**Supplementary material**

**Supplementary – Limiting soil under- and over-saturation**

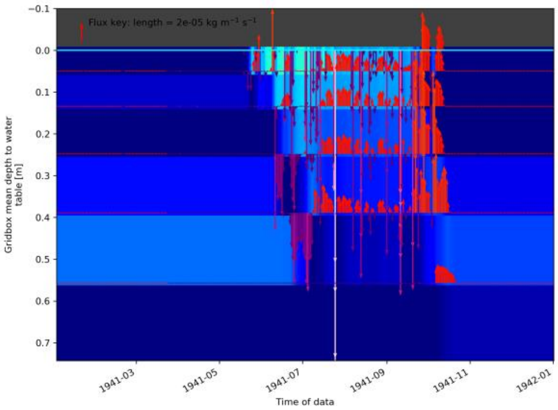


**Figure S 1:** l_soil_sat_down = false - the excess flux into a saturated layer at the beginning of winter is passed up and out of the soil, as the layer that would become saturated modifies the layer flux at its upper boundary to avoid this, passing the problem to the next layer.

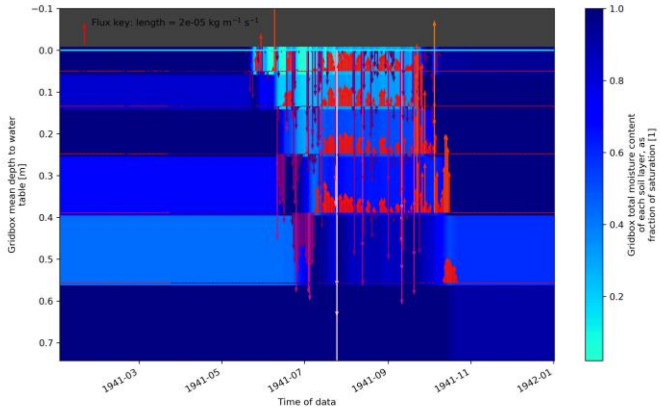


**Figure S 2:** Soilsat updown - the flux is not passed upwards, as in the case of oversaturation it is always the flux into a saturated layer which is limited, in this case, the flux in from the lower boundary.

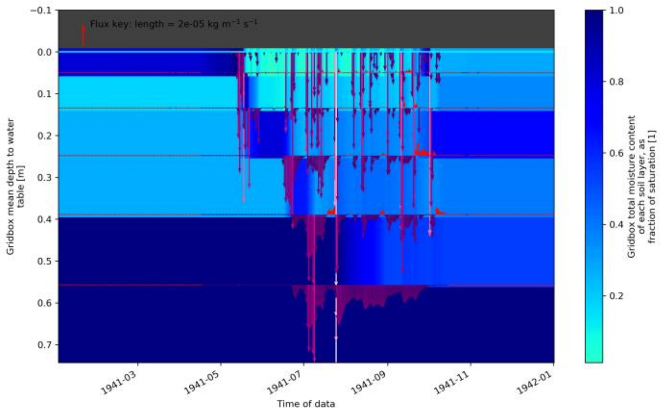


**Figure S 3:** l_soil_sat_down = true – the downwards flux into a saturated layer is mitigated by altering the water flux at the lower boundary of the saturated layer, causing the same problem in the layer below and so passing water down and out of the soil. The same effect is not seen in soilsat updown.

