Answer to referee 1

Thank you very much for your informative and detailed comments.

General comments

"The manuscript builds on the post-processing procedure DeFoReSt proposed by Pasternack et al. 2018 and presents a boosted recalibration of decadal climate predictions. The manuscript describes a well thought approach on handling drift corrections and present it reasonable well for an statistical audience. The comparison between the boosted and the non-boosted calibration is excessive and described well, but lacks hypothesis testing to determine the actual differences between the two approaches outside of the argument that it is obvious. While further work is required on the general presentation to make it more accessible to a wider audience, the authors might reconsider the choice of the journal, as the extreme focus on the statistical approach might be more appropriate for NPG. In its current shape the manuscript needs a much better illustration what has been done and why it matters. Therefore, I recommend major revisions for the manuscript and would expect a rework of the figures and potentially the structure of the arguments."

Specific points:

1. 18: "Significant advances could be achieved by recent progress in model development, data assimilation and climate observation." → has been made
   
   **Answer:** Will be corrected

2. 25: "unconditional, and conditional" → unnecessary comma
   
   **Answer:** Will be corrected

3. 37: "third/second" → why third before second?
   
   **Answer:** third before second because the ensemble mean is corrected via a 3rd order polynomial and the ensemble spread via a second order polynomial, which is described a few sentences before (31ff).

4. 47: "objective function": objective has a specific meaning in statistics (see Jeffrey’s prior) and would have to be individually proven. It is an unfortunate choice of word as it plays into the idea that statistics might be objective. As such, the word objective should be omitted in the manuscript completely.
   
   **Answer:** If the name "objective function" is misleading, we will change it to "cost function".

5. 87: "For the sake of completeness and readability these are presented in this section again." - Unnecessary sentence
   
   **Answer:** Will be deleted.

6. 124: By introducing the normal distribution with an calligraphic N and then use for the standard normal distribution greek letters, it gets quite confusing. As such this part needs to be rewritten. I would suggest to introduce $N_S$ or similar for the standard normal distribution. As the authors work beforehand with large letters for CDFs, I would recommend to use a consistent approach for the nomenclature. I am aware that the equation for the CRPS is shown in this way often in statistical leaning literature, but as GMD is not such a journal I strongly recommend intuitive naming of variables.
   
   **Answer:** We will replace the symbols $\Phi$ and $\varphi$ for the CDF and PDF of the standard normal distribution with $N_{S_C}$ and $N_{S_P}$.

7. 138ff: I would strongly recommend a schematic on which basis the authors explain the mechanism of DeFoRFeSt. Equations are fine, but as they become extremely lengthy and hard to understand for the general reader (like eq. 13), they need support and motivation.
   
   **Answer:** We will add such a schematic to the manuscript.
8. 185: Figure 1: name it consistent with Fig. 1 or rename all Figs to Figures.

Answer: Will be corrected.

9. 202ff: The problem at this point is that the boosting algorithm forms an essential part for the understanding of the manuscript. I would strongly recommend the design of a schematic to make clear what exactly is done in the boosting process (apart from the equation, but the algorithmic strategy). This part of the manuscript needs effort to make it better understandable for the wider audience, especially as the authors do not publish here for a statistical, but a general model related audience.

Answer: We will add a schematic flow chart describing the boosting algorithm analogously to Messner et al. 2017.

10. 202: "R-function poly" please make it a proper reference

Answer: Will be corrected.

11. 205: "R-package crch" please make it a proper reference

Answer: Will be corrected.

12. 206: "http://cran.r-project.org/" should go into the references

Answer: Will be corrected.

13. 218: The way it is written the choice of nu requires a sensitivity test. So either it requires the motivation for choosing nu = 0.05 to be rewritten, or a demonstration and discussion of its effect.

Answer: We will add a better motivation to the manuscript.

14. 226: The description of the cross-validation is not sufficient. A CV requires the statement on how the non-training data is afterwards evaluated (without taking into account the training data, otherwise it is not a CV but a Jackknife). The authors point to equation 21, but it is just the basis for the validation (which is described in line 216 with the Pearson correlation). So it would be required to state exactly what process is used for validation, which data is used for this step and which exact metric is applied to make the statement on a validated result.

Answer: We will add a more detailed description.

15. 238ff: Again the authors try in this section to explain everything by equations without explaining to the readers what consequences each of the decisions made have. The authors talk about extreme toy model experiments (l. 238), but do not state in what manner it is extreme. Then the authors introduce 5 parameters determining the experiments, but fail apart from short descriptions (like (un)conditional bias) to explain the reader what this actually means (and yes I am aware that most will know what it means in the direct community, but I think the authors should make the effort to explain it better as it builds a foundation of their argument). So I would recommend here to create a figure explaining the consequences of each of the parameters to give the modelling community an entry point to follow the experiments to find analogues between the toy model and the usually used GCMs or similar (this has been done in Pasternack et al. 2018, but perhaps a even more simplified/schematic version of Figure like Fig. 1 there will help). Giving the reader only an entry point by table 1 is not enough

Answer: We will add a more detailed description and two figures showing the effect of the imposed systematic errors of the toy model. Moreover we will change the phrase ’...we consider two extreme toy model experiments...' to ’...we consider two toy model experiments with different potential predictabilities...’

16. 267ff: The authors show a very large figure with many elements in 4 main colours for the different parameters, but just spend three sentences without putting it in context and give the plot any meaning (e.g. comparison, interpretation apart from first three coefficients vs. last three). As such either the plot has not more information, then it is doubtful whether the plot has any use for the manuscript, or the many different whisker plots are important and it is not represented in the
text. Just showing them is not enough, especially as later it is not referenced back to the figure when similar coefficient plots are made.

**Answer:** Showing this Fig. 2 is relevant for the toy model construction since it supports the decision to use the same magnitude for the coefficients of the start and lead time dependent systematic errors. However, since it is not used for any further evaluations we will put it to the appendix related to the table A1 which shows the final coefficients for the toy model construction.

17. 281: Estimating the 0.025 and 0.975 percentile from just 100 experiments is not a good way to demonstrate significances. The authors should either choose more experiments or go to alpha = 10. Or the description is so misunderstandable that in fact more than 100 values to estimate the percentiles are used. In that case the section has to be rewritten.

**Answer:** Indeed, using 100 experiments is not enough for calculating the 0.025 and 0.975 percentile. We will repeat that with 1000 experiments and update the corresponding text passages and figures in the manuscript.

18. 283: (see 4) : What is referenced here?

**Answer:** Will be corrected.

19. 285ff: Is there a reason, why in the DeFoReSt mode close to all metrics from Fig 3-10 show a U-shape over the lead years?

**Answer:** Regarding Figs. 3-10, particularly the ESS and the intra-ensemble variance omit a certain inverse U-shape. The reason might be, that DeFoReSt tends to be more underdispersive for the first and last lead year due to the missing additive correction term for the ensemble spread.

20. 288: It is not explained why the uncertainties of the ESS are not visible (either small or not calculable).

**Answer:** We have decided not to show any uncertainties for the ESS, since we just wanted to show the general effect of boosted recalibration and DeFoReSt and to ensure a better visibility.

21. 330ff: Two consecutive sentences start with "Here,"

**Answer:** Will be corrected.

22. 334 Why is there a bootstrapping in this section but not in the section above?

**Answer:** Unlike Sec. 4 we evaluate in Sec. 5 the CRPSS also w.r.t. a raw model. Thus, we decided to apply a bootstrapping approach to avoid any advantages of the post-processed models.

23. 334 Why is there a bootstrapping in this section but not in the section above?

**Answer:** Unlike Sec. 4 we evaluate in Sec. 5 the CRPSS also w.r.t. a raw model. Thus, we decided to apply a bootstrapping approach to avoid any advantages of the post-processed models.

24. 340ff: Why is there no comparison to the coefficients in Fig. 2?

**Answer:** The coefficients in Fig. 2 were used to derive the scale of the coefficients associated to 4th to 6th polynomials for the pseudo-forecasts. Here, unlike Fig. 11 and 13 no model selection was applied, i.e. a comparison is not very reasonable.

25. 348: "have also some impact." This should be analysed with a significance test and statements made accordingly

**Answer:** We will change the statement "have also some impact" to "have also been identified by the boosting algorithm as relevant".

26. 376: Are there significant differences between global and NA 2m-Temperature? Why is North Atlantic framed here as independent compared to the global and the comparison between those kept so short? It seems like it is written currently
that one example would be sufficient. So why are the two not conclusively compared with each other in one section? So could there be a different story apart from just showing the statistical model applied to data?

**Answer:** DeFoReSt and boosted recalibration have been developed within MiKlip project. Here, the NA as well as the global 2m-temperature are the key variables within this project. Moreover these regions distinguish themselves by their potential predictability. Thus analog to the toy model experiments we show the mechanisms of theses recalibration approaches to MiKlip predictions with smaller and higher potential predictability. Furthermore, regarding the different identified predictor variables for the NA and global 2m-temperature (Figs. 11 and 13) one can see that other processes are relevant due to a different spatial scale of these examples.

27. Fig3-5 should be combined in one figure with 9 panels

**Answer:** Will be corrected.

28. Fig7-9 should be combined in one figure with 9 panels

**Answer:** Will be corrected.

29. Fig 6+10 potentially better to have them in one plot with 2 panels

**Answer:** We would like to keep these plots separate, since they are discussed in different sections. Thus, to ensure a better readability it may be better to show these figures separately.

30. Fig11: MiKlip → MiKlip

**Answer:** Will be corrected

31. Fig12+14: Even when it is a stylistic choice: Why have the authors chosen a different colour-scheme compared to all the other figures in this manuscript?

**Answer:** We wanted to distinguish the toy model results optically from the results based on MiKlip data.

**Answer to referee 2**

Thank you very much for your informative and detailed comments.

1. General comments

"The authors present an extension to their previously introduced recalibration approach for decadal climate forecasts. The existing method is extended with a model selection approach using boosting to infer a parsimonious model from the data. Strengths and limitations of this approach are tested using synthetic data and an application to global mean and North Atlantic temperature forecasts is presented. While the boosting method presents a welcome addition to make the approach more generally useful across a diversity of applications (not limited to decadal forecasting) and therefore certainly merits publication, the article lacks in a few key aspects detailed below. Therefore, I suggest to accept the article subject to major revisions."

1.1 Interpretation of the results

"The authors focus on descriptive verification measures to discuss the results from boosted recalibration. In addition, I suggest the authors expand the discussion of the inner workings of the method and the configuration that is identified as optimal with boosting. From a methods perspective, I wonder if the boosted recalibration models are of lower complexity compared with DeFoReSt (i.e. if boosting actually manages to efficiently constrain the number of parameters). Also, the selected models appear still quite complex given the limited data at hand to train these. Have you explored early stopping rules for the boosting approach (generally skill improves rapidly in the first iterations and levels out afterwards, potentially another criterion for stopping provides better generalization ability through reduced models)? From an application perspective, some more discussion on the
identified nature of the error that is corrected with boosted recalibration would be useful, boosted recalibration is less effective if the systematic error has very simple structure as appears to be the case here."

**Answer:** The basic feature of the boosting algorithm is to allow a priori for a complex structure of the model used for recalibration but use the complexity only as needed. Thus our procedure is able to adjust complexity according to the problem at hand based on out-of-sample prediction error. This is realized by the automatic selection of the most relevant predictor variables by iteratively updating the log-likelihood. For each iteration step only one coefficient (the one that improves the fit most) is updated and thus complexity is successively increased. Here, the maximum number of iteration steps must be specified beforehand. However, if the chosen iteration step is small enough certain model coefficients are remaining zero. In order to find the best performing model an adequate iteration step has to be identified (model selection step) using a cross-validation setup. For this purpose we split the data into 5 parts and for each part, recalibrated predictions are computed from boosting model at the corresponding iteration step that were fitted on the remaining 4 parts. Afterwards the log-likelihood over all 5 recalibrated parts were summed up. This procedure is repeated for every iteration step. The iteration step with the lowest log-likelihood is considered as the one which provides the statistical model with the best predictive performance. Due to this procedure predictor variables of the statistical model that are not relevant are remaining zero. This can be seen in Figs. 11 and 13 which demonstrate which predictor variables are identified as relevant. Here, one can see that both for the North Atlantic as well as for the global 2m-temperature the complexity of boosted recalibration is around 15 identified predictor variables whereas DeFoReSt uses 22 predictor variables. We will add a schematic overview of the boosting algorithm and further explanation of the cross-validation approach to the manuscript.

