[Reply to Topical Editor report]

We are very grateful to the topical editor and the reviewers for their useful comments and the decision "Publish subject to minor revisions". Below we provide point-by-point responses to the comments.

Dear Author,
Please consider the following points raised by the reviewer.
"The author made a good effort to make the paper more accessible and I appreciate the responses to my comments. Except one perhaps, if they already have a parallelised version, why not comment on the connectivity needed between processors (need all zonal values for zonal expansion but parallel over m, need xx for the meridional, parallel over xx ?).
But I appreciate this may be the subject of future work to refine, so looking forward to more information on this in the future.

Thank you very much for the comments. In this paper, we use the DFS shallow water model. This model is parallelized with OpenMP, not with MPI, and can utilize only one node of a supercomputer. We have already developed the DFS global atmospheric model, which is parallelized with both OpenMP and MPI and can utilize many nodes. So, we will describe the parallelization for the DFS global atmospheric model in the future paper.

I find some of the notation too much (e.g. upper case N,M and equation 15 (needed?), many of these could be dropped in my view, just stating that the prognostic variables are expressed as truncated series, this would make many equations a lot less busy.

We also think that there are many equations and it is better if some of these can be dropped. On the other hand, we would like to distinguish between the original variable $T(\lambda,\theta)$ and the truncated variable $T^{N,M}(\lambda,\theta)$, because it is important that the values of $T^{N,M}(\lambda,\theta)$ are generally different from those of $T(\lambda,\theta)$. In our new DFS method, expansion coefficients are determined so that the difference between $T^{N,M}(\lambda,\theta)$ and $T(\lambda,\theta)$ is minimized.

I think what is missing is a quick summary "cook book", which could take the form of (if I understood correctly):
(1) Define expansion in double Fourier series, with zonal expansion [equation (8)] and meridional expansion [equation (10)]
(2) Apply recursive formulas for the inverse transform from spectral to gridpoint [equations 13-14)
(3) For the direct transform (grid to spectral) using the new DFS method:

a) define new expansion (tilde values, equation 23), and evaluate coefficients by forward transform directly from the values at the gridpoints (see appendix B)
b) minimise (with variation of the unknown spectral expansion coefficients of 10) over the sphere the squared residual R defined in equation 25: R is the distance between the series approximations defined in 10 and 23 leading to conditions 26 and with 27 to equation 28, noting the orthogonality condition with respect to R.
c) Evaluate the unknown spectral coefficients using the resulting matching conditions with the known expansion coefficients (tilde), derived by inserting 10 (in lhs of 28) and 23 (in rhs of 28) leading to equations 30 and the evaluation procedure 31.

Thank you for the advice. The essential summary (cook book) of the new DFS method is useful, and we have added the essential summary (cook book) of the new DFS method in Sect. 3.10. In the last paragraph of the introduction (Sect. 2), we have added "Section 3 …, and includes the essential summary of the new DFS method". In the first part of Sect. 3, we have added "The essential summary (cook book) of the new DFS method is in Sect. 3.10". In Appendix I, we have added "(See Sect. 3.10 for the spectral transform with the new DFS method.)"

Notably, above is slightly different to what is in the current manuscript, in particular the derivation of 29 with appendix D and the current definition of R. I find confusing even if it leads to the same result. Is my interpretation above incorrect ?

The reviewer's interpretation is basically right. Moreover, although $T_m^{c(s)}(\theta)$ and $\tilde{T}_m^{c(s),N}(\theta)$ are generally different, Eq. (D4) is satisfied. By substituting Eqs. (10) and (23) into Eq. (29), we lead Eq. (30). In Appendix E, we forgot to describe that Eqs. (10) and (23) are used, and we have changed "By using Eqs. (11)" to "By using Eqs. (10), (11), (23)". We have also corrected mistakes in Eq. (E2).

Other comments:
The new title is very long, I am wondering if "Improved double Fourier series for the shallow water equations on the sphere" would be more fitting.

Thank you for the advice. Surely, the new title is long, and the shorter one is preferable. We have changed the title back to "Improved double Fourier series on a sphere and its application to a semi-implicit semi-Lagrangian shallow water model". The words "semi-implicit semi-Lagrangian" can be removed. However, since the explanation of time integration using a semi-implicit semi-Lagrangian scheme is included in the paper, we keep "semi-implicit semi-Lagrangian" in the title.

The rest are details that belong in the abstract. In the abstract: "These small oscillations..."

Thank you for the advice. We have removed “These small oscillations …” in the abstract. We have also changed “except that small oscillations” to “except that very small oscillations”, because these oscillations are very small and seem to have almost no problem in practical use. We have also modified the parts related to the small oscillations a little in Sects. 6 and 7.

I would also remove the sentence in the conclusions : "The Eulerian shallow water model using the new DFS method without horizontal diffusion is also likely to be stable, although we have not tested it yet."

Thank you for the advice. We have removed this sentence.

Overall the text has changed/moved a lot, and it is difficult to verify all the revised cross references, but I assume editorial work will check this once more.

We have checked all the cross references.

Otherwise I found the appendices helpful, albeit many. In the end it is justified as this serves as a documentation of a new dynamical core and a reference in comparison to the spectral transform models based on spherical harmonics.

Thank you for the comment. We also agree with this comment.

I would like to thank the author for his hard work on addressing the different test cases and questions related to the accuracy of the wave operator."

Thank you for the comment. We are grateful to all the comments from the reviewers and the editor, which have been very useful to improve the paper.

Non-public comments to the Author:
I very much subscribe with the reviewer about the need for an essential summary (a "cook book" as the reviewer says). I think it would greatly improve the manuscript from a pedagogical point of view, ie explaining the method in a nutshell for newcomers. This could go in the introduction, the conclusions, or both with the right choice of words and references.

Thank you for the advice. We have put the essential summary (cook book) of the new DFS method in Sect. 3.10. In the introduction (Sect. 2), Sect. 3, and Appendix I, we have added references to the essential summary as described above.

Please verify all the revised cross references, for citations and equations etc..

We have verified all, and have corrected many minor mistakes. There was a mistake in the calculation of kinetic energy spectrum, so we have also modified Figs. 8, 10, 13 and S2. (We hope that there are no more mistakes.)