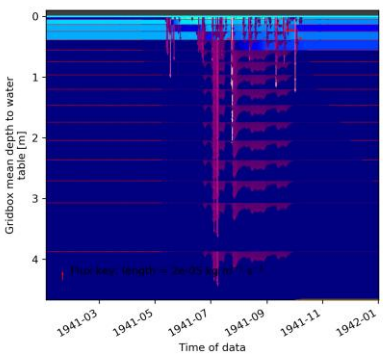


**Figure S 4:** l_soil_sat_down = true - water is able to fully infiltrate the frozen soil.

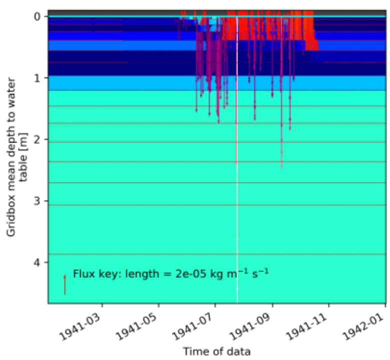


**Figure S 5:** l_soil_sat_down = false - water is unable to infiltrate a frozen soil.

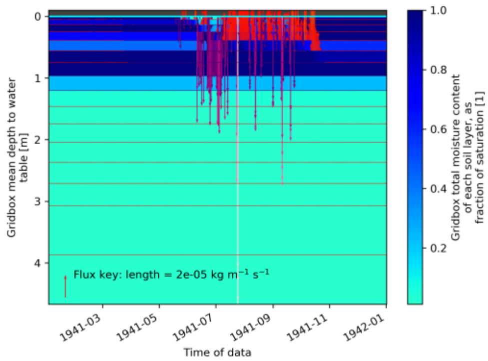


**Figure S 6:** Soilsat updown - note the similarity to soilsat up.

It is worth noting the differences and similarities between the schemes in Figure S 5, Figure S 6 and Figure S 6. For a fresh spinup, water is only able to infiltrate the frozen soil and saturate the lower soil layers in soilsat down. This means that care needs to be taken to set sthuf (the initial soil wetness) correctly for soilsat up and soilsat updown. Conversely, soilsat up and soilsat updown tend to result in wetter surface layers, though soilsat updown occasionally has a dry surface layer in winter where soilsat up would have a wet one, due to water being able to pass upwards through the saturated ice layer in soilsat up.

**Supplementary – Reasoning for choosing horizontal flows over sloped flows**

This section expands the discussion within the main text, providing reasoning as to why it was decided to connect layers horizontally, rather than having the connections sloping and connecting each layer to its corresponding layer in the other tile. The strengths and weaknesses of both schemes are also discussed.

Lateral flows of water were introduced into JULES using an approach mirroring the existing calculation of vertical fluxes, where fluxes are calculated based on the difference in matric potential between soil layers due to their level of saturation. Existing functions for calculating the layer matric potentials and hydraulic conductivities are used, and the fluxes interfaced into the existing code for the water balance for each layer. Unlike for the vertical fluxes, no implicit correction is used, as fluxes are assumed to be relatively small. Two options were considered for connecting laterally adjacent layers: a sloped scheme where layers are sequentially connected to their corresponding layer in the neighbouring tile, and a horizontal scheme where a layer is connected to any layers horizontally adjacent to it, taking into account the area of overlap of each connection (Figure 3). The sloped flow scheme was used by Heather Rumbold in JULES 3.2 in a gridded UK run (https://jules.jchmr.org/sites/default/files/Ashton_0.pdf, accessed 3rd June 2021). Here, this sloped flow code was adapted to take into account the areas and geometries associated with permafrost microtopography. The horizontal flow scheme is new to this study. In initial tests with small elevations (or large horizontal scales) both schemes appeared to generate very similar results. The horizontal scheme was eventually chosen and further extended for this work, adding horizontal exchange between the soil of the raised tile and ponded water on the low tile, and integration of the horizontal fluxes with JULES' oversaturation limiting code.

In JULES, the vertical water flux between layers, $W\ (kgm^{-2}s^{-1})$ is calculated using Darcy's law:

$$W = -K\left(\frac{\Delta\psi_m}{\Delta z} + 1\right), \tag{16}$$

where $K\ (kgm^{-2}s^{-1})$ is the hydraulic conductivity, $\Delta z\ (m)$ is the distance between the centres of the layers, and $\Delta\psi_m\ (m)$ the difference in matric potential between layers. $\psi_m$ and K are calculated using either the Brooks and Corey (BC) or the Van Genuchten (VG) relations. When calculating lateral flows, if flows are horizontal, the change in gravitational potential with distance $d\psi g /dL = 0$ and hence the 1 can be dropped, or for sloped flows, $d\psi_g/\mathrm{dl} = \sin(\phi)$, where $\phi$ is the angle of the slope.

Both sloped and horizontal flow schemes have their own strengths and weaknesses. For sloped flows each layer can be matched 1:1 with its corresponding layer. This means that it is simpler to keep track of where water is coming and going, and physically makes sense where landscape changes are continuous, and water flows primarily follow layers of higher conductivity. The weakness is that an imbalance of soil moisture is enforced, and the balancing of the water-table across both tiles becomes unachievable, in part due to the lack of a calculated positive pressure $\psi_p$ in a saturated lower layer to balance the greater $\psi_g$ of the raised layer. Figure S 7 A. illustrates the problem: if the water table is to balance across tiles via sloped flows, then water must flow from a saturated layer (pictured is the flow from layer 3), down the slope through another saturated layer which is further beneath its local water table, and up through further saturated layers to the water table. If layers have different conductivities, and indeed could be frozen, then it is non-trivial to solve the rate at which water flows from one tile to the other, leading to the need for iterative schemes. Furthermore, it cannot be done by the current method used by JULES to solve the Richards equation where $\psi$ is calculated based on the layer saturation, as it requires a consideration of $\psi_p$. Conversely, for the horizontal flow scheme, a layer may overlap more than one horizontally adjacent layer (Figure S 7B.), meaning flows are harder to keep track of and correct, but there is no danger of flows being dominated by $\psi_g$. Although the rate may be incorrect, the water table can now balance across tiles, even without considering saturated to saturated flow paths, through horizontal flow from saturated to unsaturated layers. The VG relations are more appropriate for this scheme as they avoid a mismatched $\psi_m$ when both layers are saturated.

