



1 Configuration and Evaluation of a Global Unstructured Mesh Model 2 based on the Variable-Resolution Approach

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11 **Abstract.** Targeting a long-term effort towards a global weather and climate model with a local refinement function,
12 this study systematically configures and evaluates the performance of an unstructured model based on the variable-
13 resolution (VR) approach. Aided by the idealized dry- and moist-atmosphere tests, the model performance is examined
14 in an intermediate degree of complexity. The dry baroclinic wave simulations suggest that the 3D VR-model can
15 reproduce comparable solutions in the refined regions as a fine-resolution quasi-uniform (QU) mesh model, although
16 the global errors increase. The variation of the mesh resolution in the transition zone does not adversely affect the wave
17 pattern. In the coarse-resolution area, the VR model simulates a similar wave distribution to the low-resolution QU
18 model. Two multi-region refinement approaches, including the hierarchical and polycentric refinement modes, further
19 testify the model performance under a more challenging environment. The moist idealized tropical cyclone test further
20 enables us to examine the model ability in terms of resolving fine-scale structures. It is found that the VR model can
21 have the tropical cyclone stably pass the transition zone in various configurations. A series of sensitivity tests examines
22 the model performance in a hierarchical refinement mode, and the solutions exhibit consistency even when the VR
23 mesh is slightly perturbed by one of the three parameters that control the density function. Moreover, only the finest
24 resolution has a dominant impact on the fine-scale structures in the refined region. The tropical cyclone, starting from
25 the 2nd-refinement region and passing through the inner transition zone, gets intensified and possesses a smaller area
26 coverage in the refined regions, as compared to the QU-mesh model that has the same number of grid points. Such
27 variations are consistent with the behavior that one may observe when uniformly refining the QU-mesh model. Besides
28 the horizontal resolution, the intensity of the tropical cyclone is also influenced by the Smagorinsky horizontal diffusion
29 coefficient. The VR model exhibits higher sensitivity in this regard, suggesting the importance of parameter tuning and
30 proper model configurations.

31 1 Introduction

32 Increasing resolution is generally regarded as an effective way to improve global weather and climate modeling
33 (e.g., Jung et al., 2012; Wehner et al., 2014; Zhang et al., 2014; Yu et al., 2019). As the model resolution continuously
34 increases, much more computational resources are required, constituting a major challenge for efficient model
35 development and applications. This has led to the pursuit of locally refined, variable-resolution (VR) modeling
36 approaches. Unlike nested regional modeling with prescribed lateral boundaries, the VR approach maintains the global
37 modeling configuration while permits increased resolution for certain regions of interest. Numerical modeling
38 experience has shown that such an approach can preserve the benefits of high-resolution modeling for the region of
39 interest at a lower computational cost, as the total number of the grid points can be largely reduced (e.g., Sabin et al.,



40 2013; Rauscher and Ringler, 2014; Sakaguchi et al., 2015; Gettelman et al., 2018).

41 Although the VR models can achieve improvements in the region of interest, they may potentially suffer from
42 some problems. The nonuniform mesh degrades the global accuracy (St-Cyr et al., 2008; Weller et al., 2009). Thus, it
43 is important to maximize the model performance for the refined region, while retains or minimally degrades the
44 performance for other non-refined regions. Previous studies have investigated the impacts of local grid refinements on
45 the solution error for global atmospheric models (e.g., Ringler et al., 2008; Park et al., 2013), mainly based on the
46 single-region refinement. A suite of 2D shallow-water VR-model tests demonstrates that the solution error is primarily
47 controlled by the coarsest region (Ringler et al., 2011). Model sensitivities to the width of the grid transition zone and
48 the densifying ratio are primary foci, and have been examined by utilizing various test cases. Based on the spherical
49 centroid Voronoi tessellation (SCVT), Ringler et al., (2011) suspected that the width of the transition zone may lead to
50 increased errors. Liu and Yang (2017) suggested that the width of the transition zone may cause less additional errors
51 as compared to the increase of the densifying ratio. Rapid resolution variations may also lead to wave distortion and
52 artificial wave reflection in the transition zone. Within a unified global model framework, these problems in the
53 transition zone and the low-resolution region may potentially deteriorate the performance in the refined region. Given
54 the cost of accuracy, a primary question arises, that is, whether a VR model can generate a comparable simulation in
55 the refined region compared to the high-resolution quasi-uniform (QU) model, especially in a more diverse VR
56 environment.