### 1.2 Link between the toy-model experiments and the application

"The authors quite clearly demonstrate the strengths and limitation of the boosted recalibration compared with the reference approach (DeFoReSt) using their toy model experiments. There is, however, no direct link drawn to the application of boosted recalibration with global mean and North Atlantic surface temperature forecasts. In particular, I would like to know if the lack of improvement from boosted recalibration compared with DeFoReSt is consistent with the adjustments that are applied (e.g. what errors are generally corrected)."

**Answer:** With the toy model experiments we show that boosted recalibration outperforms DeFoReSt, if the polynomial order of the systematic errors goes beyond the restrictions of the DeFoReSt design. If that is not the case, both recalibration methods perform equally. Regarding the global mean and North Atlantic surface temperature forecasts one can see in Figs. 11 and 13 that boosted recalibration mostly identified predictor variables with a polynomial order smaller than 3. Thus, the fact that DeFoReSt and boosted recalibration perform equally for recalibrating MiKlip temperature forecasts is in accordance to the toy model results. We will emphasize the connection between toy model and temperature results more in the manuscript.

### 1.3 Significance assessment

"The significance assessment introduced on L280 does not reflect that the scores between DeFoReSt and boosted recalibration may be highly correlated due to the same forecast observation pairs being used. The 2.5-97.5% interval on the mean scores therefore likely underestimates the significance of the results. Instead, I propose to use a Diebold-Mariano test or a t-test on the score differences. I expect that using such a more powerful test would allow you to demonstrate e.g. that DeFoReSt significantly outperforms boosted recalibration when the error dependency matches the assumptions in DeFoReSt at least for short lead times."

**Answer:** Actually, we do not expected that DeFoReSt outperforms boosted recalibration, because the systematic error in the Miklip data is unknown and therefore does not have to be equal to the DeFoReSt-scenario. Boosted recalibration is able to cover systematic errors up to the 6th polynomial order, which also includes the the DeFoReSt-scenario, but is more flexible due to boosting. One can see in Fig. 11 and 13 that the identified polynomials do not go beyond the 3rd order, which is caught by DeFoReSt just as well. To compare these two post-processing methods we applied a bootstrapping approach. Within the applied bootstrapping approach, we calculate the score 1000 times, each with a different sample (replacements are allowed) from the original time series. The corresponding samples for the scores of DeFoReSt and boosted recalibration are not the same, i.e. a correlation between these scores is avoided. However, if these scores would base each in the same sample a high
correlation between those is possible and a Diebold-Mariano test or a t-test would be meaningful, indeed. We will point this out more clearly in the manuscript.

2. Minor comments

1. L72: 1.5° and 40
   Answer: Will be corrected.

2. L74: The full-field initialization
   Answer: Will be corrected.

3. L151-2: the punctuation is somewhat weird, maybe this could be changed: "...drift adjusted ensemble mean forecast (i.e. a deterministic forecast without specific uncertainty quantification)."
   Answer: Will be corrected.

4. L192-4: now is used three times
   Answer: Will be corrected.

5. L209: Maybe mention that you chose maximum likelihood in the following for better readability.
   Answer: Will be corrected.

6. L310: toy model setup with low potential predictability
   Answer: Will be corrected.

7. L314: The ESS (see Fig. 8a-c) reveals that
   Answer: Will be corrected.

8. L325: Typo? Shouldn’t this read “the low predictability leads to an increased CRPS” (not reduced CRPSS)?
   Answer: Actually not. In a setup with low potential predictability the benefit of boosted recalibration over DeFoReSt is smaller compared to a setup with high potential predictability. Thus the CRPSS is reduced.

9. L332: Repetition, use “We discuss...” instead
   Answer: Will be corrected.

10. L337: Typo. 10-year validation period
    Answer: Will be corrected.

11. L368: What fraction of the skill is due to the (linear) trend in global mean surface temperature?
    Answer: This is a very interesting question, indeed. It not possible to answer this briefly. We are currently working on a study where we use a recalibrated climatology as reference for the skill evaluation. The purpose is to analyze to what extent the predictive skill of recalibrated decadal predictions is superior to a statistical model with the same statistical properties as the applied recalibration strategy.

12. L402: Pasternack et al. (2018) show that
    Answer: Will be corrected.

13. L402: DeFoReSt leads to improved ensemble...or DeFoReSt leads to an improvement in ensemble...
    Answer: Will be corrected.
14. L409-: Long sentence. Maybe start with “Common parameter estimation and model selection approaches such as step-wise regression and LASSO are designed for predictions of mean values. Non-homogeneous boosting jointly adjusts mean and variance and automatically...regression.”

Answer: Will be corrected.

15. L423: this is not supported by your figure. Boosted recalibration is not (significantly)superior to DeFoReSt if errors are ‘simple’ according to Figure 6.

Answer: Will be corrected.

16. L438: equally

Answer: Will be corrected.

17. Figure 1: Why not show all the initialization times? The figure would be easily readable even with many more lines and the alignment of the differently colored blocks may become more apparent.

Answer: We will replace that figure with an new one showing all initialization times.

18. Fig. 3-5 and 7-9: Consider combining figures 3-5 and 7-9 each into one multi-panel plot to avoid splitting the figures across pages in the final publication. Also, the information shown is somewhat redundant and I encourage the authors to drop the sharpness plot for simplicity and for the following reasons: i) the sharpness of the raw model is of no use as it is not calibrated, ii) qualitative statements about the sharpness in DeFoReSt and boosted calibration can easily be derived from a visual comparison of the MSE and ESS plots. The legend should be shown only once for all 6 (9) panels of the multi-panel plot and axes should be labelled only once per row / column. Finally, consider using a square-root (or log) transform on the y-axis to take away the focus from large differences with large scores.

Answer: Will be corrected. However, we still would like to keep the sharpness figures. Indeed one could derive the sharpness from the ESS and the MSE but we think that is may be more convenient to have a visual impression of the sharpness.

19. Fig. 4: there is indication of extra overconfidence at the beginning and end of the forecast with DeFoReSt (with setups 1-3 and DeFoReSt). This appears to be an artefact of the method. Could you please discuss this?

Answer: Regarding the ESS of the raw model, one can see that for lead year 1 and 10 particularly the setups 1-3 are strongly over- or underconfident. Thus we would explain the inverse U-shape of the pseudo-forecasts after recalibration with DeFoReSt with the fact that DeFoReSt tends to be more underdispersive for the first and last lead year due to the missing additive correction term for the ensemble spread. This example shows that boosted recalibration can account better for forecasts which are either strongly overdispersive or strongly underdispersive.

20. Fig. 6, 10: Excessive white space. Please adjust the y-axis to better focus on the available data.

Answer: Will be corrected.
List of relevant changes made in the manuscript

1. Page 8, Lines 508 – 510: Added a better motivation for choosing $\nu = 0.05$.

2. Page 9, Lines 522 – 523: Added a more detailed description which clarifies that we apply a CV and not a Jack Knife approach.


4. Page 11, Lines 581 – 583, Page 23-26: We repeated the toy model experiment 1000 times. For the original manuscript the toy model experiments were repeated only 100 times.


7. Page 18: Added a schematic which describes mechanism of DeFoRFeSt.

8. Page 19: Extended the original figure, which demonstrates the CV mechanism, by all applied start years.

9. Page 20: Added a flow chart which describes mechanism of the applied boosting approach.

10. Page 23-25: We combined Figs. 3-5 and Figs- 7-9 into one figure with 9 panels.

11. Page 34: This figure was moved to the appendix.

Minor changes (e.g., typing errors, etc.) are not listed here.
Recalibrating Decadal Climate Predictions
–
What is an adequate model for the drift?

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Abstract. Near-term climate predictions such as decadal climate forecasts are increasingly being used to guide adaptation measures. To ensure the applicability of these probabilistic predictions, inherent systematic errors of the prediction system must be corrected. In this context, decadal climate predictions have further characteristic features, such as the long time horizon, the lead-time dependent systematic errors (drift) and the errors in the representation of long-term changes and variability. These features are compounded by small ensemble sizes to describe forecast uncertainty and a relatively short period for which typically pairs of re-forecasts and observations are available to estimate calibration parameters. With DeFoReSt (Decadal Climate Forecast Recalibration Strategy), Pasternack et al. (2018) proposed a parametric post-processing approach to tackle these problems. The original approach of DeFoReSt assumes third order polynomials in lead time to capture conditional and unconditional biases, second order for dispersion, first order for start time dependency. In this study, we propose not to restrict orders a priori but use a systematic model selection strategy to obtain model orders from the data based on non-homogeneous boosting. The introduced boosted recalibration estimates the coefficients of the statistical model, while the most relevant predictors are selected automatically by keeping the coefficients of the less important predictors to zero. Through toy model simulations with differently constructed systematic errors, we show the advantages of boosted recalibration over DeFoReSt. Finally, we apply boosted recalibration and DeFoReSt to decadal surface temperature forecasts from the MiKlip Prototype system. We show that boosted recalibration performs equally well as DeFoReSt and yet offers a greater flexibility.

1 Introduction

Decadal climate predictions focus on describing the climate variability for the coming years. Significant advances have been made could be achieved by recent progress in model development, data assimilation and climate observation. A need for up-to-date and reliable short-term climate information for adaptation and planning accompanies this progress (e.g., Meredith et al., 2018). In this context, international (e.g., DCPP and WCRP grand challenge) and national projects like the German initiative Mittelfristige Klimaprognosen (MiKlip) have developed model systems to produce a skillful decadal ensemble climate prediction (Pohlmann et al., 2013a; Marotzke et al., 2016). Typically, climate predictions are framed probabilistically to address the inherent uncertainties caused by imperfectly known initial conditions and model errors (Palmer et al., 2006).
Despite the progress being made in decadal climate forecasting, such forecasts still suffer from considerable systematic
errors like unconditional and conditional biases and ensemble over- or underdispersion. Those errors generally depend on
forecast lead-time since models tend to drift from the initial state towards its own climatology. Furthermore, there can be a
dependency on initialization time when long term trends of the forecast system and observations differ (Kharin et al., 2012).
In this regard, Pasternack et al. (2018) proposed a Decadal Forecast Recalibration Strategy (DeFoReSt) which accounts for
the three above mentioned systematic errors. While DCPP recommends to calculate and adjust model bias for each lead time
separately to take the drift into account, Pasternack et al. (2018) uses a parametric approach to describe systematic errors as
a function of lead time. DeFoReSt uses third order polynomials in lead time to capture conditional and unconditional biases,
second order for dispersion and a first order polynomial to model initialisation time dependency. Third order polynomials for
the drift have been suggested by Gangstø et al. (2013) and have later been used by Kruschke et al. (2015). Hence, DeFoReSt
is an extension of the drift correction approach proposed by Kruschke et al. (2015), accounting also for conditional bias and
adjusting the ensemble spread. The associated DeFoReSt parameters are estimated by minimization of the CRPS, analog to
the nonhomogeneous Gaussian regression approach by Gneiting et al. (2005).

Although DeFoReSt with third/second order polynomials turned out in past applications to be beneficial for both, full field
initialized decadal predictions (Pasternack et al., 2018) and anomaly initialized counterparts (Paxian et al., 2018), as well as
decadal regional predictions (Feldmann et al., 2019), it is worthwhile challenging the a priori assumption by using a systematic
model selection strategy. In this context, full field initializations show larger drifts in comparison to anomaly initializations
even though drift of the latter is not negligible, particularly when taking initialization time dependency into account (Kruschke
et al., 2015).

For post-processing of probabilistic forecasts with non-homogeneous Gaussian regression Messner et al. (2017) proposed
the non-homogeneous boosting to automatically select the most relevant predictors. Originally, boosting has been developed
for automatic statistical classification (Freund and Schapire, 1997), but has been used as well for statistical regression (e.g.
Friedman et al., 2000; Bühlmann and Yu, 2003; Bühlmann et al., 2007).

