Response to the minor comments on **"Implementation of a Gaussian Markov random field sampler for forward uncertainty quantification in the Ice-sheet and Sea-level System Model v4.19"** ***by*** **Kevin Bulthuis and Eric Larour**

The authors did an excellent job addressing the reviewer's concerns regarding the manuscript. Here are some final (more minor comments)

We thank the referees and the editor for their last comments and have updated the manuscript based on the last minor comments.

1. It would be great if the authors would incorporate some of the references mentioned in the responses to the reviewers into the introduction. None of discussions are reflected in this section, which I think should be updated.

   We agree with the editor and the referee that the references mentioned in the responses to the reviewers should be mentioned in the introduction. We have changed the introduction to mention that Gaussian random fields have already been employed in glaciology (Isaac, 2015; Babaniyi, 2021; Brinkerhoff, 2021) while the SPDE approach has already been employed in the context of Bayesian inverse problems in glaciology (Isaac, 2015; Petra, 2014).

2. Equation (1) is not very clear. Could the authors be a bit more concrete, where do these parameters enter in the model, what the ultimate goal is with this model?

   Equation (1) is just a general model in statistics to represent a random field. It is simply used to decompose the random field into a deterministic component (its mean or trend) and a stochastic component (which is really what we are interested in in this paper). But we recognized that in the context of this paper, introducing notations (like μ and ϵ) that are not used in the remainder of the paper can bring some confusion. To avoid complicating the text, we have decided to remove this equation from the paper. The introductory paragraph in Section 2 now simply writes as "In this work, we model a spatially varying uncertain input parameter as a random field $\{x(\boldsymbol{s}), \boldsymbol{s} \in D\}$ indexed over the computational domain D. In the following, we assume this random field to be a Gaussian random field with zero mean."

3. It would be great if the authors could give some intuition or add a reference for the uncertainty representation in Equation (20).

   The uncertainty representation in Equation (20) follows the perturbation approach in Larour et al. (2012), though not written explicitly as we did in the paper. This equation can be seen as perturbing each input quantity with a multiplicative noise proportional to a local (measurement) error margin. We have added the reference and this interpretation in the text.

References

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