57 A primary motivation of increasing resolution is to resolve fine-scale atmospheric fluid structures. Tropical
58 cyclone is a useful testbed and has been frequently used to examine resolution sensitivity. High-resolution global models
59 show their improvement in tropical cyclone statistics as the resolution increases (e.g., Walsh et al., 2012; Strachan et
60 al., 2013; Bacmeister et al., 2014). The global VR approach has also been explored to simulate tropical cyclones for
61 cost-effective climate simulations (e.g., Zarzycki and Jablonowski, 2014). Unlike the adverse influence caused by the
62 boundary forcing in a nested regional model, the VR model captures smoother cloud patterns and smoother mid-level
63 jet structures, producing enhanced tropical cyclone activities (Hashimoto et al., 2015). Based on the VR meshes, the
64 Community Atmosphere Model well maintains the tropical cyclone crossing the transition zone without discernable
65 wave reflection or grid imprinting (Zarzycki et al., 2014). The simulations of such synoptic-scale system crossing the
66 transition zone are of vital importance to the model stability, especially under higher resolution conditions.

67 While previous studies (e.g., Rauscher et al., 2013; Harris et al., 2016; Huang et al., 2016) have reported benefits
68 of VR modeling, how to properly utilize this technique is still a challenging task and deserves ongoing exploration (e.g.,
69 Guba et al., 2014; Düben and Korn, 2014; Hendricks et al., 2016). In this study, we systematically configure and
70 evaluate the Global-to-Regional Integrated forecast SysTem (GRIST) Atmosphere framework based on the VR
71 approach. GRIST is a global model built on an unstructured Voronoi-Delaunay mesh. In previous studies, the
72 component performance in shallow water environment (Zhang, 2018; Wang et al., 2019), 3D dry dynamics (Zhang et
73 al., 2019) and idealized moist-atmosphere environment (Zhang et al., 2020) have been assessed in detail, mainly based
74 on the QU-mesh simulations. Neither the dry dynamical core (dycore hereafter) nor the dycore-tracer-physics (DTP
75 hereafter) split-coupling mechanism were systematically exposed to the more challenging VR configuration. It remains
76 unknown whether the model is able to behave properly when the local mesh variation is considered. The idealized test
77 cases drive the model towards some well-expected behaviors, facilitating some basic understanding of VR modeling.
78 The major objects of this study are:

- 79 (i) To validate the behavior of the dynamical model in its VR configuration, and to understand the strength and
80 weakness under various refinement meshes;
- 81 (ii) To provide a general guidance of utilizing different-style VR refinements for more realistic modeling in future.
82 To achieve these goals, we first evaluate the VR model performance in dry baroclinic wave test cases based on



83 three different initial point sets. We focus on the ability of the VR model to match the QU-mesh solutions in the refined
84 regions, and the model stability in terms of waves crossing through the transition zones. To balance the accuracy and
85 computational cost, we test the model in two multi-region refinement modes, which are more flexible for enlarging the
86 effective coverage of the fine-resolution regions given a certain number of grid points. The performance under the
87 moist-physics configuration is examined by using an idealized tropical cyclone test. A series of numerical tests was
88 performed to examine the model reliability under more challenging conditions.

89 The remainder of this paper is organized as follows. Section 2 describes the model and mesh generation. Section
90 3 examines the performance of the VR model against the QU one based on dry-atmosphere simulations. The model
91 performance under higher-resolution conditions with various mesh refinement modes is also investigated. Section 4
92 documents model sensitivity based on moist-atmosphere simulations. Section 5 presents a summary.