Unlike other parameter estimation strategies based on iterative minimization of a cost function by simultaneously updating
the full set of parameters, boosting only updates one parameter at a time; the one that leads to the largest
decrease in the cost function. As all parameters are initialized to zero, those parameters corresponding to
terms which do not lead to a considerable decrease in the cost function – hence are not relevant – will not
be updated and thus will not differ from zero; the associated term has thus no influence in the predictor. Here, we extend the
underlying non-homogeneous regression model of DeFoReSt to higher order polynomials and use boosting for parameter esti-
mation. Additionally, cross-validation identifies the optimal number of boosting iteration and serves thus for model selection.
The resulting boosted non-homogeneous regression model is hereafter named boosted recalibration.

A toy model producing synthetic decadal forecasts-observation pairs is used to study the effect of using higher order poly-
nomials and boosting on recalibration. Moreover, we compare boosted recalibration and DeFoReSt to recalibrate forecasts from
the MiKlip decadal prediction system.
The paper is organized as follows: Sec. 2 introduces the MiKlip decadal climate prediction system and the corresponding reference data used, Sec. 3 describes the decadal forecast recalibration strategy DeFoReSt and introduces boosted recalibration, an extension to higher order polynomials, parameter estimation with non-homogeneous boosting and cross validation for model selection. A toy model developed in Sec. 4 is the basis for assessing recalibration with boosted recalibration and DeFoReSt. The subsequent Section 5 uses both approaches to recalibrate decadal surface temperature predictions from the MiKlip system. Analogously to Pasternack et al. (2018), we assess the forecast skill of global mean surface temperature and temperature over the North Atlantic subpolar gyre region (60°-10°W, 50°-65°N). The latter has been identified as a key region for decadal climate predictions (e.g. Pohlmann et al., 2009; van Oldenborgh et al., 2010; Matei et al., 2012; Mueller et al., 2012). Section 6 closes with a discussion.

2 Data and methods

2.1 Decadal climate forecasts

Basis for this study are retrospective forecasts (hereafter called hindcast) of surface temperature from the Max-Planck-Institute Earth System Model in a low-resolution configuration (MPI-ESM-LR). The atmospheric component of the coupled model is ECHAM6 at a horizontal resolution of T63 with 47 vertical levels up to 0.01 hPa (Stevens et al., 2013). The ocean component is MPIOM with a nominal resolution of 1.5° and 40 vertical levels (Jungclaus et al., 2013). This setup together with a full-field initialization of the atmosphere with ERA40 (Uppala et al., 2005) and ERA-Interim (Dee et al., 2011), as well as a full-field initialization of the Ocean with the GECCO2 reanalysis (Köhl, 2015) is called the MiKlip Prototype System. The full-field initialization nudges the full atmospheric or oceanic fields from the corresponding reanalysis to the MPI-ESM, not just the anomalies. A detailed description of the Prototype system is given in Kröger et al. (2018). In the following, we use a hindcast set from the MiKlip Prototype System with 50 hindcasts, each with 10 ensemble members integrated for 10 years started every year in the period 1961 to 2010.

2.2 Reference data

The Met-Office’s Hadley Centre and the Climatic Research Unit at the University of East Anglia produced HadCRUT4 (Morice et al., 2012), an observational product used here as a reference to verify the decadal hindcasts. The historical surface temperature anomalies with respect to the reference period 1961-1990 are available on a global 5°-by-5° grid on a monthly basis since January 1850. HadCRUT4 is a composite of the CRUTEM4 (Jones et al., 2012) land-surface air temperature dataset and the HadSST3 (Brohan et al., 2006) sea-surface temperature (SST) dataset.

2.3 Assessing reliability and sharpness

To assess the performance of boosted recalibration w.r.t. DeFoReSt, we use the same metrics as in Pasternack et al. (2018). For the sake of completeness and readability these are presented in this section again.
Calibration or reliability refers to the statistical consistency between the forecast probability distributions and the verifying observations. Hence, it is a joint property of the predictions and the observations. A forecast is reliable if forecast probabilities correspond to observed frequencies on average. Alternatively, a necessary condition for forecasts to be reliable is given if the time mean intra-ensemble variance equals the mean squared error (MSE) between ensemble mean and observation (Palmer et al., 2006).

A common tool to evaluate the reliability and therefore the effect of a recalibration is the rank histogram or Talagrand diagram which were separately proposed by Anderson (1996); Talagrand et al. (1997); Hamill and Colucci (1997). For a detailed understanding, the rank histogram has to be evaluated by visual inspection. Analog to Pasternack et al. (2018), we use the Ensemble Spread Score (ESS) as a summarizing measure. The ESS is the ratio between the time mean intra-ensemble variance $\bar{\sigma}^2$ and the mean squared error between ensemble mean and observation, $MSE(\mu,y)$ (Palmer et al., 2006; Keller and Hense, 2011):

$$ESS = \frac{\bar{\sigma}^2}{MSE(\mu,y)},$$

with

$$\bar{\sigma}^2 = \frac{1}{k} \sum_{j=1}^{k} \sigma^2_j,$$

and

$$MSE(\mu,y) = \frac{1}{k} \sum_{j=1}^{k} (y_j - \mu_j)^2.$$  

Here, $\sigma^2_j, \mu_j$ and $y_j$ are the ensemble variance, the ensemble mean and the corresponding observation at time step $j$, with $j = 1, ..., k$, where $k$ is the number time steps.

Following Palmer et al. (2006), $ESS = 1$ indicates perfect reliability. The forecast is overconfident when $ESS < 1$, i.e., the ensemble spread underestimates forecast error. If the ensemble spread is greater than the model error ($ESS > 1$), the forecast is overdispersive and the forecast spread overestimates forecast error. To better understand the components of the $ESS$, we also analyze the mean squared error $MSE$ of the forecast separately.

Sharpness, on the other hand, refers to the concentration or spread of a probabilistic forecast and is a property of the forecast only. A forecast is sharp, when it is taking a risk, i.e., when it is frequently different from the climatology. The smaller the forecast spread, the sharper the forecast. Sharpness is indicative of forecast performance for calibrated and thus reliable forecasts, as forecast uncertainty reduces with increasing sharpness (subject to calibration). To assess sharpness, we use properties of the width of prediction intervals as in Gneiting and Raftery (2007). Analog to Pasternack et al. (2018), we use the time mean intra-ensemble variance $\bar{\sigma}^2$ to assess the prediction width.

Scoring rules, like the Continuous Ranked Probability Score (CRPS), assign numerical scores to probabilistic forecasts and form attractive summary measures of predictive performance, since they address reliability and sharpness simultaneously (Gneiting et al., 2005; Gneiting and Raftery, 2007; Gneiting and Katzfuss, 2014).
Given, $F$ is the predictive cumulative probability distribution function (CDF) and $F_0$ denotes the Heaviside function for the verifying observations $o$ with $F_0(y) = 1$ for $y > o$ and $F_0(y) = 0$ otherwise verifying observation, the CRPS is defined as

$$CRPS(F, o) = \int_{-\infty}^{\infty} (F(y) - F_0(y))^2 dy.$$  \hspace{1cm} (4)

where $F_0(y)$ is the Heaviside function and takes the values 0 or 1 if $y$ is less than or greater equal than the observed value $o$. Under the assumption that the predictive distribution $F$ CDF is a normal distribution with mean $\mu$ and variance $\sigma^2$ Gneiting et al. (2005) showed that (4) can be written as

$$CRPS(FN(\mu, \sigma^2), o) = \sigma \left\{ \frac{o - \mu}{\sigma} \right\} \Phi \left( \frac{o - \mu}{\sigma} \right) - \frac{1}{\sqrt{\pi}} \right \},$$  \hspace{1cm} (5)

where $N_{SC}(\cdot)\Phi(\cdot)$ and $N_{SP}(\cdot)\Phi(\cdot)$ denote the probability distribution function (CDF) CDF and the PDF probability density function (PDF), respectively, of the standard normal distribution. The CRPS is negatively oriented. A lower CRPS indicates more accurate forecasts; a CRPS of zero denotes a perfect (deterministic) forecast.

The Continuous Ranked Probability Skill Score (CRPSS) is, as the name implies, the corresponding skill score. It’s skill score (CRPSS) - A skill score relates the accuracy of the prediction system to the accuracy of a reference prediction (e.g., climatology). Thus, with hindcast scores a given $CRPS_F$ for the hindcast distribution and reference scores a given $CRPS_R$ for the reference distribution the CRPSS can be defined as

$$CRPSS = 1 - \frac{CRPS_F}{CRPS_R}.$$  \hspace{1cm} (6)

Positive values of the CRPSS imply that the prediction system outperforms the reference prediction. Furthermore, this skill score is unbounded for negative values (because hindcasts can be arbitrarily bad) but bounded by 1 for a perfect forecast.

3 Model selection for DeFoReSt

We first review the decadal climate forecast recalibration strategy (DeFoReSt) proposed by Pasternack et al. (2018) and illustrate subsequently how a modelling strategy based on boosting and cross validation leads to an optimal selection of polynomial orders in the non-homogeneous regression model used for recalibration.

3.1 Review of DeFoReSt

DeFoReSt assumes normality for the recalibrated predictive probability distribution function PDF $f^{Cal}(X; t, \tau)$ for a predicted parameter variable $X$ for each initialization time $t \in \{1961, 1962, 1963, \ldots, 2010\}$ and lead time $\tau \in \{1,2,3,\ldots,10\}$. $f^{Cal}(X; t, \tau)$ thus describes the recalibrated forecast PDF uncertainty of a given parameter variable $X$. $X \sim f^{Cal}(X; t, \tau)$ or - expressed in terms of the ensemble – $f^{Cal}(X; t, t)$ the distribution of the recalibrated ensemble members around the recalibrated ensemble mean as a function of initialization time $t$ and forecast lead-year $\tau$. Mean $\mu_{Cal}(t, \tau)$ and variance $\sigma_{Cal}^2(t, \tau)$ of
the recalibrated PDF distribution \( f^{\text{Cal}}(X; t, \tau) \) are now modelled as linear functions of the ensemble mean \( \hat{\mu}(t, \tau) \) and ensemble variance \( \hat{\sigma}^2(t, \tau) \) as

\[
\begin{align*}
\mu_{\text{Cal}}(t, \tau) &= \alpha(t, \tau) + \beta(t, \tau) \hat{\mu}(t, \tau) \\
\ln(\sigma^2_{\text{Cal}}(t, \tau)) &= \gamma(t, \tau) \hat{\sigma}^2(t, \tau).
\end{align*}
\]

Note, different from Pasternack et al. (2018), the logarithm in Eq. (8) ensures positive recalibrated variance \( \sigma^2_{\text{Cal}}(t, \tau) \) irrespectively of the value of \( \gamma \). Hence, the recalibrated parameter \( X_{\text{Cal}} \) is now conceived as a random variable distributed according to

\[
X_{\text{Cal}}(t, \tau) \sim N(\alpha(t, \tau) + \beta(t, \tau) \hat{\mu}(t, \tau), \exp(\gamma(t, \tau) \hat{\sigma}^2(t, \tau))).
\]

\( \alpha(t, \tau) \) accounts for the (unconditional) bias depending on lead year (i.e., the drift). Similarly, \( \beta(t, \tau) \) accounts for the conditional bias. Thus, the expectation of the recalibrated variable \( E(X_{\text{Cal}}(t, \tau)) = \alpha(t, \tau) + \beta(t, \tau) \hat{\mu}(t, \tau) \) can be conceived as a conditional and unconditional bias and drift adjusted ensemble mean forecast (or "deterministic": we call a deterministic forecast a forecast without specifying uncertainty) forecast. Moreover, DeFoReSt assumes that the ensemble spread \( \sigma(t, \tau) \) is sufficiently well related to the forecast uncertainty such that adequate adjustment can be realized by multiplying \( \gamma(t, \tau) \).

