Review of the GMDD manuscript gmd-2020-226:
Extending Legacy Climate Models by Adaptive Mesh Refinement for Single Component Tracer Transport: A Case Study with ECHAM6-HAMMOZ (ECHAM30-HAM23-MOZ10)

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Summary:
The manuscript describes how an existing transport algorithm can be augmented with an adaptive mesh refinement (AMR) approach. In particular, the Flux-Form Semi-Lagrangian (FFSL) transport scheme of the model ECHAM6-HAMMOZ has been modified without changing the underlying spectral transform dynamical core of ECHAM. This allows the newly developed FFSL AMR transport scheme to resolve a tracer mixing ratio with higher resolution in regions of interests while utilizing interpolated wind information from ECHAM's coarser-resolution Gaussian grid. Currently, the better resolved AMR tracer distribution is not communicated back (only one-way coupling) to the dynamical core. However, two-way coupling will become important for any future practical applications of the code. The manuscript describes the algorithmic changes of the existing FFSL transport scheme and provides assessments of the AMR transport algorithms via idealized tracer transport test cases. These idealized test cases seem to utilize a standalone AMR model that is not connected to ECHAM. In addition, a dust transport example with parameterized sources and sinks is presented that mimics a more realistic flow situation with ECHAM. However, the dust example leaves it open what the 'correct' solution is since even the non-adapted control simulations at the resolutions T31 and T63 have almost no resemblance (no convergence). This makes it rather impossible to judge whether AMR provides any benefits in this more realistic example. It also raises the question how AMR would be used for multiple tracers that most likely all need to be refined in different areas (e.g. will each tracer have its own AMR grid?). Add a comment about such aspects.

Overall, the research is very interesting and should be published (after revisions). However, the manuscript contains various mathematical errors in the equations (e.g. quantities with different physical units are used in sums, incorrect equations for the PPM subgrid distribution). This raises the question whether these are typos or whether the implementation is also incorrect. The manuscript also needs some additional explanations of the algorithm as detailed below. For example, it is unclear whether/how time-averaged winds are computed which are a key component of the original FFSL algorithm by Lin and Rood (1996). I also would like to see the cosine bell transport test in its most challenging configuration, which is the transport of the tracer at the 45° angle to the equator. This will more clearly assess the 2D transport characteristics of the chosen dimensionally-split AMR approach. Currently, the cosine bell is only tested for pure north-south or west-east flows in a 1D manner which leads to high convergence rates (between 2^{nd} and 3^{rd} order). I assume that the convergence rate will drop to first order for a 2D flow, and that the cosine bell will suffer from rather severe shape deformations. This will provide a more holistic assessment of the pros and cons of the dimensionally-split approach.

Detailed comments:

1) The abstract (line 10) states that the AMR data structure is introduced, but this description is missing in the manuscript. Add this information.
2) Line 48-49: be more specific what is meant by 'spectral' method since there are many variants. Here, the spectral transform method is meant. Use 'The FFSL scheme' in line 49 to make this sentence clearer.
3) Line 66: sentence starting with 'By' is not a sentence, rephrase
4) Line 68: An important component of the original FFSL scheme by Lin and Rood (1996) is the use of limiters to avoid numerical oscillations and negative tracer mixing ratios. Later in the manuscript, it is stated that limiters are not used. Please provide insight whether/how such unwanted characteristics in the AMR algorithm are avoided without any limiters.
5) Line 69: How is the AMR tracer transport discretized in the vertical direction (e.g. vertical remaps). This is important for the dust example (could be included in this section).
6) Line 104: A more precise explanation is that c is a dimensionless tracer mixing ratio. The phrase 'concentration' implies a physical unit.
7) Eqs. (4)-(7): Unit mismatches in equations and incorrect definition of the advective operators in Eq. (5). In Eq. (4) F is defined as a flux difference with units kg/(m^3 s) and is then added to the air density (with units of kg/m^3) in Eqs. (6) and (7). There is no notion of the computation of a time-averaged (time integrated) flux as in Lin and Rood (1996) which is a major error/omission. In addition, the advective operators (Eq. (5)) use wrong math notation. The divergence of the scalar *u* does not exist (also wrong in line 122), and even if *u* was meant to symbolize the horizontal velocity vector $\vec{v} = \binom{u}{v}$ the equations are still incorrect. As before, a unit mismatch is present in the two terms on the right hand sides (RHS) of Eq. (5). In addition, only $\rho\frac{\partial u}{\partial x}$ contributes to the definition of the advective operator in the x direction, and only $\rho\frac{\partial v}{\partial y}$ can be used in the y direction. This should be expressed in spherical geometry. Do these errors impact the implementation?
8) Line 142: typo, should read 'accounts for'
9) Line 153: the use of the phrase 'could' is confusing. Do you mean 'would'? Is there a condition if case 'could' was intentional?
10) Line 170: the definition of x is incorrect. 'x' needs to represent a normalized coordinate that varies between -1/2 and +1/2 and cannot be defined as the longitude (varying between 0-2π) or the sine of the latitude. Correct the definition of x.
11) Eqs. (11) and (12): More explanations are needed to clarify the computations of the departure points. How is u_a computed? At which spatial positions are u and v assessed? Is there any grid staggering? Are the velocities time-centered or time extrapolated? If yes, how is this accomplished? u and v are typically not constant along long trajectories. Please comment on the specifics. Are iterations needed to compute the trajectories for the semi-Lagrangian transport?
12) Fig. 3: add labels that show the i+1/2 and i-1/2 positions.
13) Line 190, Eq. (14): it seems as if ΔA needs a subscript. Correct.
14) Eq. (15): incorrect equations for the PPM subgrid distributions. The middle term on the RHS needs to be linear (just x and not x^2 in the upper equation. In the lower equation, the same math error exists for the linear terms. In addition, the normalized coordinates instead of λ and μ need to be used (see point 10). The explanations of PPM also become somewhat sloppy here since Colella and Woodward (1984) do not use the *a* and *b* notation for the coefficients

and the reader will be guessing how to find the information. An easier way is to point to Carpenter et al. (1990). However, I suggest adding the precise definition of the a and b coefficient, and also come back to the point whether/how (if any) limiters are used for the subgrid distribution.

15) Eq. (16): Unit mismatch between symbols F and ρ. The ρ in line 217 misses the superscript n+1.
16) Section 2.4: is it correct that only the wind is interpolated/updated at each time step whereas the AMR tracer distribution is kept from time step to time step? How close to the pole can the refinement go, e.g. just one grid spacing north/south of the poles as suggested later in Fig. 15? It would be helpful to remind the reader in section 2, that the AMR tracer is never averaged back to the Gaussian grid and thereby does not influence the dynamical core computations. Is my understanding correct, that the tracers are still also computed on a coarser Gaussian grid in addition to the AMR transport? It seems to be a must for quantities like moisture tracers in real applications.
17) Section 2.5: What is the allowable refinement ratio, e.g. just 1:2? Be clearer what the 'refinement of intermediate steps' means. It is still not clear, even after reading section 3.1. Where exactly are the additional refinement regions for intermediate steps?
18) Section 2: add some comments about the AMR data structure. Is this an AMR application that can currently only run on 1 CPU?
19) Section 3 and 4: add the time step information for all test cases.
20) Section 3.2.1: add the assessment of the most challenging 45° rotation angle which exposes the characteristics of the 2D transport. Does the cosine bell test use analytically initialized wind speeds on the AMR grid (which are analytically updated when the grid moves) or interpolated winds from a coarser (Gaussian?) grid? Are these simulations embedded in ECHAM or run with a standalone version of the AMR code? They seem to be standalone applications since no reference is made to Gaussian grid resolutions, correct?
21) Line 370: cosbell should read cosine bell
22) Section 3.2.2: provide information on the wind initialization for this test case (analytical or interpolated).
23) Fig. 14: Why does the curve in the right figure start with a mass variation of 4×10^{-12} instead of 0?
24) Fig. 15: it seems as if the refinement criterion was inadequate (too sensitive) since almost the complete domain is refined at day 12. This is especially true in regions with very little tracer variations. Why was this example chosen instead of a more tailored refinement criterion that focuses the AMR grid on the spirals?
25) Line 442: what is meant by 'uniform refinement'? Do you mean uniform resolution? There seems to be a contradiction in lines 443 and 444. Line 443 states that experiment 3 uses a wind interpolation. Line 444 refers to an exact (analytical?) wind field for experiment 3. Please clarify.
26) Section 4.1: Comment on the vertical transport of the tracer. How is it handled?
27) Line 530: typo, needs to read the tendency of the 'tracer density', not tracer concentration.
28) Eq. (23) and lines 536-539: Does the phrase 'hybrid' refer to a hybrid sigma-pressure η coordinate? Eq. (23) is not valid for the such a hybrid system and also does not require the definition of p in line 553. Does the divergence operator imply a 3D divergence or horizontal divergence? The **u** vector is undefined (2D or 3D). In a hybrid sigma-pressure system η the tracer transport equation (here written with the symbol q for the tracer mixing ratio) is