93 **2 Model and mesh generation**

94 **2.1 Model and configurations**

95 The model evaluated here is a developing version of the Global-to-Regional Integrated forecast SysTEM (GRIST)
96 Atmosphere framework, the version is named after the last two numbers of the solar calendar year (i.e., GRIST-A20).
97 The code is able to exactly regress to the state reported in Zhang et al. (2020), although significant code refactoring is
98 continuously added. It uses an unstructured-mesh formulation, which permits the use of SCVT (Ringler et al., 2008;
99 Jacobsen et al., 2013) that enables VR modeling. The dycore framework is described and evaluated in Zhang et al.
100 (2019). The moist atmospheric model is equipped with a general physics-dynamics coupling workflow that has a DTP
101 split-coupling mechanism, and details were described in Zhang et al., 2020. The dry-mass coordinate allows a switch
102 between the hydrostatic and nonhydrostatic solvers, and exactly (to machine roundoff) conserves the dry air mass. The
103 3D tracer transport module has several horizontal and vertical options for various applications (cf., Zhang et al., 2020).
104 In this study, the Two-step Shape-Preserving Advection Scheme (TSPAS; Zhang et al., 2017; Yu, 1994) was used as the
105 horizontal one, and a Runge-Kutta based third-order upwind scheme (Wicker and Skamarock, 2002) is used as the
106 vertical one.

107 In this study, we employ both the nonhydrostatic and hydrostatic dynamical core. Results of the high-resolution
108 QU-mesh model are used as a reference for evaluating the performance of the VR model. We use a physically-based
109 Smagorinsky horizontal diffusion (Smagorinsky, 1963) with the second-order Laplacian operators. It is activated in all
110 dry and moist experiments except for the dry test on the lowest-resolution QU mesh (i.e., G6 in this study). For the
111 moist simulations, we couple the model with a simple-physics package that has important driving mechanisms for
112 tropical cyclones (Reed and Jablonowski, 2012), which provides a higher degree of complexity than the dry simulations.
113 A summary of the mesh resolution and timesteps for each experiment is given in the supplement file (Table S1, S2).

114 **2.2 Generation of the variable-resolution mesh**

115 Technically, running an unstructured-mesh model in a VR configuration only requires altering the mesh file (when
116 idealized test cases are considered). A detailed description of the properties and generation of the SCVT is provided in
117 Ringler et al. (2011) and Ju et al. (2011). In this study, we focus on two key elements of the SCVT: generators and
118 density function. A spherical Voronoi tessellation is a spatial subdivision of a sphere Ω based on a set of distinct points
119 on Ω . For each point x_i , $i = 1, \dots, n$, the corresponding Voronoi region V_i , $i = 1, \dots, n$, is defined by:

$$120 V_i = \{x \in \Omega \mid \|x - x_i\| < \|x - x_j\| \text{ for } j = 1, \dots, n \text{ and } j \neq i\}, \quad (1)$$

121 where $\|\cdot\|$ denotes the geodesic distance. Each point x_i is called a generator and its corresponding Voronoi region V_i



122 is also called the Voronoi cell. A spherical Voronoi tessellation mesh becomes a SCVT mesh when the generators are
123 also the centroids of the corresponding Voronoi regions, which are derived from a density function defined on Ω . In
124 this study, the SCVT mesh is constructed via an iterative process based on the Lloyd's algorithm (Du et al., 1999). In
125 our implementation, the iteration continues until the circumcenter of each Delaunay triangle falls within the triangle.
126 Instead of the time-consuming serial construction, we generate the VR tessellation based on a parallel algorithm
127 (Jacobsen et al., 2013).

128 2.2.1 Generators

129 In this study, we use three different ways to construct the original generators.

- 130 (i) **Icosahedron bisection.** The icosahedron-based SCVT meshes benefit the excellent uniform properties due to
131 bisections of a regular icosahedron. We use the vertexes of the polyhedron after icosahedral bisections as the
132 initial generators for constructing the SCVT. The mesh resolution is referred to as Grid-level/ G_n , where n
133 denotes the number of bisections. After each bisection, the grid points of the meshes are approximately four
134 times more than the former one.
- 135 (ii) **Icosahedron bisection with a final-step trisection.** Given the requirement for less computational cost, less
136 generators are desired to be used for the VR mesh than the uniform high-resolution mesh. If the VR mesh (one
137 based on refining the QU mesh) uses the same number of points as the one-level-lower QU mesh, the
138 densifying ratio between the fine- and coarse-resolution regions has to be quite large to achieve fine enough
139 resolution in the refined region. Too large densifying ratio may lead to much larger simulation errors. Here,
140 we conduct a final-step trisection (B3) instead of the bisection to achieve an intermediate resolution between
141 two neighboring G-level resolutions. For instance, the resolution of the QU G5B3 mesh (~80 km) is three
142 times finer than G5 (~240 km), between G6 (~120 km) and G7 (~60 km). The number of the added primal
143 cells (mainly hexagons) from G5 to G5B3 is equal to four times the number of the dual cells (triangles) in G5.
- 144 (iii) **Spherical uniform random (SUR) set of points.** Initial points are created uniformly on the sphere Ω by the
145 Monte Carlo method (Metropolis and Ulam, 1949). In this way, the set of generators can be obtained by using
146 an arbitrary number of initial points, not restricted to the fixed number of points as in the icosahedron bisection.