The functional forms of \( \alpha(t, \tau), \beta(t, \tau) \) and \( \gamma(t, \tau) \) are motivated from Gangstø et al. (2013), Kharin et al. (2012), Kruschke et al. (2015), and Sansom et al. (2016). Gangstø et al. (2013) suggested a third order polynomial in \( \tau \) as a good compromise between flexibility and parameter uncertainty; the linear dependency on \( t \) was used in various previous studies (Kharin et al., 2012; Kruschke et al., 2015; Sansom et al., 2016). A combination of both led to DeFoReSt as described in Pasternack et al. (2018):

\[
\begin{align*}
\alpha(t, \tau) &= \sum_{l=0}^{3} (a_{2l} + a_{(2l+1)} t) \tau^l, \\
\beta(t, \tau) &= \sum_{m=0}^{3} (b_{2m} + b_{(2m+1)} t) \tau^m, \\
\gamma(t, \tau) &= \sum_{n=0}^{2} (c_{2n} + c_{(2n+1)} t) \tau^n.
\end{align*}
\]

The ensemble inflation \( \gamma(t, \tau) \) is, however, assumed to be quadratic at most. Pasternack et al. (2018) assumed that a higher flexibility may not be necessary.

\( \alpha(t, \tau), \beta(t, \tau) \) and \( \gamma(t, \tau) \) are functions of \( t \) and \( \tau \), linear in the parameters \( a_l, b_m \) and \( c_n \). The parameters are estimated by minimizing the average CRPS over the training period following Gneiting et al. (2005) using the associated scoring function

\[
\Gamma(G_1) := \text{CRPS} =
\frac{1}{k} \sum_{j=1}^{k} \sqrt{\exp(\gamma(t, \tau) \sigma_j^2)} \left\{ Z_j \left[ 2 N_{S\Phi}(Z_j) - 1 \right] + 2 N_{S\Phi}(Z_j) - \frac{1}{\sqrt{\pi}} \right\},
\]

where \( Z_j \) is the observed value for the training period.
where

\[
Z_j = \frac{O_j - (\alpha(t, \tau) + \beta(t, \tau) \hat{\mu}_j(t, \tau))}{\sqrt{\exp(\gamma(t, \tau)^2 \hat{\sigma}_j^2(t, \tau))}}
\]  

(14)

is the standardized forecast error for the \( j \)th forecast in the training data set. Optimization is carried out using the algorithm of Nelder and Mead (1965) as implemented in \( \text{R} \) (R Core Team, 2018).

Initial guesses for parameters need to be carefully chosen to avoid convergence into local minima of the cost function. Here, we obtain initial guesses for \( a_l \) and \( b_m \) from a standard linear model using the ensemble mean \( \hat{\mu}(t, \tau) \) and polynomials of \( t \) and \( \tau \) as terms in the predictor according to Eqs. (7), (10) and (11). Initial guesses for \( c_0, c_1 \) and \( c_2 \) are all zero which yields unit inflation as \( \ln(\sigma^2_{\text{cal}}(t, \tau)) = 0 \) leads to \( \sigma^2_{\text{cal}}(t, \tau) = 1 \). Convergence to the global minimum is facilitated, however, cannot be guaranteed.

An alternative to minimization of the CRPS is maximization of the likelihood. Here, CRPS grows linearly in the prediction error, in contrast to the likelihood which grows quadratically (Gneiting et al., 2005). Thus a maximization of the likelihood is more sensitive to outliers and extreme events (Weigend and Shi, 2000; Gneiting and Raftery, 2007). This implies a prediction recalibrated using likelihood maximization is more likely to be underconfident than a prediction recalibrated using CRPS minimization (Gneiting et al., 2005).

We use cross-validation with a 10-year moving validation period as proposed by Pasternack et al. (2018) to ensure fair conditions for assessing the benefit of DeFoReSt. This means, the parameters \( a_l, b_m \) and \( c_n \) needed for recalibrating one hindcast experiment with 10 lead years (e.g. initialization 1963, forecasting years 1964 to 1973) are estimated via those hindcasts which are initialized outside that period (e.g. here hindcasts initialized 1962; 1974; 1975,...). This procedure is repeated for every initialization year \( z \in \{1960, 1961, 1962, \ldots, 2010\} \). Figure 2 shows an illustration of this setting.

### 3.2 Boosted recalibration and cross-validation

In Eq. 8, we followed Pasternack et al. (2018) with a multiplicative term \( \gamma(t, \tau) \) to adjust the spread. From now on, we follow the suggestion and notation from Messner et al. (2017) and include an additive term \( (\gamma(t, \tau)) \) and multiplicative term \( (\delta(t, \tau)) \).

The model for the calibrated ensemble variance (Eq. (8)) changes to

\[
\ln(\sigma^2_{\text{cal}, \text{boost}}(t, \tau)) = \gamma(t, \tau) + \delta(t, \tau) \hat{\sigma}^2(t, \tau).
\]  

(15)

Note the change in definition for \( \gamma(t, \tau)! \)

\( \alpha(t, \tau), \beta(t, \tau), \gamma(t, \tau) \) and \( \delta(t, \tau) \) are modelled using a similar approach as in Eqs. 10–12 where we now use orthogonalized polynomials to address for the lead time dependency of these corrections terms. In light of a model selection, this has the advantage that the individual predictors are now uncorrelated. Moreover, for boosted recalibration we now use orthogonalized polynomials of order 6 in lead time \( \tau \), assuming that this is sufficiently large to capture all features of lead time dependent drift \( (\alpha(t, \tau)) \), conditional bias \( (\beta(t, \tau)) \) and ensemble dispersion \( (\gamma(t, \tau) \) and \( \delta(t, \tau)) \); the dependence on initialization time \( t \) is kept...
linear:
\[
\alpha(t, \tau) = \sum_{l=0}^{6} \left( a_{2l} + a_{(2l+1)} P_l(\tau) \right),
\]
\[
\beta(t, \tau) = \sum_{m=0}^{6} \left( b_{2m} + b_{(2m+1)} P_m(\tau) \right),
\]
\[
\gamma(t, \tau) = \sum_{n=0}^{6} \left( c_{2n} + c_{(2n+1)} P_n(\tau) \right),
\]
\[
\delta(t, \tau) = \sum_{p=0}^{6} \left( d_{2p} + d_{(2p+1)} P_p(\tau) \right).
\]

Here, \( P_l(\tau), P_m(\tau), P_n(\tau) \) and \( P_p(\tau) \) are orthogonalized polynomials of order \( l, m, n \) and \( p \), which are provided by the \texttt{R}-function \texttt{poly} (\textbf{R Core Team}, 2018).

We apply boosting for non-homogeneous regression problems as proposed by Messner et al. (2017) for estimating \( a_t, b_m, c_n \) and \( d_p \). The algorithm iteratively seeks the minimum of a loss function (negative log-likelihood or CRPS) by identifying and updating only the most relevant terms in the predictor. This is realized with the \texttt{R}-package \texttt{crch} for non-homogeneous boosting (Messner et al., 2016, 2017) available from http://cran.r-project.org/ (CRAN) which uses a minimization of the negative log-likelihood by default instead of minimizing the CRPS. Judging from our experience, for the problem at hand, the difference in using one or the other loss functions appears to be small. The above mentioned effect of outliers and extremes on dispersivity described by Gneiting et al. (2005) should be rather small here, since annual aggregated values are recalibrated. Thus, we use the negative log-likelihood as cost function in the following.

In each iteration, the negative partial derivatives
\[
r = -\frac{\partial l(\mu, \sigma)}{\partial \mu}; \quad s = -\frac{\partial l(\mu, \sigma)}{\partial \sigma},
\]

of the negative log-likelihood for a single observation \( y \)
\[
l(\alpha + \beta \mu, \gamma + \delta \sigma; y) = -\log \left( \frac{1}{\gamma + \delta \sigma} N_{S_t} \phi \left( \frac{y - \alpha - \beta \mu}{\gamma + \delta \sigma} \right) \right),
\]
is obtained. Where \( N_{S_t} \phi \) is the \textbf{PDF} probability density function of the normal distribution, \( \mu \) the ensemble mean and \( \sigma \) the ensemble standard deviation corresponding to the initialization time \( t \) and lead time \( \tau \) of the observation \( y \). Pearsons correlation coefficient between each predictor term (e.g., \( t \) or \( t \tau^2 \)) and the partial derivatives \( r \) and \( s \) (Eq. (20)) estimated over every available \( t \in \{1961, 1962, 1963, \ldots, 2010\} \) and \( \tau \in \{1, 2, 3, \ldots, 10\} \) is used to identify and update the most influential term in the predictor. The parameter associated to the term with the highest correlation is updated by their correlation coefficient multiplied with a predefined stepsize \( \nu \). Schmid and Hothorn (2008) showed that the choice of \( \nu \) is only of minor importance and suggested a value of 0.1. A smaller value for \( \nu \) leads to an increase in precision in the updated coefficients at the expense of computing time. This allows a more detailed analysis of the relative importance of predictor variables. \( \nu = 0.05 \) turns out to be a reasonable compromise between precision and computing time in this setting. Nonetheless, from personal experience
we use $\nu=0.05$, which turns out to be more appropriate for our study. A distinct feature of boosting for non-homogeneous regression is, that both mean and standard deviation of a forecast distribution are taken into account, but for each iteration step only one parameter (either associated to the mean $\mu_{\text{Cal,boost}}$ or variance $\sigma_{\text{Cal,boost}}$) is updated: the one leading to the largest improvement of the cost function/objective function. Only those parameters associated to the most relevant predictor terms are updated; parameters of less relevant terms remain zero. The algorithm is described in more detail in Messner et al. (2017). The algorithm is originally described in Messner et al. (2017); for reasons of convenience we show with Fig. 3 a schematic flow chart of the boosting algorithm adopted to means of boosted recalibration.

If the chosen iteration steps is small enough, a certain number of less relevant predictor terms have coefficients equal to zero, which prevents the model from overfitting. A cross-validation (CV) approach is used to identify the iteration with the set of parameter estimates with maximum predictive performance. Currently, CV is carried out after each boosting iteration. The data is split into 5 parts, each part consist of approx. 10 years in order to reflect conditions of decadal prediction. For each part, a recalibrated prediction is computed, with the model trained on the remaining 4 parts. Afterwards these 5 recalibrated parts are used to calculate the full negative log-likelihood. Here, the full negative log-likelihood results from summing Eq. (21) for all available $t$ and $\tau$ and the associated observations $y$. The iteration step with minimum negative log-likelihood is considered best. We allow a maximum number of 500 iterations.

Analog to standard DeFoReSt, the previously described modelling procedure (boosting and CV for iteration selection) is carried out in a cross-validation setting (second level of CV) for model validation. A 10-year moving validation period (see Sec. 3.1) leads to cross-validation. For example, to recalibrate the hindcast initialized 1963 including lead years 1964 to 1973, all hindcasts which are not initialized within that period (e.g. $t \in \{1960,1974,1975,1976,\ldots,2010\}$) are used for boosting DeFoReSt.

4 Calibrating toy model experiments

To assess the model selection approach for DeFoReSt we consider two toy model experiments with different potential predictabilities we consider two extreme toy model experiments to generate pseudo-forecasts, as introduced by Pasternack et al. (2018). They are designed as follows

1. the predictable signal is stronger than the unpredictable noise,
2. the predictable signal is weaker than the unpredictable noise.

These experiments are controlled by five further parameters:

$\eta$ determines the ratio between the variance of the predictable signal and the variance of the unpredictable noise, it controls potential predictability, see Pasternack et al. (2018). We investigate two cases: $\eta = 0.2$ (low potential predictability) and $\eta = 0.8$ (high potential predictability).

$\chi(t, \tau)$ specifies the unconditional bias added to the predictable signal,
$\psi(t, \tau)$ specifies analogously the conditional bias, and

$\omega(t, \tau)$ specifies the conditional dispersion of the forecast ensemble.

$\zeta(t, \tau)$ controls analogously the unconditional dispersion and has not been used in Pasternack et al. (2018).