The horizontal flow scheme raises the question of what to do with the upper layers of the raised tile, which may have no horizontally adjacent layer. If the water table in the elevated tile is above the surface of the lower tile, and above the level of the surface of any ponded water (if present), water will be able to laterally egress the soil (Figure S 7, Figure 3 C.). Again, this is not possible in the sloped connection scheme. Similarly, if a pond is present on the low tile and the surface of the pond is above the level of the water table in the high tile, water can flow from the pond laterally into the soil (Figure S 7 D.). For these flows it is therefore necessary to determine the level of the local water table. For the high tile, for a particular connection, the local pressure head in the high tile $\psi_{ph}$ $(m)$ is the height of the water table above the midpoint of the vertical overlap. The overlap and midpoint may be different to the layer thickness and midpoint due to the layer being partially saturated, or if only part of the layer is above the pond height (Figure S 7 C. and D.). For water egress, $\Delta\psi = \psi_{ph}$ as there is no matric suction from air. For water ingress, $\Delta\psi$ is the height of the pond above the midpoint of the connection, plus the matric suction of the layer the water is entering.

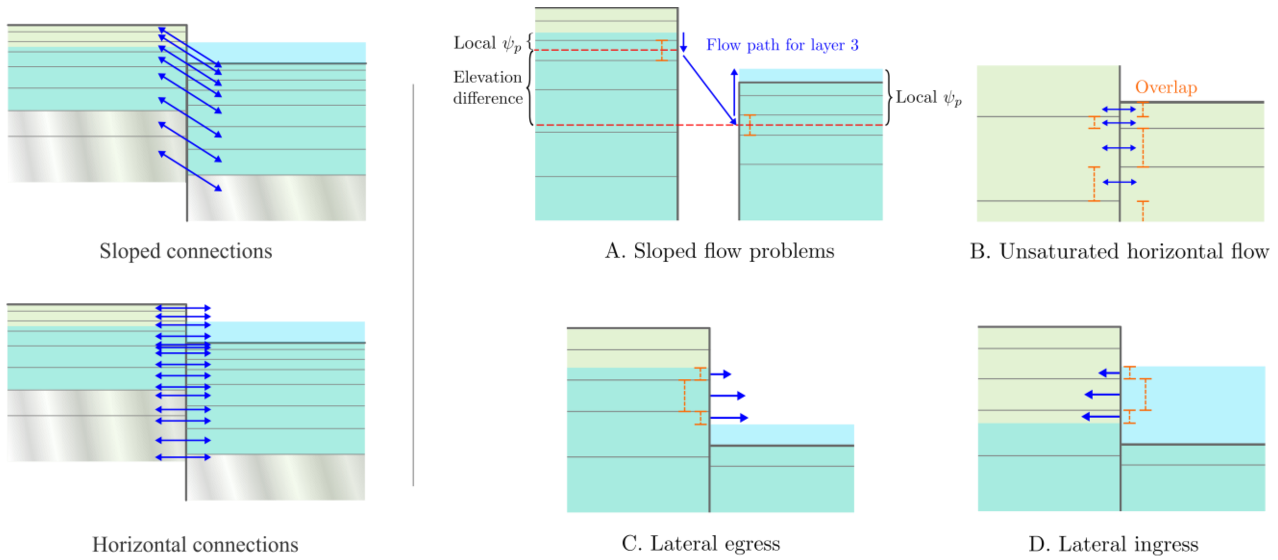


**Figure S 7:** Paths of flow – see above for details.

While the local pressure head must be calculated in order for horizontal flows between the soil and the air or the pond and the soil to be modelled, flows are always from a saturated or partially saturated region to an unsaturated one. The model therefore does not implement saturated lateral flows, which are conceptually important for flows between polygons (Wales et al., 2020), meaning that advective flows of heat may not be properly represented. However, the model will still be able to act to balance the water table and moisture potentials between tiles, though the rate with which equilibrium is reached may be different. Any discrepancy in rate is however expected to be less than the usual timescales over which the water table changes, and therefore not a problem.