Comments to the manuscript

General comments

This paper deals with the Double Fourier Series (DFS) on the sphere where new DFS functions are used to represent the variables on the global domain. Discretization procedures for the spatial differentiations, elliptic equations, and the shallow water models are shown in some detail using the trigonometric identities. Combining the DFS method with the semi-Lagrangian time-differentiation, the paper provided simulation results for a couple of shallow water model test cases, including the standard test cases of Williamson et al. (1992). The author emphasizes that the DFS functions in the present study improves the simulation results over the DFS models in the previous studies. However, it is hard to be convinced of the improvement of the new DFS method due to less rigorous assessment of the solution method for differential operators such as elliptic equations, least square method, and also limited test case results. Specific comments are shown below.

Specific major comments

[1] One of the most important aspect of the paper is that the DFS expansion coefficients ($T^c_{n,m}, T^s_{n,m}$) are calculated based on the least square method, as is shown section 2.3. I am afraid, however, that the derivation procedure does not seem to be the least square method which is required for determining the expansion coefficients. The residual function here is defined using the difference between the spectral representation of the function (T^c_m, T^s_m) two different sets of DFS. The fact that, for the spherical harmonics model (SHM), the spectral coefficients are determined in the least square sense on the spherical domain is explained as below:

$$E = \int_0^{\pi} \left[T^c_m(\theta) - \sum_{n=m}^{N} c_n P^m_n(\theta) \right]^2 \sin\theta d\theta$$

$$\frac{\partial E}{\partial c_n} = 0 \ \cdots \ \text{least squared error}$$

$$2\int_0^{\pi} P^m_n(\theta) \left[T^c_m(\theta) - \sum_{n=m}^{N} c_n P^m_n(\theta) \right] \sin\theta d\theta = 0$$

$$\therefore \ c_n = \int_0^{\pi} P^m_n(\theta) T^c_m(\theta) \sin\theta d\theta$$

$$\left[\because \ \int_0^{\pi} P^m_n(\theta) P^m_{n'}(\theta) \sin\theta d\theta = \delta_{nn'} \right]$$

That is, c_n is obtained with least squared error on the sphere.

The least square method for the SHM in the present study is different from above equations.

[2] It looks like that the equations (24a)-(24d) are just algebraic equations resulted from simply multiplying $\sin\theta$ or $\sin^2\theta$ or $\sin^4\theta$ to the same equation. For instance, in the case of odd *m* (>=3), it follows:

$$\begin{aligned} T_m^c(\theta) &= \sum_n T_{n,m}^c \sin^2\theta \sin n\theta \\ &= \sum_n \tilde{T}_{n,m}^c \sin n\theta \\ \sin^2\theta T_m^c(\theta) &= \sum_n T_{n,m}^c \sin^4\theta \sin n\theta \\ &= \sum_n \tilde{T}_{n,m}^c \sin^2\theta \sin n\theta \end{aligned}$$

$$\sum_n T_{n,m}^c \sin^4\theta \sin n\theta = \sum_n h_{n,m}^c \sin n\theta$$

$$\Rightarrow \underbrace{\begin{bmatrix} 5-diagonal \\ matrix \\ \\ \\ \end{bmatrix} \begin{bmatrix} T_{1,m}^c \\ T_{3,m}^c \\ T_{5,m}^c \\ \vdots \end{bmatrix}}_{\text{lhs of }(24d)} = \begin{bmatrix} h_{1,m}^c \\ h_{3,m}^c \\ h_{5,m}^c \\ \vdots \end{bmatrix}$$

$$\sum_n \tilde{T}_{n,m}^c \sin^2\theta \sin n\theta = \sum_n h_{n,m}^c \sin n\theta$$

$$\Rightarrow \underbrace{\begin{bmatrix} 3-diagonal \\ matrix \\ \\ \\ \end{bmatrix} \begin{bmatrix} \tilde{T}_{1,m}^c \\ \tilde{T}_{3,m}^c \\ \tilde{T}_{5,m}^c \\ \vdots \end{bmatrix}}_{\text{rhs of }(24d)} = \begin{bmatrix} h_{1,m}^c \\ h_{3,m}^c \\ h_{5,m}^c \\ \vdots \end{bmatrix}$$

It should be explained why above equations are the same as those the author derived.

[3] The largest wavenumber (truncation wavenumber) in (8b) should be determined considering the grid structure, grid[0] or grid[1] or grid[-1] to make completeness of spectral expansion issue clear (refer to Cheong et al. 2004).

[4] Section 2.12 presents the Laplacian operator and the Poisson's equation. The accuracy of the new DFS method for these basic operators and others such as biharmonic diffusion operator should be addressed with detailed error magnitude. Also important is the global mean associated with the Poisson's equation.

[5] One of the most basic test case is the cosine-bell advection, which is not included in this manuscript. The test case is simple but useful to demonstrate the advantage and disadvantages of a numerical method.

[6] It is very nice to see that the simulations are carried out without numerical instability even without horizontal diffusion. The author may address why it is possible. Is it due to the diffusive property of the semi-Lagrangian?

[7] **Figure 2**. The problem setting is quite strange. In principle, any scalar function with $m>0$ should vanish at poles. Nevertheless, the 'original' function is given to have value of unity at north pole. Therefore, the computation and comparison are not meaningful.

[8] **Figure 5**. Result of DFS0 appears to be too much smooth compared to DFS_old. Why is it?

Specific minor comments

[1] The right hand side of (25) should be represented with matrix-vector multiplication as in the left hand side.

[2] Terms associated with $\tilde{T}_m^{c,J}$ and $\tilde{T}_m^{s,J}$ in (36) do not appear in (37). The reason should be explained.

[3] Equation (B1) can be found in Cheong 2000a.

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