The coefficients for Bias (drift), conditional bias and effects in the ensemble dispersion are chosen such that they are close to those obtained from calculated by calibrating MiKlip Prototype surface temperature data with HadCRUT4 observations. Thus $\chi(t, \tau), \psi(t, \tau), \omega(t, \tau)$ and $\zeta(t, \tau)$ based on the same polynomial structure as used for the calibration parameters $\alpha(t, \tau), \beta(t, \tau), \gamma(t, \tau)$ and $\delta(t, \tau)$ (see (16) - (19)) (a detailed description of the toy model design is given in Appendix A). In the following, when we discuss the polynomial lead time dependency of the toy models systematic errors we refer to the polynomial order of $\alpha(t, \tau), \beta(t, \tau), \gamma(t, \tau)$ and $\delta(t, \tau)$. Note that the corresponding polynomials are also orthogonalized as in (16) - (19).

For an assessment of the model selection approach, we are using seven different toy-model setups per value of $\eta$. Each setup uses different orders of polynomial lead time dependency for imposing the above mentioned systematic deviations on the predictable signal. One toy model setup is designed such that the corresponding systematic deviations could be perfectly addressed by DeFoReSt. Additionally, there are other setups with systematic deviations based on a lower/higher polynomial order than what is used for DeFoReSt. Thus we compare pseudo-forecasts from setups which require model structures for recalibration given in Tab. 1.

<table>
<thead>
<tr>
<th>Setup</th>
<th>$\alpha(t, \tau) =$</th>
<th>$\beta(t, \tau) =$</th>
<th>$\gamma(t, \tau) =$</th>
<th>$\delta(t, \tau) =$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(a_0 + a_1 t)P_0(\tau) + \ldots$</td>
<td>$(b_0 + b_1 t)P_0(\tau) + \ldots$</td>
<td>$(c_0 + c_1 t)P_0(\tau) + \ldots$</td>
<td>$(d_0 + d_1 t)P_0(\tau) + \ldots$</td>
</tr>
<tr>
<td>1</td>
<td>$(a_2 + a_3 t)P_1(\tau)$</td>
<td>$(b_2 + b_3 t)P_1(\tau)$</td>
<td>$(c_2 + c_3 t)P_1(\tau)$</td>
<td>$(d_2 + d_3 t)P_1(\tau)$</td>
</tr>
<tr>
<td>2</td>
<td>$(a_4 + a_5 t)P_2(\tau)$</td>
<td>$(b_4 + b_5 t)P_2(\tau)$</td>
<td>$(c_4 + c_5 t)P_2(\tau)$</td>
<td>$(d_4 + d_5 t)P_2(\tau)$</td>
</tr>
<tr>
<td>3</td>
<td>$(a_6 + a_7 t)P_3(\tau)$</td>
<td>$(b_6 + b_7 t)P_3(\tau)$</td>
<td>$(c_6 + c_7 t)P_3(\tau)$</td>
<td>$(d_6 + d_7 t)P_3(\tau)$</td>
</tr>
<tr>
<td>DeFoReSt</td>
<td>$\sum_{t=1}^{3}(a_{2t} + a_{2t+1} t)P_t(\tau)$</td>
<td>$\sum_{m=1}^{3}(b_{2m} + b_{2m+1} t)P_m(\tau)$</td>
<td>$\gamma(t, \tau) = 0$</td>
<td>$\sum_{p=1}^{2}(d_{2p} + d_{2p+1} t)P_p(\tau)$</td>
</tr>
<tr>
<td>4</td>
<td>$(a_8 + a_9 t)P_4(\tau)$</td>
<td>$(b_8 + b_9 t)P_4(\tau)$</td>
<td>$(c_8 + c_9 t)P_4(\tau)$</td>
<td>$(d_8 + d_9 t)P_4(\tau)$</td>
</tr>
<tr>
<td>5</td>
<td>$(a_{10} + a_{11} t)P_5(\tau)$</td>
<td>$(b_{10} + b_{11} t)P_5(\tau)$</td>
<td>$(c_{10} + c_{11} t)P_5(\tau)$</td>
<td>$(d_{10} + d_{11} t)P_5(\tau)$</td>
</tr>
<tr>
<td>6</td>
<td>$(a_{12} + a_{13} t)P_6(\tau)$</td>
<td>$(b_{12} + b_{13} t)P_6(\tau)$</td>
<td>$(c_{12} + c_{13} t)P_6(\tau)$</td>
<td>$(d_{12} + d_{13} t)P_6(\tau)$</td>
</tr>
</tbody>
</table>

Table 1. Overview of the different toy model setups and the corresponding polynomial lead time dependencies.

As mentioned before, the functions $\chi(t, \tau), \psi(t, \tau), \zeta(t, \tau)$ and $\omega(t, \tau)$ in the toy model experiments are based on the parameters estimated for calibrating the MiKlip Prototype ensemble global mean surface temperature against HadCRUT4 observations. Here, $\chi(t, \tau), \psi(t, \tau), \zeta(t, \tau)$ and $\omega(t, \tau)$ are based on ratios of polynomials up to $3^{rd}$ order w.r.t. lead time. Based
on our experience we assume that systematic errors with higher than \(3^{rd}\) order polynomials could not be detected sufficiently well within the MiKlip Prototype experiments. Therefore, the coefficients for the \(4^{th}\) to \(6^{th}\) order polynomials are deduced from the coefficient magnitude of the \(1^{st}\) to \(3^{rd}\) order polynomial. Here, Fig. A1 shows the coefficients which were obtained from calibrating the MiKlip Prototype–global mean surface temperature with cross-validation (see Pasternack et al. (2018)), assuming a \(3^{rd}\) order polynomial dependency in lead years for \(\alpha(t,\tau), \beta(t,\tau), \gamma(t,\tau)\) and \(\delta(t,\tau)\). Here, it turns out that those coefficients associated with terms describing the lead time dependence exhibit roughly the same order of magnitude (see Fig. A1). Thus, we assume the coefficients associated to \(4^{th}\) to \(6^{th}\) order polynomials being of the same order of magnitude. An overview of the applied coefficient values is given in Appendix A.

Analogously to the MiKlip experiment, the toy model uses 50 start years, each with 10 lead years, and 15 ensemble members. The corresponding pseudo-observations run over a period of 59 years in order to cover lead year 10 of start year 50. The corresponding imposed systematic errors for the unconditional and conditional bias (related to \(\chi(t,\tau)\) and \(\psi(t,\tau)\)), unconditional and conditional dispersion (related to \(\zeta(t,\tau)\) and \(\omega(t,\tau)\)) are shown exemplary for start year 1 and start year 50 in Figs. 4 and 5. Here, the effect of an increasing polynomial dependency in the lead time in the setups 1 to 6 can be seen in form of an increased variability. For the DeFoReSt setup, the systematic error manifests itself as a superposition of setup 1 to 3 for \(\chi(t,\tau)\) and \(\psi(t,\tau)\) and of setup 1 to 2 for \(\omega(t,\tau)\) (\(\zeta(t,\tau)\) is equal zero for the DeFoReSt setup). Regarding the influence of the start year this effect amplifies for \(\chi(t,\tau)\) and \(\zeta(t,\tau)\) with increasing start time and diminishes for \(\chi(t,\tau)\) and \(\omega(t,\tau)\) due to their inverse definition (see eqs. A10 and A12).

For each toy model setup we calculated the Ensemble Spread Score ESS, the Mean Squared Error MSE, time mean intra-ensemble variance and the Continuous Ranked Probability Skill Score CRPSS of pseudo-forecasts recalibrated with boosting. Reference for the skill-score are forecasts recalibrated with DeFoReSt. All scores have been calculated using cross-validation with an annually moving calibration window with a width of 10 years (see Pasternack et al. (2018)).

To ensure a certain consistency 1000 pseudo-forecasts are generated from the toy model and evaluated as described above. The scores presented are all mean values over these 1000 experiments. In particular, to assess a significant improvement of boosted recalibration over DeFoReSt w.r.t. CRPSS the 2.5\% and 97.5\% percentiles are also estimated from this 1000 experiments.

4.1 Toy model setup with high potential predictability (\(\eta = 0.8\))

Figs. 6a-c show the MSE for 7 different setups (see Sec. 4). Panel 6a shows the result without any post-processing (raw pseudo-forecasts), panel 6b with DeFoReSt and panel 6c with boosted recalibration. Here, the performance of both post processing methods is strongly superior to the raw pseudo-forecast output. As DeFoReSt uses third order polynomials in lead time to capture conditional and unconditional biases, it performs equally well as the boosted calibration for the first four setups; for setups using higher order polynomials boosted calibration is superior.

Regarding the ESS (Figs. 6d-f) shows that the raw pseudo-forecasts are widely fluctuating between under- and overdispersiveness (ESS-values from 0.1 to 1.7), depending on the associated complexity of the imposed systematic errors (different setups). Corresponding to this the post processed pseudo-forecasts are more reliable with ESS-values close to
1. The *boosted recalibration* approach is superior to the recalibration with *DeFoReSt* for every lead year. The improvement is largest for setups 4-6, because *DeFoReSt* is limited to third order polynomials and cannot account for higher polynomial orders of these setups.

The post-processing methods are further compared by calculating the time mean intra-ensemble variance (see Figs. 6g-i). For every setup the intra-ensemble variance of the raw pseudo-forecasts is higher than the intra-ensemble variance of corresponding post-processed forecasts. Comparing *DeFoReSt* with the boosted recalibration reveals that the sharpness of the first approach is larger for setups 1 to 3 and the 'DeFoReSt setup’, leading particularly for the first 3 setups to an overconfidence (see 6e). However for setups 4 to 6 *DeFoReSt* exhibits a smaller sharpness, which still results in combination with the increased *MSE* (see 6b) to underdispersiveness.

A joint measure for sharpness and reliability is the *CRPS* and its skill-score, the *CRPSS*. Fig. 7 shows the *CRPSS* of the different pseudo-forecasts with *boosted recalibration*, where pseudo-forecasts recalibrated with *DeFoReSt* are used as reference, i.e. positive values imply that *boosted recalibration* is superior to *DeFoReSt*. Colored dots in Fig. 7 denote significance in the sense that the 0.025 and 0.975 quantiles from the 1000 experiments do not include 0. Regarding setups 1 to 3 and the 'DeFoReSt setup’, the *CRPSS* is neither significantly positive nor negative for all lead years, except lead year 1 of setup 3. On the other hand, for setups 4 to 6 the *boosted recalibration* outperforms the recalibration with *DeFoReSt* with values of the *CRPSS* between 0.1 and 0.4. Again, this is likely due to *DeFoReSt* assuming third order polynomials in lead time to capture conditional and unconditional biases, second order for dispersion and therefore does not account for systemic errors based on higher orders. However, Fig. 7 suggests that *boosted recalibration* can account for systematic errors with various levels of complexities.

### 4.2 Toy model setup with low potential predictability (**\( \eta = 0.2 \)**)

Figs. 8a-c show the *MSE* of the different pseudo-forecasts for a toy model setup with a low potential predictability. One can see that both post processing approaches lead to a strong improvement compared to the raw pseudo-forecasts; both approaches work roughly equally well for all setups. Compared to the previous section (**\( \eta = 0.8 \)**), the *MSE* of the pseudo-forecasts has increased due to a smaller signal-to-noise-ratio.

Regarding the *ESS* (see Fig. 8d-f), reveals that compared to the pseudo-forecasts with high predictability the raw simulations from different toy models are underdispersive for almost all lead years (*ESS*-values smaller than 1). The pseudo-forecasts show again an increased reliability after recalibration, with *ESS*-values close to 1. For every lead year, *boosted recalibration* is superior to *DeFoReSt*; the latter leads to slightly overconfident recalibrated forecasts.

Figs. 8g-i show the time mean intra-ensemble variance of the raw and recalibrated pseudo-forecasts. For every setup the intra-ensemble variance of the different pseudo-forecasts has decreased due to recalibration (with and without boosting). Comparing *DeFoReSt* with *boosted recalibration* reveals a smaller intra-ensemble variance for every setup, leading to an overconfidence for every lead year as observed in Fig 8e.