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}q\right)+\nabla\cdot\left(\frac{\partial p}{\partial \eta}q\vec{u}\right)+\frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}q\right)=0$$

The vertical pressure derivative stands for a pseudo density, velocity vector symbolizes the horizontal velocity vector, and the vertical velocity is $\dot{\eta}$. Please clarify and correct Eq. (23) as needed. Do you refer to a pure σ vertical coordinate (not hybrid) and if yes, is this the default in ECHAM? The phrase 'the hybrid coordinate prescribes a vertical pressure distribution' is confusing. Clarify and rephrase.

29) Line 544: since no limiters are used comment on the presence of under- and overshoots, and negative tracer values.
30) Line 584: what is the position of the model top for the 31-level setup?
31) Line 590: typo, should read October 1 to October 31
32) Line 597: provide approximate grid resolutions for T31 and T63 (in degrees or km).
33) Section 4.3.3: The chosen refinement/coarsening thresholds stated in line 562 seem to be inadequate (way to small/sensitive) for the dust simulations. All the dust figures show color bars with labels between 0.00001-0.00071, which are orders of magnitude bigger than the AMR criterion. Explain the motivation for the small AMR thresholds. They will refine areas that are irrelevant. For example, Fig. 22 (left) shows large refinement areas where there is no obvious presence of the tracer. The light yellow color scheme is also very difficult to see on top of the white background. I suggest adjusting the color scheme for all dust simulations to improve the clarity/readability of the figures.
Line 635 suggests a motivation for a small threshold, but why was it enough to e.g. go with a threshold like 10^{-6} instead if the chosen 10^{-11}? Provide more insight.
34) Page 32: Fig. 21 can be deleted. The left column of Fig. 21 is a repetition of the data in Fig. 20 (left column) and the right column is indistinguishable (by eye) from the left column. It is sufficient to state this in one sentence.
35) Fig. 22: incorrect figure caption
36) Section 4.3.3: The dust example is problematic since there is no reference solution (T31 and T63 simulations differ greatly). Was the uniform-resolution dust simulation also conducted at higher resolutions like T127 to understand this better? If there is no trusted uniform-resolution reference solution, it is unclear how to judge any AMR simulation and to see the added value. For example, I cannot see the AMR improvements in Fig. 23 (right) since they have no resemblance with the T63 simulation (Fig. 20 right) in the refined patch. This assumes that T63 is the 'more correct' simulation. Make this clearer in the discussion.
37) Line 667: without any 2-way interaction, the practical value of the AMR tracer transport is limited. It would be good to highlight the current study as a first step towards to full functionality of the AMR approach.