Referee Comment on 'A new sampling capability for uncertainty quantification in the Ice-sheet and Sea-level System Model v4.19 using Gaussian Markov random fields'

October 2021

1 Summary

In this manuscript, the authors implement a method by which samples from a Gaussian Random Field with Matern covariance can be drawn. This method is based on the stochastic PDE approach. Using this sampling mechanism, the manuscript shows a handful of experiments demonstrating the sensitivity of mass flux to input fields (e.g. thickness, traction) perturbed according to this random noise. Tha manuscript also demonstrates an autoregressive approach to generating time-correlated noise, and shows the experimental distribution of mass loss due to an uncertain surface mass balance perturbed in this way.

The methods presented here are already mature, both from the perspective of the sampling technique and the ice sheet model. There's no data assimilation or attempts to tune the hyperparameters of the sampling method, so there's nothing to discuss with respect to inference. The patterns of sensitivity are about as one would expect. It's good that this capability exists, and will be useful in future studies. The paper is mostly written clearly, and does a decent job of placing this work in the appropriate context. As such, I only have some minor technical points, discussed below. However, this brevity is because there just isn't much scientific impact presented to comment on.

Minor Points

- 78 This WLOG statement isn't true: of course there's a loss of generality from assuming a mean of zero. However, it's reasonable to argue that it doesn't matter because you later plan to use these as relative perturbations to some a priori inferred mean.
- 106 It's worth noting here that Gaussian random fields are already being employed in Isaac, 2015 (already referenced in this paper at another location)

and also in Brinkerhoff, 2021 (https://arxiv.org/abs/2108.07263), both of which use low-rank approximations to yield the problem tractable.

- 144 It's worth mentioning the downsides to the SPDE approach as well: in the inverse context, it provides no immediate solution for how to represent posterior covariance.
- 160 It's not immediately obvious that the matrix square root (particularly of K) should be easy to compute, or that it should retain sparsity. If it doesn't retain sparsity, then the scalability of this method could be substantially limited.
- **Eq. 16** Is there a more rigorous means to quantify what potential errors that this mass lumping step induces?
- **192** I don't understand this paragraph. if $\phi = 1$ then shouldn't nothing change at all (contrary to the statement that it leads to a random walk)? It seems to me that the equations would yield

$$x_t = x_t + \epsilon_t$$
$$\epsilon_t \sim \mathcal{N}(\mu = 0, \sigma^2 = 0)$$

I may be misunderstanding, but perhaps clarification would be helpful here.

- 285 Can 'converged' be rigorously defined here?
- **Fig. 16** Might this be better represented as a single plot, with *t* as the independent-variable?