

Interactive comment on “Free–Surface Flow as a Variational Inequality (*evolve_glacier v1.1*): Numerical Aspects of a Glaciological Application” by Anna Wirbel and Alexander Helmut Jarosch

Anonymous Referee #1

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review of "Free–Surface Flow as a Variational Inequality (*evolve_glacier v1.1*): Numerical Aspects of a Glaciological Application" by Anna Wirbel and Alexander Helmut Jarosch

This paper presents interesting insights for the solution of the kinematics equation of the upper stress-free surface of a glacier under the constraint that the ice thickness cannot be a negative value. This result in a variational inequality that is solved using the finite element method. The paper test different stabilization schemes to solve this equation and also different time step discretization schemes. Different benchmarks are presented to evaluate the different discretization schemes and validate the implemen-

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tation of the method. My overall feeling about the paper is that it deals in a rigorous way to an important numerical problem in glaciology that is not well addressed by many models such that this contribution will help de the community improving their models. But, if I am quite convinced that the material presented here is rigorous and of good quality, the presentation itself of all this material is a bit confusing. At the end, I am not quite sure of how the constraint $S > B$ is enforced. The problem is that the central topics of the paper (solving the free surface equation as a variational inequality) is hidden by a number of technical details (e.g remeshing). For example, in 4.2 which is about how the free surface is solved, it is just said that it is done by solving Eq. (3) and then the remaining part of the paragraph is regarding technical aspects. But how is solved this equation with the constraint $S > B$ is never clearly stated. I would really suggest to focus on the main text on the numerical methods used to solve the free surface equation with the constraint $S > B$ (reduced-space method?) and to separate it from the technical implementation of this equation under a more general model frameworks (solving Stokes, dealing with 2D and 3D meshes, allowing remeshing, etc...). The part 2 should therefore be extended (Mathematical formulation and numerical implementation) such that the variational formulation is given as well as the way the constraint $S > B$ is enforced.

I have some more specific comments below:

- abstract: acronym PETS and SNES should be given (it seems that the explanation of how it is solved is here?) but better would be to explain with words which methods is used to solve it (which tools/modules is used could be given later in the text). In general the abstract should be more focussed on the main points of the paper and avoid the technical aspects (is the fact that there is subdomains, a feature quite classical in FEM, relevant in the abstract?). Moreover, the term/verb "partition" has a special meaning in FEM (parallel computing with partitioned mesh). Here you are referencing to different bodies on which you are solving different equations?

- in the introduction, you should clearly specify that the main topics is the free surface equation under the constraint $S > B$, but that you eventually also need a Stokes modules

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to compute the velocity entering this equation as well as a module to evolve the mesh.

- eq. (2): the effective viscosity is not given. Anyway, is this equation really needed as it is a boundary condition for the Stokes solver, not the free surface?

- line 23, page 4: Crank–Nicholson- -> Crank–Nicholson

- part 3: I found part 3 a bit confusing. For example, page 4, lines 24-30, what does "an update" means? Are you remeshing or just moving the nodes (which ones? only vertically?) such that the surface nodes stay on the surface? And what do you mean by "significant"? How often the mesh is updated and how should be clearly stated. It seems that you are deforming your mesh such that the surface nodes stay on the surface but then remeshing when the displacements of these nodes is too large, but not sure. To my point, to get the correct solution, one has to perform an update of the mesh at each time the free surface variable S has been modified. Not really sure if you are doing that when reading the sentence lines 27-29.

- page 6, line 21: I would suggest to use nodes instead of gridpoints.

- page 7, line 3: are the Stokes equations also solved in subdomain 2? Anyway, velocity computed on domain 2 should be ~ 0 so that Eq. (3) should reduce to (4) even without the subdomain strategy?

- page 7, line 6: can you explain what you mean by a "velocity-dependent" buffer zone"?

- page 7, lines 8-13: an example of where you explain how you solve the free-surface equation under the constraint $S > B$ but just giving the technical details of the modules used, not really the method. All these materials all along the manuscript should be grouped in part 2.

- part 4.3: if you are using a vertically extruded mesh or a completely unstructured mesh should be mentioned here. Also, the last sentence is not very clear (what is the difference between creating a new mesh and remeshing?). A "msh" file is a bit technical. My understanding is that the STL surface has been modified, then the "msh"

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file is necessarily modified and therefore a new mesh has to be created. So this should be done each time the variable S has been modified? Also, how the previous solution (velocity, but also S) is interpolated on the new mesh should be discussed.

- page 9, line 6: It seems that $S=S_0$ is more an initial condition than a Dirichlet boundary condition? It is only set at $t=0$ not for any t ?

- page 9, around equation (5): I would suggest to avoid mixing of $_0$ and $_{init}$? S_0 , A_0 and V_0 would be more consistent, meaning the value at $t=0$. Also, in (5) h should write h_0 , but to avoid new variable to be introduced $h+z(t)$ could be replaced by $S(t)=S_0 + (u_z + a) t$?

- page 11, lines 1-5: what is the boundary condition at the bedrock for the Stokes?

- page 11, line 11: give the SMB that you have used such that the test can be reproduced by other users. In general, the exhaustive setups of these tests that aim to serve here as benchmarks should be given.

- page 12, line 6: 5.2.1 -> 5.2?

- page 12, line 9: specify that Eq. (B2) is given in Appendix 2.

- page 12? line 15: give the value for the computational time step. It seems from this sentence that the Stokes and the free-surface solver are not solved at each time step. This should be explained before in the manuscript. Regarding the advancing and retreat cases, I would suggest to discuss the two situations already in the introduction. At the end, you are presenting both situations which have their own difficulties in terms of numerics (constraint $S > B$ when retreating, front oscillation when advancing).

- page 12, line 22: how much the initial volume of the pyramid is a function of the mesh resolution? There is a dot over 6 in 0.376?

- Figure 5: relative difference instead of absolute would be more pertinent as nevertheless the choice of these values has no real meaning?

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- Figure 6 (and others): I would suggest to avoid the grey background and also the frame around panels. I would also suggest to use letter (a, b, c...) to label the different panels of a given figure. "N = 1000 resolution" looks strange (for the resolution N=1000).
- Figure 8: ice thickness would be more informative, or at least the initial S should be drawn on these two panels giving S?
- page 18, line 3: and at other places. Why computational time step as it has a physical meaning. So time step alone would be better.
- page 19, line 28: again, are the velocity and free surface solved using different time steps?
- page 21, line 16: are display?
- page 21, line 18: region (configuration(ii) (see Fig. 13 and Fig. 14), results -> region (configuration (ii), see Fig. 13 and Fig. 14), results
- part 7.2 misses an analysis if the oscillations are created from the free surface equation and/or the Stokes velocity solution in line of what has been shown in John et al. (2018a)? With a perfect velocity field, would one get surface oscillations? With a perfect free surface solver, would one get oscillations?
- Eq. (A1): define S_0 , S_{mid} (S_{mid} defined after B1 should be defined here)
- Eq. (A3): what is h here (I guess not the same as in (5))? h_K was used somewhere in the text.
- make title A1, A2, etc... similar
- page 28, line 5: $t, t+1$ should be defined before

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