# 1 Response to reviewer 1

We thank the reviewer once again for his feedback, which has helped significantly improve the quality of the manuscript.

## 1.1 Major comments

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***Reviewer statement 1***: *Assuming that $\nabla$ is a regular del operator ($\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$), Eq. 1 for the nondivergent filtered velocity field, $\nabla \cdot \mathbf{u} = 0$, is fine, but what does $\nabla \mathbf{u} \cdot \mathbf{u}$ in (2) then mean? Operator $\nabla$ does not apply to a vector $\mathbf{u}$ as $\nabla \mathbf{u}$, so apparently $\nabla \mathbf{u} \cdot \mathbf{u}$ is supposed to read $\nabla(\mathbf{u} \cdot \mathbf{u})$, but this is wrong, as one expects this term to be $(\mathbf{u} \cdot \nabla)\mathbf{u}$ (provided $\nabla \cdot \mathbf{u} = 0$).*

**Response**: We thank the reviewer for this comment. Overall it depends on how the $\cdot$ operator is defined, but we agree with that the chosen notation was confusing and we now switched to the more standard tensor (rather than vector) notation. The nonlinear term is now written as

$$u_j \frac{\partial u_i}{\partial x_j}.$$

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***Reviewer statement 2***: *Furthermore, $\nabla$ is also applied to tensors ($\boldsymbol{\tau}$ and $\boldsymbol{\tau}^{\mathrm{SGS,dev}}$) as $\nabla \cdot \boldsymbol{\tau}$ and $\nabla \cdot \boldsymbol{\tau}^{\mathrm{SGS,dev}}$, while the operation $\nabla\cdot$ for tensors is generally not defined. Besides this, terms $\nabla \cdot \boldsymbol{\tau}$ and $\nabla \cdot \boldsymbol{\tau}^{\mathrm{SGS,dev}}$ (whatever they mean) must enter (2) with the same sign, as $\boldsymbol{\tau}$ and $\boldsymbol{\tau}^{\mathrm{SGS,dev}}$ are quantities of the same physical nature (kinematic stresses). But in fact, concluding from how these variables are defined, $\boldsymbol{\tau} = -2\nu\mathbf{S}$ and $\boldsymbol{\tau}^{\mathrm{SGS,dev}} = -2\nu^{\mathrm{SGS}}\mathbf{S}$, they are rather kinematic momentum fluxes (negative of stresses, cf. Eqs. 5 and 6).*

**Response**: We thank the reviewer for his comment. We have now fixed the sign convention by introducing a minus sign in the viscous term definition. We however decided to keep these terms defined as *stresses*, since this is what they are from physical perspective, irrespective of the sign, and since this is also how they are typically referred to in the literature (see e.g. Pope (2000)). With regards to the divergence operator applied to a second order tensor, we respectfully disagree with the reviewer's comment since

$$\nabla \cdot \boldsymbol{\tau} = \boldsymbol{e}_k \frac{\partial}{\partial x_k}[\boldsymbol{e}_i \boldsymbol{e}_j \tau_{ij}] = \delta_{ki} \boldsymbol{e}_j \frac{\partial \tau_{ij}}{\partial x_k} = \boldsymbol{e}_j \frac{\partial \tau_{ij}}{\partial x_i} \ .$$

Note however that the equations have now been rewritten in index notation with the momentum-flux divergence term taking its standard form (see Eq. 1 in the revised manuscript).

***Reviewer statement 3***: *Another problem I have is related to the choice of notation for horizontal velocity vector in (4). Notation* $\mathbf{u}$ *is already reserved for the 3D filtered velocity vector in Eqs. 1 and 2, which implies that* $|\mathbf{u}|$ *can be nothing else than* $\sqrt{u^2+v^2+w^2}$*, so it would be necessary to introduce a horizontal 2D velocity vector* $\mathbf{v}=(u,v)$ *with* $|\mathbf{v}|=\sqrt{u^2+v^2}$ *and make corresponding adjustments in the remaining equations of Sect. 2.1.*

**Response**: The quantity $|\tilde{\mathbf{u}}| \equiv \sqrt{u^2+v^2}$ has now been defined.

## 1.2 Minor comments

***Reviewer statement 1***: *Table 1. Total number of grid/cell points (e.g.,* $160^3$*) is not a measure of grid resolution.*

**Response**: In Table 1, "grid resolution" is replaced by $N_x \times N_y \times N_z$.

***Reviewer statement 2***: *Figure 1. Primes (') are typically reserved for denoting fluctuations, not their RMS values. May be a source of confusion.*

**Response**: The RMS values of streamwise, cross-stream and vertical velocity fluctuations are now denoted by $u'_{\mathrm{RMS}}$, $v'_{\mathrm{RMS}}$ and $w'_{\mathrm{RMS}}$, respectively.

***Reviewer statement 3***: *Table 2. Are you sure that you need four decimal places to characterize the relative error?*

**Response**: The relative error is now characterized up to the second decimal place.

***Reviewer statement 4***: *Table 3. Same problem. Here you even use five decimal places...*

**Response**: Please see the response to statement 3.

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***Reviewer statement 5***: *Table 4. See two previous points.*

**Response**: Please see the response to statement 3.

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***Reviewer statement 6***: *Line 294. You need to be more specific about the way double-primed quantities have been evaluated and comment on meaning of their signs.*

**Response**: We now refer to these quantities as ”conditionally-averaged velocity fluctuations” and no additional notation is introduced. The sign approach is standard, i.e. a positive velocity fluctuation with respect to the conditionally-averaged mean will have a positive sign, and viceversa for a negative fluctuation with respect to the conditionally-averaged mean.

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***Reviewer statement 7***: *Line 321. Apparently, it should be* $C_{\mathrm{S}} = 0.1678$*.*

**Response**: The value for the Smagorinsky constant at line 321 is now $C_{\mathrm{S}} = 0.1678$.

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