***Response to the Referee #2 comment on* "A new sampling capability for uncertainty quantification in the Ice-sheet and Sea-level System Model v4.19 using Gaussian Markov random fields" *by* Kevin Bulthuis and Eric Larour**

The authors added support for Gaussian random fields with Matern-type covariance functions in ISSM (Ice-sheet and Sea-level System Model) and in this paper, they show the resulting new capabilities ISSM offers for forward UQ. The authors describe how to draw samples from a Gaussian distribution with Matern covariance and use these samples to run forward UQ for a few ice sheet flow models.

The topic of the paper is interesting. In terms of novelty as mentioned above, the mathematical techniques described in this paper are all well-known, hence there is not much one can comment on this aspect. The validation tests ("sanity checks") are nice and may be useful, especially to the users of ISSM. Below I list a few comments/concerns:

1. The examples are interesting but lack sufficient details. For instance, there are several things explained in words but there is no concrete problem/mathematical description, only several references to previous work are provided. The authors spent significant effort to explain the sampling procedure, which is known, but unfortunately rushed through the numerical experiments.

   We agree with the referee with the fact that the description of the problem may seem succinct, especially for people not familiar with ice-sheet modeling. However, the focus of this paper is on the sampling procedure (even though it is known for people in UQ) rather than the description of the ice-sheet dynamics, which we believe is not required to understand the numerical experiments. We will be pleased to clarify some points on the numerical experiments if the referee has any specific question on them.

2. How are the parameters (mean, correlation, etc.) chosen for the Gaussian distributions one samples from for the forward UQ? Without a proper data assimilation or inversion process for these ice sheet problems, not sure how realistic these distributions are and certainly not sure how much these can be trusted for prediction and UQ.

   The mean values for the Gaussian distributions are based on reference values derived from bedrock and surface elevation data for the ice thickness, a thermal model and surface temperature data for the ice hardness and an inverse method for the basal drag. Regarding the choice of the parameters of the SPDE, our goal was not to provide realistic distributions or predictions but rather to illustrate how the implemented sampler can be used for forward UQ analysis. For this reason, the parameter range (30 km) has been chosen rather arbitrarily but is believed to represent supposed variations in ice thickness. To assess the impact of the parameter range, we also provided results for other parameter ranges. In order to provide trusty predictions, these parameters should indeed ideally be constrained with a proper data assimilation or inversion process. Such methods are however beyond the scope of this manuscript but we plan to constrain these parameters in a future work.

   We have added the following sentences at the end of Section 4.2: For this application, we have chosen the values of the parameter range arbitrarily without any proper data assimilation or inversion process. The use of an unconstrained parameter range is justified by the fact that we do not seek to provide new probabilistic mass-balance estimates for the Pine Island Glacier but seek only to demonstrate the interest of the implemented stochastic sampler for forward UQ analysis.

3. What was the dimension of the unknowns or quantity of interest? Are the sampling and forward UQ processes described scalable and computationally tractable for large-scale ice sheet problems?

   We thank the referee for this question. Indeed, the paper lacks some details regarding the dimension (or size) of the problem. For the problem considered in Section 4, the mesh is made of 2085 elements and 1112 nodes, which corresponds to a relatively small dimension compared to other ice-sheet problems. For our purpose, a higher spatial resolution was not required. Therefore, the dimension of the discretized random field is of 1112. The quantity of interest is of dimension 13 (for the mass fluxes through the 13 gates), even though the calculation of each mass flux requires the prior calculation of the ice thickness and the velocity field. In our example, the cost of the sampler is relatively negligible (only a few tenths of seconds per sample). The cost of solving the ice-sheet model is of a few seconds per run. We have added these values to the manuscript.

   Regarding the scalability of the method, the sampler scales well for larger ice-sheet problems. The SPDE approach in itself is highly scalable and efficient for large-scale problems (see for instance Isaac et al. (2015)) . The computational cost of the sampler is in any case (much) lower that the cost of solving the ice-sheet model. Therefore, the cost for the forward UQ analysis is determined by the computational cost of the ice-sheet solver, which can indeed be computationally prohibitive if the dimension of the problem at hand is too high (especially for transient simulations). However due to the nature of the Monte Carlo simulations, all simulations can be run in an embarrassingly parallel way. If computational resources are limited, then other methods for UQ propagation like advanced Monte Carlo sampling approaches or surrogate modeling might be of interest to be investigated.

4. How exactly the convergence of the samples is assessed?

   This question has also been raised by referee#1. The statement 'reasonable convergence' might seem a little bit vague. We can estimate the estimation error for the mean and standard deviation of the mass flux via bootstrapping. The bootstrap error is of a few hundredths of percent for the mean value and a few percents for the standard deviation. We have indicated these values at the end of line 285.

5. It is unclear what the novelty in this paper is. Is the goal to present these new capabilities ISSM provides and compare the new results with previous forward UQ studies? If so, the abstract of the paper is a bit misleading. This suggests that the authors propose a new sampling technique for UQ.

   Indeed, the novelty of this paper is not in the sampling technique in itself, which is already well-established in UQ and has been used in a number of studies including applications in glaciology (though with a different goal). In particular, the use of the words "new sampling capability" might be a bit misleading and might suggest that we are actually proposing a new sampling technique in uncertainty quantification. The goal of this paper is more to introduce a (new) stochastic sampler for Gaussian Matérn random fields within ISSM and to illustrate the interest of this sampler for forward UQ analysis in ice-sheet models. While the sampling method is well-established in UQ, we thought this paper is of interest for ice-sheet modelers who are less familiar with uncertainty quantification. We have modified the abstract and replaced "(new) sampling capability" by "stochastic sampler" in the main text in order to make the manuscript more consistent with this goal.

6. Also, from the title (and the abstract), it is not clear that only forward UQ is being considered.

   Indeed, the title may be a bit misleading. To highlight the fact that only forward UQ is being considered and that we are not proposing a new sampling technique (but only implementing an existing one), we have changed the title of this paper as "Implementation of a Gaussian Markov random field sampler for forward uncertainty quantification in the Ice-sheet and Sea-level System Model v4.19".

References

Isaac, T., Petra, N., Stadler, G., and Ghattas, O. Scalable and efficient algorithms for the propagation of uncertainty from data through inference to prediction for large-scale problems, with application to flow of the Antarctic ice sheet, J. Comput. Phys., 296, 348–368, https://doi.org/10.1016/j.jcp.2015.04.047, 2015.