In the low potential predictability setting (**\( \eta = 0.2 \)**) the ensemble variance is larger as the total variance in the toy model is constrained to one. Thus reducing \( \eta \) leads to an increase in ensemble spread.
Fig. 9 shows the CRPSS of the pseudo-forecasts with boosted recalibration with DeFoReSt as reference. The low potential predictability leads to a reduced CRPSS compared to the setting with $\eta = 0.8$. The improvement due to boosted recalibration is also smaller. Only the first two lead years of setups 4-6 are significantly different from zero. This suggests that the improvement due to boosted recalibration decreases with a decreasing potential predictability of the forecasts.

5 Calibrating decadal climate surface temperature forecasts

While in Sec. 4 DeFoReSt and boosted recalibration were compared by the use of different toy model data, in this section these two approaches will be applied to surface temperature of MiKlip Prototype runs with MPI-ESM-LR. Here, global mean and spatial mean values over the North Atlantic subpolar gyre ($60^\circ$-10$^\circ$W, 50$^\circ$-65$^\circ$N) region will be analyzed.

Where we discuss which predictors are identified by boosted recalibration as most relevant and we compute the ESS, the MSE the intra-ensemble variance and the CRPSS with respect to climatology for both recalibration approaches. The scores have been calculated for a period from 1960 to 2010. In this section, a 95% confidence interval was additionally calculated for these metrics using a bootstrapping approach with 1000 replicates. For bootstrapping we randomly draw a new forecast-observation-pair of dummy time series with replacement from the original validation period and calculate these scores again. This procedure has been repeated 1000 times. Please note that we draw for each model a new forecast-observation-pair of dummy time series to avoid that the metrics of these models are calculated on the basis of the same sample. Furthermore, all scores have been calculated using cross-validation with a yearly moving calibration window with a 10-year validation periodwidth of 10 years (see Sec-3.1)

5.1 Global mean surface temperature

Fig. 10 shows the coefficients estimated by boosted recalibration for global mean surface temperature. The predictors are standardized, i.e. larger coefficients imply larger relevance of the corresponding predictors for the recalibration. Model selection is based on negative log-likelihood minimization in a cross-validation setup, as proposed by Pasternack et al. (2018). Thus for every training period different coefficients are obtained. The resulting distributions are represented in a box-and-whisker-plot, which also allows an assessment of the variability in coefficient estimates.

Most relevant are the coefficients $a_0$ and $a_1$, associated with unconditional bias ($a_0$) and the linear dependence on the start year ($a_1$). This is followed by $b_0$ in the conditional bias. In general, coefficients associated with first and second order terms in the lead time dependence ($a_2$, $a_4$, $b_2$, $b_4$) are dominating. Those coefficients describing the interaction between linear start year and first or second order lead year dependency (e.g., $a_3$, $b_3$, $c_3$, $b_5$, $c_5$) have also been identified by the boosting algorithm as relevant have also some impact.

The recalibration of ensemble dispersion is mostly influenced by a linear start year dependence in the unconditional term ($c_1$) and in the conditional term $d_0$. Higher terms are of minor relevance.

The performance of the ensemble mean of the raw forecast (black), recalibrated with DeFoReSt (blue) and with boosted recalibration is measured with the MSE shown in Fig. 11a. While a strong drift (lead-year dependence) influences the MSE
for the raw forecasts, both recalibrated variants exhibit a smaller and roughly constant $MSE$ across all $\tau$. This decrease in $MSE$ is a result of adjusting the unconditional and conditional bias ($\alpha(t, \tau)$ and $\beta(t, \tau)$).

Fig. 11b evaluates the ensemble spread and shows the $ESS$. The raw pseudo-forecast is underdispersive (ESS < 1) for all lead years and needs recalibration. The recalibrated forecasts show an adequate ensemble spread in both cases (ESS close to 1) for all lead years. $Boosted$ $recalibration$ (red) outperforms $DeFoReSt$ which becomes slightly under-/overdispersive for the first/last lead years. However, the differences in ESS between $boosted$ $recalibration$ and $DeFoReSt$ are not significant.

Fig. 11c shows the intra-ensemble variance (temporal average) across lead-years $\tau$. The ensemble variances of the raw forecast and $DeFoReSt$ are roughly equal, while $boosted$ $recalibration$ adjust the ensemble variance.

Compared to raw and $DeFoReSt$, the intra-ensemble variance of $boosted$ $recalibration$ is larger for lead year 1 and smaller for lead years 3 to 10. $Boosted$ $recalibration$ is sufficiently flexible to adjust the ensemble variance to a value close to the MSE. This consistent behaviour is roughly constant over lead years.

Although, $boosted$ $recalibration$ shows mostly a smaller ensemble variance (lead years 3-10) than $DeFoReSt$, both recalibration approaches are roughly equal when the performance is assessed with the $CRPSS$ with climatological reference (Fig. 11d).

Here, the $CRPSS$ of both models is around 0.8 for all lead years w.r.t. climatological forecast. In contrast, the raw forecast is inferior to the climatological forecast for most lead years, except lead years 3-6, where the raw forecast has positive skill, which could be attributed to the fact that temperature anomalies are considered. This implies that the observations and the raw forecast have the same mean value 0. This mean value seems to be crossed by the raw forecast mainly between lead 4 and 5.

### 5.2 North Atlantic mean surface temperature

Fig. 12 shows the coefficients of the corresponding standardized predictors which were estimated using $boosted$ $recalibration$ for North Atlantic surface temperature. Analogously to the global mean surface temperature, model selection is used within a cross-validation setup and the resulting coefficient distributions are shown in a box-and-whisker-plot. Here, the terms for the unconditional ($a_i$) and conditional bias ($b_j$) for the linear start year dependency ($a_i t$ and $b_j t$) and the first polynomial order lead time dependency ($a_i P_1(\tau)$ and $b_j P_1(\tau)$) are most relevant. Moreover, the linear interaction between lead time and initialization time ($a_3 t P_1(\tau)$) was identified as a relevant factor for the unconditional bias. Regarding the coefficients corresponding to the unconditional ($c_k$) and conditional ($d_l$) ensemble dispersion, one can see that the linear start and lead year dependencies ($c_1 t$, $c_2 P_1(\tau)$ and $d_1 t$, $d_2 P_1(\tau)$), as well as the interaction ($d_3 t P_1(\tau)$) between these two coefficients have the most impact.

Fig. 13a shows the $MSE$ of the raw forecast (black), $DeFoReSt$ and $boosted$ $recalibration$, where both recalibrated forecasts perform roughly equal. The raw forecast is inferior to both post processed forecast, mostly due to missing correction of unconditional and conditional biases. Compared to global mean temperature (Fig. 11a), $MSE$ for the North Atlantic temperature is generally larger. Thus potential predictability for the North Atlantic surface temperature is smaller than in the global case.
Regarding the reliability both recalibrated forecasts show also an *ESS* close to one for all lead years for the North Atlantic surface temperature (Fig. 13b), which is similar to the outcome of the global mean temperature (Fig. 11b). Again *boosted recalibration* outperforms *DeFoReSt*, the latter becomes slightly underdispersive for later lead years. However, the differences in *ESS* for both recalibration approaches are not significant. The raw forecast’s reliability is obviously inferior here, as it is significantly underdispersive for lead years 1 to 3 and overdispersive for lead years 5 to 6.

The mentioned lower potential predictability for the North Atlantic manifests also in a 10-times larger ensemble variance, cf. Fig. 13c. Noteworthy is here, that due to the smaller potential predictability in this region, the ensemble variance of both recalibrated forecasts is similar across the lead time and different from the raw forecast. A lower predictability of the North Atlantic surface temperature yields also a smaller *CRPSS* w.r.t. climatology for both recalibrated forecasts, Fig. 13d. Again, both recalibrated forecasts perform roughly equal for all lead years and are also significantly to the raw forecast.

### 6 Conclusions

Pasternack et al. (2018) proposed the recalibration strategy for decadal prediction (*DeFoReSt*) which adjusts non-homogeneous regression (Gneiting et al., 2005) to problems of decadal predictions. Characteristic problems here are a lead time and initialization time dependency of unconditional, conditional biases and ensemble dispersion. *DeFoReSt* assumes third order polynomials in lead time to capture conditional and unconditional biases, second order for dispersion, first order for initialization time dependency. Although, Pasternack et al. (2018) shows that *DeFoReSt* leads to an improvement of ensemble mean and probabilistic decadal predictions, it is not clear whether these polynomials with predefined orders are optimal. This calls for a model selection approach to obtain a recalibration model as simple as possible and as complex as needed. We thus propose here not to restrict orders a priori to such a low order but use a systematic model selection strategy to determine optimal model orders. We use the non-homogeneous boosting strategy proposed by Messner et al. (2017) to identify the most relevant terms for recalibration. The recalibration approach with boosting (called *boosted recalibration*) starts with order six polynomials in lead time and first order in initialization time to account for the unconditional and conditional bias, as well as for ensemble dispersion.

**Common parameter estimation and model selection approaches such as stepwise regression and LASSO** are designed for predictions of mean values. Non-homogeneous boosting jointly adjusts mean and variance and automatically selects the most relevant input terms for post-processing ensemble predictions with non-homogeneous (i.e., varying variance) regression. Besides other common parameter estimation and model selection approaches like stepwise regression or LASSO (Tibshirani, 1996), which are designed for predictions of mean values, non-homogeneous boosting adjusts mean and variance, it automatically selects the most relevant input terms for post-processing ensemble predictions with non-homogeneous (i.e., varying variance) regression. Boosting iteratively seeks the minimum of a costless function (here the log-likelihood) and updates only the one coefficient with the largest improvement of the fit; if the iteration is stopped before a convergence criterion is fulfilled those coefficients not considered until then are kept at zero. Thus, boosting is able to handle statistical models with a large number of variables.
We investigated *boosted recalibration* using toy model simulations with high ($\eta = 0.8$) and low potential predictability ($\eta = 0.2$) and errors with different complexities in terms of polynomial orders in lead time were imposed. *Boosted recalibration* is compared to *DeFoReSt*. The *CRPSS*, the *ESS*, the time mean intra-ensemble variance (a measure for sharpness) and the *MSE* assess the performance of the recalibration approaches. Scores are calculated with 10 year block-wise cross-validation (Pasternack et al., 2018) and with 100 pseudo-forecasts for each toy model simulation.

Irrespective of the complexity of systematic errors and the potential predictability, both recalibration approaches lead to an improved reliability with *ESS* close to one. Sharpness and *MSE* can also be improved with both recalibration approaches. Given a high potential predictability ($\eta = 0.8$), *boosted recalibration* – although allowing for much more complex adjustment terms – performs equal to *DeFoReSt* if systematic errors are less complex than a 3rd order polynomial in lead time, implied by the *CRPSS* of the pseudo-forecasts recalibrated with *boosted recalibration* and *DeFoReSt* as reference. Moreover, a significant improvement for almost all lead years can be observed if the complexity of systematic errors is larger than 3rd order order polynomials in lead time. The gain w.r.t. *DeFoReSt* can hardly be observed for a low potential predictability ($\eta = 0.2$), as the *CRPSS* shows only for two lead years a significant improvement for the above mentioned complexities. This is due to a generally weaker predictable signal, and thus a weaker impact of systematic error terms in higher order of the polynomial. The improvement due boosting increases with the imposed predictability. However, the presented toy model experiments suggest the use of *boosted recalibration* due to higher flexibility without loss of skill.

Analogously to Pasternack et al. (2018), we recalibrated mean surface temperature of the MiKlip Prototype decadal climate forecasts, spatially averaged over the North Atlantic subpolar gyre region and a global mean. Pronounced predictability for these cases has been identified by previous studies (e.g., Pohlmann et al., 2009; van Oldenborgh et al., 2010; Matei et al., 2012; Mueller et al., 2012). Nonetheless, both regions are also affected by a strong model drift (Kröger et al., 2018). For the global mean surface temperature, we could identify the linear start year dependency of the unconditional bias as a major factor. Moreover, it turns out that polynomials of lead year dependencies with order greater than 2 are of minor relevance.

