

# ***Interactive comment on “Assessment of numerical schemes for transient, finite-element ice flow models using ISSM v4.18” by Thiago Dias dos Santos et al.***

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## **1 Overview**

This work provides an overview of some numerical developments in the ISSM finite-element (FEM) framework designed to improve ISSM's treatment of thickness advection. In general, naive implementations of FEM schemes can have difficulties with hyperbolic advection, suffering either from oscillations or instabilities as a result. Over the years, a number of approaches to improve FEM performance for hyperbolic transport have been developed, mostly for the context of high-speed compressible flows with

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shocks. In this work, the authors implement a range of such schemes and test them for a set of representative ice sheet modeling examples. The presentation is thorough and includes a good overview of the basic ideas in play in this work. The examples do a reasonable job of demonstrating the effectiveness of the various approaches. It should be noted that many of the schemes were developed in the context of compressible flow scenarios in which discontinuities and shocks are common. Such methods are potentially not well-suited for ice flows which don't display a tendency to steepen into shocks, so this work will occupy a useful place in the literature.

## **2 General Points**

It would be helpful if you also included the current baseline ISSM results (i.e. using none of the treatments described in this work) as a comparison to demonstrate the usefulness of these approaches. If one of these is the current standard in ISSM, you should state that.

The sub-element driving-stress parameterization is similar to that used in BISICLES (Cornford, et al, 2013) and in PISM (Feldmann, et al, 2014), which use one-sided differences to compute surface gradients on each side of the grounding line. You should cite these (I realize I'm a co-author on one, but they are relevant here). The basic idea is that the discretization of the basal friction should match the discretization of the driving stress – the discontinuity in the basal friction gets matched by a discontinuity in the driving stress.

The experiment in which you apply melt in partially-grounded cells to produce a response is a bit problematic, since you're essentially basing the experiment on a numerical error which converges to zero as you refine the mesh. It reveals the sensitivity of the different schemes to numerical errors, so it's useful, but I think you should clarify that it's not a reasonable choice for realistic marine ice sheet simulations (which was

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the conclusion I took away from Seroussi and Morlighem, 2018).

In general, plots of convergence work much better as log-log plots, since straight lines indicate consistent rates of asymptotic convergence. This is most relevant for plots like those in Figures 3, 6, 8, 11, 13, and 14. You're currently using semilog plots, which is helpful, but not as effective as log-log plots would be. This is natural for plots of errors (figures 3), For cases where you're demonstrating convergence to a particular value (Figures 6, 8, 11, 13, 14), it can possibly be more useful to plot either difference from an "exact" value, or even just the change between the the current value and the value from the next-coarser mesh (i.e.  $\phi_h - \phi_{2h}$ ).

Another general suggestion – I think it's clearer if you use "finer" and "coarser" when describing resolution (rather than "higher" and "lower").

Also, it would be useful if you showed examples of meshes and solutions for the example problems in the supplementary material.

### 3 Specific points

1. line 4: The ice thickness evolution equation isn't strictly hyperbolic. Because the ice thickness appears in the momentum equation used to solve for the ice velocity, it acts more like an advection-diffusion equation. There is advective transport (fast sliding), but also diffusion of thickness. This is partly why implicit methods are so useful.
2. line 39: "Wiggles" is fine, but "oscillations" is a more-standard term...
3. line 55: "flux corrections" → "flux-correction"
4. line 116 (or so): "integrating by part" → "integrating by parts"

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5. Section 2.5: A potentially simpler way to think of the Zalesak FCT scheme is as a hybridization of a higher-order (but potentially oscillatory and non-max/min preserving) scheme and a lower-order diffusive (but max/min-preserving) scheme. In smooth regions, you use the higher-order scheme. Where necessary (because the higher-order scheme produces new max/min values and therefore oscillations), you fall back to the lower-order scheme, but you add just enough of the flux from the higher-order (anti-diffusive) scheme to make things as accurate as possible without inducing oscillations (via the creation of new maxima or minima).
6. line 256: As I mentioned, you should cite the BISICLES and PISM references here for completeness.
7. line 279: The free-slip condition isn't just no normal flow, but also Neumann conditions on the tangential velocities:  $\frac{\partial v_x}{\partial y} = 0$ .
8. line 300: Note that the steady-state initial GL position for SSA in MISMIP3D is downstream of that produced by higher-fidelity schemes (full-Stokes, Blatter-Pattyn first-order, L1L2, etc). I'd suggest specifying that the projected GL location of 600km is only true for SSA models.
9. line 338: You refer to the "Amundsen Sea Sector", but then use the (common) abbreviation "ASE". (I'm guessing you don't want to use the abbreviation for "Amundsen Sea Sector", however)
10. line 385 (Figure 2): The convergence of ice speed in Figure 2 at the grounding line and ice front tells a consistent story in the under-resolved (5km and 2km) cases – too much friction near the grounding line (SEP 1) means that GL ice velocities are too slow. Reducing friction (SEP2) but not properly discretizing the driving stress (NSED) means an unbalanced driving stress and thus a too-high speed at the GL. If they're discretized consistently (SEP2+SED2), then they achieve something closer to the correct balance even in the under-resolved

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cases. Something similar is happening on the floating side of the GL, propagating out to the ice front speeds.

11. Figure 3: As mentioned above, this figure would be much clearer as a log-log plot because it would make convergence regimes and their associated convergence rates apparent.
12. line 396: I think that "2km" is more compact and clearer than "2000 m"
13. Figure 5: It would be helpful if you rescaled the y-axis so that the lines used more of the plot range. (something like [-200:200], perhaps?)
14. Figure 6: this would also be more effective as a log-log plot – perhaps using absolute values? The few coarse-resolution positive values are less important than the convergence tendencies as you refine your mesh...
15. Figures 6, 8, 11, and 13 would be more accessible if you used the same color legend for the right and left plots – it would make comparing them much simpler. Alternatively, you could keep the red-blue colors and collapse them onto a single (larger) plot.
16. line 425: "1,3000" → "1,300"
17. line 147: What exactly were the convergence issues for the DG implementation? Is the momentum solve not converging? Something else? It's perhaps not surprising that it's potentially not as robust as implementations which have seen a lot more use.
18. line 397 (and elsewhere): You mention that artificial diffusion helps provide stability in the presence of strong discontinuities and shocks (and then mention that DG and SUPG may produce oscillations in the presence of shocks), but it's not clear to me how relevant that really is for ice sheets. It's likely that methods developed

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to provide stabilization in flow regimes which include shocks and discontinuities are overkill for the (presumably) less-demanding ice-sheet case.

19. line 518: "when Backward Euler approach" → "when the Backward Euler approach"
20. line 519: " solution using DG scheme" → "solution using the DG scheme"
21. line 526: This would be a good place to point out that ice sheets don't see the shocks and discontinuities that most of the stabilization schemes were designed to handle.
22. line 535: "recommend to avoid" → "recommend avoiding"
23. line 535: "as in all transient" → "as all", or perhaps "because all"
24. line 543: "strong recommend" → "strongly recommend"
25. line 543: "although a carefully attention"

## References

- Cornford, et al, "Adaptive mesh, finite volume modeling of marine ice sheets", *Journal of Computational Physics*, 232:1, 2013.
- Feldmann, et al, "Resolution-dependent performance of grounding line motion in a shallow model compared with a full-Stokes model according to the MISIP3d intercomparison", *Journal of Glaciology*, 60:220, 2014.

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