularly true for analytical function but not so clear for real fields. Can you comment on this on the text?
6. p.42, L.25: you completely exclude here "dynamic" grids, i.e., grids which definition evolve with time and for which the regridding weigths have to be recalculated at each timestep. No "offline" operations for those grids. Please comment on this.

Other comments:

- p.3, L.8-9 : This is not true for YAC. YC is a full coupler, it is not an interpolation library designed to generate weights to be consumed by another coupler.

- p.9, L5-7 : I think that SCRIP 1^{st} order conservative remapping is indeed similar to ESMF 1^{st} order conservative remapping, but I don't think this is true for the 2^{nd} order. Compared to the 1^{st} order, SCRIP applies weighted gradients in the longitudinal/latitudinal direction, while ESMF applies Kritsikis 2017, so SCRIP and ESMF are different for the 2^{nd} order.
- p.9, L20-21 : I guess ESMF patch algorithm would also fall in this catergory ?
- p.10, L.22 : I understand that the polynomials are integrated over each mesh of the supermesh, but then can you give some details on the procedure to go from the supermesh to the target mesh ?
- p.16, L.8 : can you detail why you write that L1 identifies errors in large-scale features and that L2 identifies errors in small-scale features ? I never understood that.
- p.20,Fig.2 : there should be an arrow between the source and target mesh definition (top right) and the bottom left of the figure, which represents the regridding per se as the mesh definition is certainly required for the regridding step
- p.24,L.13 : I think that if (N^{uni}_{type}) is the number of uniform grids, N sould be defined as
  $N = [(N^{uni}_{type})! / 2] * [N^{uni}_{ref]}]^2 *[N_{fields}]$
  and
  $N = [(N^{rrm}_{type})! / 2] * [N^{rrm}_{ref]}]^2 *[N_{fields}]$
  It happens here that N, as expressed in the paper, i.e $N = [(N^{uni}_{type})! / 2] = 3$ for $(N^{uni}_{type})=3$ and that $N = [(N^{rrm}_{type})! / 2] = 1$ for $(N^{rrm}_{type})=2$, gives the right result but this is just by chance.
- p.24,L.30 : I had problem remembering what *p* is. I understand that it is, e.g. for TempestRemap the order of the polynomials used for the reconstruction? It would probably be useful to explicitly note this “definition” of *p* in 2.3.2. and in 2.3.4. (For GMLS, p is clearly defined on p.11 L.28, which is good.)
- p.32, L.3: “for some scalar fields”: please provide example of those intensive variables such as SST
- p.32, L.12: recall which are the “mesh-based remapping schemes”, i.e., ESMF and TempestRemap.
- p.33, L.3: Please clarify in the text and in the captions where is TempestRemap on Fig. 10 c)
- p.37, Fig.12: Specify the metric in the captions
- p.37, L.15-17 compared to L.18-19: for me, those two sentences are contradictory: if ‘conserve2nd’ is only marginally better than ‘conserve’, how can you state that the superiority of the ‘conserve2nd’ is clearly demonstrated?
- p.42, L.3: When discussing extension to vector fields, you should also mention that regridding the vector components expressed in a local coordinate system linked to the grid or in the spherical reference system is wrong in principle. For proper treatment of vector fields, the source code should send the 3 components of the vector projected in a Cartesian coordinate system as separate fields. The target code should receive the 3 interpolated Cartesian components, recombine them to get a proper vector field, and project the resulting vector in its local reference system.

Minor comments:

- p.1, L.3 : I think the form 's can be used for people only. Here « component » is only a qualifier and I think you should write « one component computational mesh »
- p.3, L.30 : « interpolators » → « interpolations »?
- p.5, L.6 : « interpolator » → « interpolation »?
- p.6, L31 : I suppose that both « discontinuity detecting » and « a posteriori stabilization » both qualify the « procedure » ? If so I would write « … discontinuity-detecting and a-posteriori-stabilizing procedure »
- p.9, L.28 : I would change « weather and climate modeling » for « weather and climate applications»
- p.10, L.15-16 : put « (FV) » after « finite volume »
- p.10, L29 : for « potential function », do you mean the potential function expressing the tractory of the mesh boundary ?
- p.14, L.21 : repeated **back-and-forth** remap transfers ?
- p.14, L.24 : I think you should remove the "psi" after « the regridded field » as the regridded field is "R Ds psi"
- p.29, L12-14:, I don't think it because of the presence of discontinuities that you do a remap comparison? Please rephrase.
- p.45, L.27: I would put part of the sentence, i.e., "These low order ESMF maps are highly dissipative" with a "However," before the sentence starting with "In contrast, …" that introduces TempestRemap and then go on with the remark on the fact that the highly dissipative ESMF maps that can be corrected by the $O(h^2)$ TempestRemap maps.
- p.45, L.27: I would also give a general definition of *p* and *h* to make sure that someone reading only the introduction and the conclusion can understand the main findings of the paper.