Regarding the probabilistic forecast skill (*CRPSS*), *DeFoReSt* and *boosted recalibration* perform roughly equally, implying that the polynomial structure of *DeFoReSt*, chosen originally from personal experience, turns out to be quite appropriate. Both recalibration approaches are reliable and outperforming the climatological forecast with a *CRPSS* near 0.8. *This in line with the results from the toy model experiments which shows that *DeFoReSt* and *boosted recalibration* perform similar if systematic errors are less complex than a 3rd order polynomial in lead time.*

For the North Atlantic region, the linear start year and lead year dependencies of the unconditional and conditional biases show the largest relevance; also the linear interaction between lead time and initialization time of the unconditional bias has a certain impact. The coefficients corresponding to the unconditional and conditional ensemble dispersion, show a minor relevance compared to the errors related to the ensemble mean.

Also for the North Atlantic surface temperature both post-processing approaches are performing roughly equal; they are reliable and superior to climatology w.r.t. *CRPSS*. However, the *CRPSS* for the North Atlantic case is generally smaller than for the global mean.
This study shows that *boosted recalibration*, i.e. recalibration model selection with nonhomogeneous boosting allows a parametric decadal recalibration strategy with an increased flexibility to account for lead time dependent systematic errors. However, while we increased the polynomial order to capture complex lead time dependent features, we still assumed a linear dependency in initialization time. As this model selection approach reduces parameters by eliminating irrelevant terms, this opens up the possibility to increase flexibility (polynomial orders) also in terms related to the start year.

Based on simulations from a toy model and the MiKlip decadal climate forecast system we could demonstrate the benefit of model selection with boosting (*boosted recalibration*) for recalibrating decadal predictions, as it decreases the number of parameters to estimate predictors without being inferior to the state-of-the-art recalibration approach *DeFoReSt*.

*Code and data availability.* The *HadCRUT4* global temperature data set used in this study is freely accessible through the Climatic Research Unit at the University of East Anglia (http://www.cru.uea.ac.uk). The MiKlip Prototype data used for this paper are from the BMBF-funded project MiKlip and are available on request. The post-processing, toy model and cross-validation algorithms are implemented using GNU licensed free software from the R Project for Statistical Computing (http://www.r-project.org) and can be found under https://doi.org/10.5281/zenodo.3975758 (Pasternack et al., 2020).
\[
X_{Cal} = \mathcal{N}(\alpha(t, \tau) + \beta(t, \tau)\hat{\mu}(t, \tau), \exp(\gamma(t, \tau)\hat{\sigma}(t, \tau)^2))
\]

\(\alpha\): bias and drift. \(\beta\): variability of \(\hat{\mu}\). \(\gamma\): adjusts ens. spread.

**Figure 1.** Schematic overview of the effect of DeFoReSt for an exemplary decadal toy model with ensemble mean (colored lines), ensemble minimum/maximum (colored dotted lines) and associated pseudo-observations (black line). Note that different colors indicate different initialization times. Before recalibration (top figure) the ensemble mean shows a lead time dependent mean or unconditional bias (drift) which is tackled by \(\alpha(t, \tau)\). Moreover the ensemble mean \(\hat{\mu}\) executes a conditional bias, i.e. that the variances of \(\hat{\mu}\) and observations disagree. This is tackled with \(\beta(t, \tau)\). Decadal predictions can also be over- or underdispersive, i.e. that the ensemble spread over- or underestimates the error between observations and ensemble mean. This example shows an overdispersive forecast. Within DeFoReSt the coefficient \(\gamma(t, \tau)\) accounts for the dispersiveness of the forecast ensemble. The bottom figure shows the exemplary decadal toy model after applying DeFoReSt with the inherent corrections of lead and start time dependent unconditional bias, conditional bias and dispersion.
Figure 2. Schematic overview of the cross-validation setting for a decadal climate prediction, initialized in 1964 (red dotted line). All hindcasts which are initialized outside the prediction period are used as training data (black dotted lines). A hindcast which is initialized inside the prediction period is not used for training (gray dotted lines).
Initialize coefficients: \( \Lambda = 0 \) and \( \Xi = 0 \)

Compute negative partial derivatives of \( l(\mu, \sigma) \) w.r.t. to \( \mu \) and \( \sigma \) (see Eq. 20)

Find the predictor variable \( X_j \) with the highest correlation to \( r \) and \( Z_k \) with the highest correlation to \( s \):

\[
j_\ast = \arg \max_j \rho(X_j, r) \quad \text{and} \quad k_\ast = \arg \max_k \rho(Z_k, s)
\]

Tentatively update coefficients:

\[
\Lambda^\ast = \Lambda \quad \text{and} \quad \Xi^\ast = \Xi, \quad \text{with} \quad \\
\Lambda_{j^\ast}^\ast = \Lambda_{j^\ast} + \nu \rho(X_{j^\ast}, r) \quad \text{and} \quad \\
\Xi_{k^\ast}^\ast = \Xi_{k^\ast} + \nu \rho(Z_{k^\ast}, s)
\]

Really update the coefficient that improves the current fit most.

If \( l(X^T \Lambda^\ast, \sigma) < l(\mu, Z^T \Xi^\ast) \)

- Set \( \Lambda = \Lambda^\ast \)
- Set \( \Xi = \Xi^\ast \)
- \( I = I + 1 \)

\( I < \text{mstop} \) yes

Stop

\( I = I + 1 \) no

Figure 3. Schematic flow chart for boosting algorithm proposed by (Messner et al., 2016). For the ensemble mean and the ensemble variance we use the expressions \( \mu_{\text{cal,boost}}(t, \tau) = X^T \Lambda \) and \( \ln(\sigma_{\text{cal,boost}}^2(t, \tau)) = Z^T \Xi \), where \( X = (1, X_1, X_2, ...) \) and \( Z = (1, Z_1, Z_2, ...) \) are vectors of predictor terms and \( \Lambda = (a_0, b_0, a_1, b_1, ...) \) and \( \Xi = (c_0, d_0, c_1, d_1, ...) \) are vectors of the corresponding coefficients. Here, \( 0 \) is a vector of zeros, \( \text{mstop} \) is the a predefined maximum number of boosting iteration steps \( I \) and \( \rho(X_j, r) \) as well as \( \rho(Z_k, s) \) are the correlation coefficients calculated by \( X_j \times r \) and \( Z_k \times s \) over the respective training data.
Figure 4. $\chi(t, \tau)$ (a) and $\psi(t, \tau)$ (b) which are related to the unconditional and conditional bias, as well as $\zeta(t, \tau)$ (c) and $\omega(t, \tau)$ (d) which are related to the unconditional and conditional dispersion of the ensemble spread for the different toy model setups (colored lines) as a function of lead year $\tau$ with respect to start year $t = 1$.
Figure 5. $\chi(t, \tau)$ (a) and $\psi(t, \tau)$ (b) which are related to the unconditional and conditional bias, as well as $\zeta(t, \tau)$ (c) and $\omega(t, \tau)$ (d) which are related to the unconditional and conditional dispersion of the ensemble spread for the different toy model setups (colored lines) as a function of lead year $\tau$ with respect to start year $t = 50$. 

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Figure 6. Mean squared error (MSE) of different toy model setups with high potential predictability ($\eta = 0.8$, colored lines). a) raw pseudo-forecast, b) post-processing with DeFoReSt and c) post-processing with boosted recalibration. Analog to that order show d) to f) the Ensemble spread score (ESS) and g) to i) the Intra-ensemble variance (temporal average).
Figure 7. CRPSS of different toy model setups with high potential predictability ($\eta = 0.8$, colored lines) post-processed with boosted recalibration. The associated toy model setups post-processed with DeFoReSt are used as reference for the skill-score. CRPSS larger zero implies boosted recalibration performing better than DeFoReSt. Colored dots in Fig. 7 denote significance in the sense that the 0.025 and 0.975 quantiles from the 100 experiments do not include 0.
Figure 8. Mean squared error (MSE) of different toy model setups with high potential predictability (η = 0.2, colored lines). a) raw pseudo-forecast, b) post-processing with DeFoReSt and c) post-processing with boosted recalibration. Analog to that order show d) to f) the Ensemble spread score (ESS) and g) to i) the Intra-ensemble variance (temporal average).
Figure 9. CRPSS of different toy model setups with low potential predictability ($\eta = 0.2$, colored lines) post-processed with boosted recalibration. The associated toy model setups post-processed with DeFoReSt are used as reference for the skill-score. CRPSS larger zero implies boosted recalibration performing better than DeFoReSt. Colored dots indicate lead years with either significant positive or negative values based on a 95% confidence interval from bootstrapping (100 repititions).
Figure 10. Coefficient estimates for recalibrating global mean 2m-Temperature of the MiKlip prototype system. Colored boxes represent the inter-quartile range (IQR) around the median (central, bold and black line) for coefficient estimates from the cross-validation setup; Whiskers denote maximum 1.5IQR. Coefficients are grouped according to correcting unconditional bias (blue), conditional bias (red), unconditional dispersion (orange) and conditional dispersion (green). Values refer to coefficients $a_0, b_0, c_0, d_0, ..., a_6, b_6, c_6, d_6$ and not to the product between these coefficients and the corresponding predictors (e.g. $a_2 P_1(\tau)$ refers to $a_2$). Please note, the value $c_0$ is around -2.5, but for a better overview the vertical axis is limited to the values range between -1 and 1. Vertical dashed bars highlight coefficients related to lead time dependent terms.
Figure 11. a) MSE, b) Reliability, c) Ensemble Variance and d) CRPSS of global mean surface temperature without any correction (black line), after recalibration with DeFoReSt (blue line) and boosted recalibration (red line). The CRPSS for the raw forecasts (black line) is for lead year 1 smaller than -1 and therefore not shown. The vertical bars show the 95% confidence interval due 1000-wise bootstrapping.
Figure 12. Identified coefficients for recalibrating the mean 2m-Temperature over the North Atlantic of prototype. Here, the coefficients are grouped by correcting uncond. bias (blue bars), cond. bias (red bars), uncond. dispersion (orange bars) and cond. dispersion (green bars). The coefficients are standardized, i.e. higher values implying a higher relevance. Values refer to coefficients $a_0, b_0, c_0, d_0, ..., a_6, b_6, c_6, d_6$ and not to the product between these coefficients and the corresponding predictors (e.g. $a_2 P_1(\tau)$ refers to $a_2$).
Figure 13. a) MSE, b) Reliability, c) Ensemble Variance and d) CRPSS of surface temperature over the North Atlantic without any correction (black line), after recalibration with DeFoReSt (blue line) and boosted recalibration (red line). The CRPSS for the raw forecasts (black line) is for lead year 1 smaller than -1 and therefore not shown. The vertical bars show the 95% confidence interval due 1000-wise bootstrapping.
Appendix A: Toy model construction

The toy model proposed by Pasternack et al. (2018) consists of pseudo-observations $x(t + \tau)$ and associated ensemble predictions, hereafter named pseudo-forecasts $f(t, \tau)$.

Both are based on an arbitrary but predictable signal $\mu_x$. Although almost identical to Pasternack et al. (2018), we quote the construction of pseudo-observations in the following for purposes of overview.

The pseudo-observations $x$ is the sum of this predictable signal $\mu_x$ and an unpredictable noise term $\epsilon_x$,

$$x(t + \tau) = \mu_x(t + \tau) + \epsilon_x(t + \tau). \quad (A1)$$

Following Kharin et al. (2012) $\mu_x$ can be interpreted as the atmospheric response to slowly varying and predictable boundary conditions, while $\epsilon_x$ represents the unpredictable chaotic components of the observed dynamical system. $\mu_x$ and $\epsilon_x$ are assumed to be stochastic Gaussian processes

$$\mu_x(t + \tau) \sim \mathcal{N}(0, \sigma_{\mu_x}^2) \quad \text{with} \quad \sigma_{\mu_x}^2 = \eta^2 \leq 1 \quad (A2)$$

and

$$\epsilon_x(t + \tau) \sim \mathcal{N}(0, \sigma_{\epsilon_x}^2) \quad \text{with} \quad \sigma_{\epsilon_x}^2 = 1 - \eta^2. \quad (A3)$$

The variation of $\mu_x$ around a slowly varying climate signal can be interpreted as the predictable part of decadal variability, its amplitude is given by the variance $\text{var}(\mu_x(t + \tau)) = \sigma_{\mu_x}^2$. The total variance of the pseudo-observations is thus $\text{Var}(x) = \sigma_x^2 = \sigma_{\mu_x}^2 + \sigma_{\epsilon_x}^2$. Here, the relation of the latter two is uniquely controlled by the parameter $\eta \in [0, 1]$, which can be interpreted as potential predictability ($\eta^2 = \sigma_{\mu_x}^2 / \sigma_x^2$).

