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Geoscientific Model Development Discussions

Interactive comment on "Analytical solutions for mantle flow in cylindrical and spherical shells" *by* Stephan C. Kramer et al.

Anonymous Referee #2

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Summary:

With the spirit of providing a set of benchmark functions for mantle flow simulation finite element codes, the authors compute explicitly a set of analytical solutions satisfying the incompressible Stokes equations, together with free-slip or zero-slip boundary conditions over both, 2D cylindrical and 3D spherical domains. The authors describe in detail the deduction of these analytical solutions in the several scenarios and validate them by performing a study of the FEM convergence rates considering the finite element framework Fluidity, obtaining the expected convergence rates when considering the well-known Taylor-Hood elements. This is complemented with a discussion on mathematical details of finite element convergence rates to give fundamentals of the suboptimality obtained when considering the delta distribution as source term. The

authors also provide a python package (Assess) allowing for evaluations of the analytical solutions at arbitrary points of the corresponding domain. Despite some minor typos, the document is in general well written and structured. Even if I did not verify in detail all the computations carried out by the authors to obtain the analytical solutions, their deduction seems correct and the numerical experimentation correctly verifies their accuracy. Moreover, the work perfectly fits with the scope of the journal since the authors also provide a python package with the implementation that can be used for the verification of any computational model based on the resolution of the incompressible Stokes equations.

Recommendation:

At a first glance, I am inclined to suggest a major review since I am not totally convinced of its novelty. In particular, it must be verified by the authors if this set of solutions have not already been considered in existing works. For instance, one of the articles mentioned by the other review [2] already includes a discussion of analytical solutions for both cases of boundary conditions considered by the authors, plus the mixed BC case. Additionally, [4] also considers the benchmarks for the incompressible Stokes equations in a 3D spherical shell. Once this is verified, including an extensive state-of-the-art literature review, I will be happy recommending it for publication in GMD.

Additional suggestions:

I have some additional suggestions listed below that, with the spirit of complementing the already ongoing discussion with the other review,Âă as it is already a very detailed review in my opinion.

1. The abstract and introduction can be improved by including self-explained sentences and letting citations only for verification purposes. In particular, the sentences "Computational models of mantle ..." in the abstract, and "3-D spherical geometry is implicitly required to simulate global mantle dynamics" in the introduction must be complemented with a brief explanation, from the physical and numerical point of view, of

the loosing when considering a cartesian model of the globe.

2. I strongly support the suggestion of considering a more deep literature review. In particular it is also missing a discussion of the already existing benchmarks in the FE community for Stokes equations in smooth domains (e.g., [1,3]). 3.- Please review the punctuation of the entire document. In particular, equations must be treated as part of the text. For instance, equations (6), (9), (10)(this is a typo), (12), ... must be ended with a "dot". 4.- In line 20, page 4 add "s" to relation

5.- In line 17, pag 5 (and in the rest of the article when it corresponds) add "coefficients" after "solution". i.e., write "solution coefficients"Âă

References:

[1] Buffa, A., de Falco, C., Sangalli, G.: IsoGeometric Analysis: Stable elements for the 2D Stokes equation. International Journal for Numerical Methods in Fluids 65(11-12), 1407–1422 (2011).

[2] Horbach, A., Mohr, M., Bunge, HP. A semi-analytic accuracy benchmark for Stokes flow in 3-D spherical mantle convection codes. Int J Geomath 11, 1 (2020).

[3] Hoang, T., Verhoosel, C.V., Auricchio, F., van Brummelen, E.H., Reali, A.: Mixed Isogeometric Finite Cell Methods for the Stokes problem. Computer Methods in Applied Mechanics and Engineering 316, 400 – 423 (2017).Âă

[4] Liu, S., King, S.D.: A benchmark study of incompressible Stokes flow in a 3-D spherical shell using ASPECT. Geophysical Journal International 217(1), 650–667 (2019).

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