Geosci. Model Dev. Discuss.,
https://doi.org/10.5194/gmd-2020-194-AC1, 2020

Geoscientific Model Development Discussions
Open Access

Interactive comment on "Analytical solutions for mantle flow in cylindrical and spherical shells" *by* Stephan C. Kramer et al.

Stephan C. Kramer et al.

s.kramer@imperial.ac.uk

Received and published: 14 October 2020

Hi Cedric

Thanks for your very useful comment. Your paper (Thieulot, Analytical solution forviscous incompressible Stokes flow in a spherical shell, Solid Earth, 8, 1181–1191,2017 https://doi.org/10.5194/se-8-1181-2017) certainly looks relevant for our study and we should add a reference and briefly describe how it relates on revision of our paper.

Initially I also thought your isoviscous case should overlap with some of our solutions. In a sense I guess your solution starts from a different starting point by making a number of assumptions (section 2.1), to then, assuming the isoviscous case with m=-1, arrive at the density in eqn (50). In our paper on the other hand, we start from assuming a generic form of density, e.g. eqn. (43) in our paper, to derive the solution from. This indeed suggests that your eqn. (50) could be seen as a linear combination of our "generic" solutions - at least for the last 2 terms: the first term $\sim$ ln r/rˆ4, is not a form we provide a solution for, but could probably be derived in a similar way. In terms of spherical harmonics, your cos(theta) corresponds to a degree l=1, and your assumption of azimuthal symmetry implies order m=1, i.o.w. Y_lm(theta,phi) = cos(theta) for l=1, m=0.

However then I realized is that our solution also makes different assumptions about boundary conditions. We provide solutions for both the free slip (i.e. zero tangential stress, and zero normal velocity), and no slip (all velocity components zero). If I understand correctly, in your solution the tangential stress follows from section 4.6, where g=0 at the boundary, so both tangential components would be (rf'-f)sin(theta), which is nonzero at the boundary. So I think our solutions in fact do not overlap, and describe different cases for different boundary conditions. In fact, obtaining solutions for "natural" boundary conditions, in particular free-slip which comes with a number of implementation issues such as strongly imposing a normal Dirichlet that is not aligned with the Cartesian direction, and associated rotational nullspace issues, was a strong motivation for our paper.

Of course your paper provides a wider family of solutions than just the isoviscous case, by offering solutions for radially dependent viscosity. As a future extension I think it would be interesting to see whether it is possible to derive solutions for the generic polynomial viscosity term you assume using the techniques in our paper

Best wishes Stephan Kramer

Interactive comment on Geosci. Model Dev. Discuss., https://doi.org/10.5194/gmd-2020-194, 2020.