In this toy model setup, the concrete form of this variability is not considered and thus taken as random. A potential climate trend could be superimposed as a time varying mean $\mu(t) = E[x(t)]$. As for the recalibration strategy only a difference in trends is important, we use $\mu(t) = 0$ and $\alpha(t, \tau)$ addressing this difference in trends of forecast and observations.

The pseudo-forecast with ensemble members $f_i(t, \tau)$ for observations $x(t + \tau)$ is specified as:

$$f_i(t, \tau) = \mu_{ens}(t, \tau) + \epsilon_i(t, \tau), \quad (A4)$$

where $\mu_{ens}(t, \tau)$ is the ensemble mean and

$$\epsilon_i(t, \tau) \sim \mathcal{N}(0, \sigma_{\epsilon_{ens}}^2(t, \tau)) \quad (A5)$$

is the deviation of ensemble member $i$ from the ensemble mean; $\sigma_{\epsilon_{ens}}^2$ is the ensemble variance. In general, ensemble mean and ensemble variance both can be dependent on lead time $\tau$ and initialization time $t$. We relate the ensemble mean $\mu_{ens}(t, \tau)$ to the predictable signal in the observations $\mu_x(t, \tau)$ by assuming a) a systematic deviation characterized by an unconditional bias $\chi(t, \tau)$ (accounting also for a drift and difference in climate trends), a conditional bias $\psi(t, \tau)$ and b) a random deviation $\epsilon(t, \tau)$:

$$\mu_{ens}(t, \tau) = \chi(t, \tau) + \psi(t, \tau) (\mu_x(t, \tau) + \epsilon_f(t, \tau)), \quad (A6)$$
with \( \epsilon_f(t, \tau) \sim \mathcal{N}(0, \sigma_{\epsilon_f}(t, \tau)) \) being a random forecast error with variance \( \sigma_{\epsilon_f}^2(t, \tau) < \sigma_{\epsilon_x}^2 \). Although the variance of the random forecast error can in principle be dependent on lead time \( \tau \) and initialization time \( t \), we assume for simplicity a constant variance \( \sigma_{\epsilon_f}^2(t, \tau) = \sigma_{\epsilon_f}^2 \). In contrast to the original toy model design, proposed by Pasternack et al. (2018), we assume an ensemble dispersion related to the variability of the unpredictable noise term \( \epsilon_x \) with an unconditional and a conditional inflation factor (\( \zeta(t, \tau) \) and \( \omega(t, \tau) \))

\[
\sigma_{\text{ens}}^2(t, \tau) = (\zeta(t, \tau) + \omega(t, \tau) (\sigma_{\epsilon_x} - \sigma_{\epsilon_f}))^2.
\]

According to Eq. A6 the forecast ensemble mean \( \mu_{\text{ens}} \) is simply a function of the predictable signal \( \mu_x \). In this toy model formulation, an explicit formulation of \( \mu_x \) is not required, hence a random signal might be used for simplicity and it would be legitimate to assume \( E[\mu_x] = \mu(t + \tau) = 0 \) without restricting generality. Here, we propose a linear trend in time \( E[\mu_x] = \mu(t + \tau) = m_0 + m_1 t \) to emphasize a typical problem encountered in decadal climate prediction: different trends in observations and predictions (Kruschke et al., 2015). Given this setup, a choice of \( \chi(t, \tau) \equiv 0, \psi(t, \tau) \equiv 1, \zeta(t, \tau) \equiv 0 \) and \( \omega(t, \tau) \equiv 1 \) would yield a perfectly calibrated ensemble forecast:

\[
f_{\text{perf}}^\text{perf}(t, \tau) \sim \mathcal{N}(\mu_x(t, \tau), \sigma_{\epsilon_x}^2(t, \tau)).
\]

The ensemble mean \( \mu_x(t, \tau) \) of \( f_{\text{perf}}^\text{perf}(t, \tau) \) is equal to the predictable signal of the pseudo-observations. The ensemble variance \( \sigma_{\epsilon_x}^2(t, \tau) \) is equal to the variance of the unpredictable noise term representing the error between the ensemble mean of \( f_{\text{perf}}^\text{perf}(t, \tau) \) and the pseudo-observations. Hence, \( f_{\text{perf}}^\text{perf}(t, \tau) \) is perfectly reliable.

As mentioned in 4 this toy model setup is controlled on the one hand by \( \eta \) characterizing the potential predictability and on the other hand by \( \chi(t, \tau), \psi(t, \tau), \zeta(t, \tau) \) and \( \omega(t, \tau) \), which control the unconditional and the conditional bias and the dispersion of the ensemble spread.

Here, \( \chi(t, \tau), \psi(t, \tau), \zeta(t, \tau) \) and \( \omega(t, \tau) \) are obtained from \( \alpha(t, \tau), \beta(t, \tau), \gamma(t, \tau) \) and \( \delta(t, \tau) \) as follows:

\[
\chi(t, \tau) = -\frac{\alpha(t, \tau)}{\beta(t, \tau)}
\]
\[
\psi(t, \tau) = \frac{1}{\beta(t, \tau)}
\]
\[
\zeta(t, \tau) = -\frac{\gamma(t, \tau)}{\delta(t, \tau)}
\]
\[
\omega(t, \tau) = \frac{1}{\delta(t, \tau)}.
\]

The parameters \( \chi(t, \tau), \psi(t, \tau), \zeta(t, \tau) \) and \( \omega(t, \tau) \) are defined such that a perfectly recalibrated toy model forecast \( f_{i}^{\text{Cal}} \) would have the following form:

\[
f_{i}^{\text{Cal}}(t, \tau) \sim \mathcal{N}(\alpha(t, \tau) + \beta(t, \tau) \mu_{\text{ens}}(t, \tau), (\exp(\gamma(t, \tau) + \delta(t, \tau) \sigma_{\text{ens}}(t, \tau)))^2),
\]
Applying the definitions of $\mu_{\text{ens}}$ (Eq. A6) and $\sigma_{\text{ens}}$ (Eq. A7) leads to

$$f^\text{Cal}_i(t, \tau) \sim \mathcal{N}(\alpha(t, \tau) + \beta(t, \tau) (\chi(t, \tau) + \psi(t, \tau) \mu_x(t, \tau)), (\exp(\gamma(t, \tau) + \delta(t, \tau) (\zeta(t, \tau) + \omega(t, \tau) \sigma_{\epsilon_x}(t, \tau)))^2),$$  \hspace{1cm} (A14)

and applying the definitions of $\chi(t, \tau)$, $\psi(t, \tau)$ and $\omega(t, \tau)$ (Eqs. A9-A12) to (A14) would further lead to:

$$f^\text{Cal}_i(t, \tau) \sim \mathcal{N}(\alpha(t, \tau) - \beta(t, \tau) \frac{\alpha(t, \tau)}{\beta(t, \tau)} \frac{\gamma(t, \tau)}{\beta(t, \tau)} \mu_x(t, \tau) \sigma_{\epsilon_x}(t, \tau))^2),$$  \hspace{1cm} (A15)

This shows that $f^\text{Cal}$ is equal to the perfect toy model $f^\text{Perf}(t, \tau)$ (A8):

$$f^\text{Cal}(t, \tau) \sim \mathcal{N}(\mu_x(t, \tau), \sigma_{\epsilon_x}^2(t, \tau)).$$  \hspace{1cm} (A16)

This setting has the advantage that the perfect estimation of $\alpha(t, \tau)$, $\beta(t, \tau)$, $\gamma(t, \tau)$ and $\delta(t, \tau)$ is already known prior to calibration with minimization of the logarithmic likelihood.

As described in 3.2, a 6th order polynomial approach was chosen for unconditional $\alpha(t, \tau)$, $\beta(t, \tau)$, $\gamma(t, \tau)$ and $\delta(t, \tau)$, yielding

$$\alpha(t, \tau) = \sum_{l=0}^{6} (a_{2l} + a_{(2l+1)} t) P_l(\tau),$$  \hspace{1cm} (A17)

$$\beta(t, \tau) = \sum_{l=0}^{6} (b_{2l} + b_{(2l+1)} t) P_l(\tau),$$  \hspace{1cm} (A18)

$$\gamma(t, \tau) = \sum_{l=0}^{6} (c_{2l} + c_{(2l+1)} t) P_l(\tau),$$  \hspace{1cm} (A19)

$$\delta(t, \tau) = \sum_{l=0}^{6} (d_{2l} + d_{(2l+1)} t) P_l(\tau).$$  \hspace{1cm} (A20)

For the current toy model experiment, we exemplarily specify values for $a_i$, $b_i$, $c_i$ and $d_i$ as obtained from calibrating the MiKlip Prototype surface temperature over the North Atlantic against HadCRUT4 ($T_{\text{obs}}$):

$$E[T_{\text{obs}}] \sim \mathcal{N}(\alpha(t, \tau) + \beta(t, \tau) \tilde{f}_\text{Prot}(t, \tau), (\exp(\gamma(t, \tau) + \delta(t, \tau) \sigma_{f_{\text{fut}}}(t, \tau)))^2),$$  \hspace{1cm} (A21)

where $\tilde{f}_\text{Prot}$ and $\sigma_{f_{\text{fut}}}$ specifying the corresponding ensemble mean and ensemble spread. Here, $\chi(t, \tau)$, $\psi(t, \tau)$, $\zeta(t, \tau)$ and $\omega(t, \tau)$ are based on ratios of polynomials up to 3rd order w.r.t. lead time. Since systematic errors with higher than 3rd order polynomials could not be detected sufficiently well within the MiKlip Prototype experiments we deduce the coefficients for the 4th to 6th order polynomials from the coefficient magnitude of the 1st to 3rd order polynomial. Here, Fig. A1 shows the coefficients which were obtained from calibrating the MiKlip Prototype global mean surface temperature with cross-validation (see Pasternack et al. (2018)), assuming a 3rd order polynomial dependency in lead years for $\alpha(t, \tau)$, $\beta(t, \tau)$, $\gamma(t, \tau)$ and $\delta(t, \tau)$.

Those coefficients associated with terms describing the lead time dependence exhibit roughly the same order of magnitude. Thus, we assume the coefficients associated to 4th to 6th order polynomials being of the same order of magnitude. The values of the coefficients are given in Tab. A1.
Figure A1. Coefficient estimates for recalibrating global mean 2m-Temperature of the MiKlip Prototype System with a third order polynomial lead time time dependency for the unconditional and conditional bias ans dispersion. Here, non-homogeneous boosting is not applied and all polynomials are orthogonalized, i.e. \(P_3(\tau), P_2(\tau), P_1(\tau)\) refers to the order of the corresponding polynomial. Colored boxes represent the inter-quartile range (IQR) around the median (central, bold and black line) for coefficient estimates from the cross-validation setup; Whiskers denote maximum 1.5IQR. Coefficients are grouped according to correcting unconditional bias (blue), conditional bias (red), unconditional dispersion (orange) and conditional dispersion (green). Values refer to coefficients \(a_0, b_0, c_0, d_0, ..., a_6, b_6, c_6, d_6\) and not to the product between these coefficients and the corresponding predictors (e.g. \(a_2 P_1(\tau)\) refers to \(a_2\)). Vertical dashed bars highlight coefficients related to lead time dependent terms.

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</table>

Table A1. Overview of the values for the coefficients \(a_l, b_l, c_l\) and \(d_l\).
Author contributions. Alexander Pasternack, Jens Grieger, Henning W. Rust and Uwe Ulbrich established the scientific scope of this study. Alexander Pasternack, Jens Grieger and Henning W. Rust developed the algorithm of boosted recalibration and designed the toy model applied in this study. Alexander Pasternack carried out the statistical analysis and evaluated the results. Jens Grieger supported the analysis regarding post-processing of decadal climate predictions. Henning W. Rust supported the statistical analysis. Alexander Pasternack wrote the manuscript with contribution from all co-authors.

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