147 2.2.2 Density function

148 The advantage of the SCVT is their freedom to specify the density function. For any two Voronoi regions indexed
149 by i and j , the conjecture is:

$$150 \frac{dx_i}{dx_j} \approx \left[\frac{\rho(x_j)}{\rho(x_i)} \right]^{1/4}, \quad (2)$$

151 where $\rho(x_i)$ is the density function evaluated at x_i , and dx_i measures the local mesh resolution (Ju, 2007). By
152 specifying the density function, the SCVT meshes are able to precisely control the distribution of the local resolution,
153 which provides a convenient way for various refinement modes.

154 A QU mesh forms when the density is constant on the sphere, and the Voronoi region is approximately equivalent
155 to each other. The basic density function for the VR SCVT in this study is expressed as:

$$156 \rho(x_i) = \frac{1}{2(1-\gamma)} \left[\tanh \left(\frac{\beta - \|x_{rc} - x_i\|}{\alpha} \right) + 1 \right] + \gamma, \quad (3)$$

157 where $\|x_{rc} - x_i\|$ denotes the geodesic distance between the location of the refinement center and each generator lying
158 on the surface of the sphere. Here, α indicates the width of the transition zone between the fine-resolution and coarse-
159 resolution regions; β defines the coverage radius of the fine-resolution region; γ measures the densifying ratio of the



160 mesh resolution between the fine and coarse regions. The densifying ratio should better have an moderate value, e.g.,
161 no greater than 1:4 (Liu and Yang, 2017), to constrain the solution error. To balance the demand for less grid points and
162 smaller errors, we fix $\gamma = (1/4)^4$ in this study, marked as X4.

163 The basic density function is flexible to adjust the mesh because of its three-parameter space, but it can only be
164 used for a single-region refinement. Here, we adjust the basic density function for multi-region refinement. The multi-
165 region refinement is divided into two different modes based on the refinement centers. In a hierarchical refinement
166 mode, we add a uniform intermediate-resolution region between the inner fine-resolution and the outer coarse-
167 resolution regions. Eq. (3) can be generalized to a form that allows to control the resolution of the intermediate region:

$$168 \quad \rho(x_i) = \frac{1}{2(1-\gamma)} \left[\frac{1-\lambda}{1-\gamma} \tanh\left(\frac{\beta_1 - \|x_{rc} - x_i\|}{\alpha_1}\right) + \frac{\lambda-\gamma}{1-\gamma} \tanh\left(\frac{\beta_2 - \|x_{rc} - x_i\|}{\alpha_2}\right) + 1 \right] + \gamma. \quad (4)$$

169 λ is designed to control the resolution of the intermediate-resolution region, also referred to as the 2nd-refinement
170 region (dx_{r_2}) that is located between the 1st-refinement (dx_{r_1}) and the coarse-resolution regions (dx_c):

$$171 \quad \frac{dx_{r_1}}{dx_{r_2}} \approx \lambda^{1/4}. \quad (5)$$

172 The function of γ is similar to that in the previous single-region refinement, except that the fine-resolution region is
173 referred to as the 1st-refinement region here:

$$174 \quad \frac{dx_{r_1}}{dx_c} \approx \gamma^{1/4}. \quad (6)$$

175 Corresponding to γ , we refer to the meshes that are generated based on λ values of $(1)^4$, $(1/2)^4$, and $(1/3)^4$ as
176 XL1, XL2, and XL3 meshes, since the resolutions of the 1st-refinement and the 2nd-refinement region vary according
177 to the inner densifying ratios of 1, 2, and 3, respectively. Generally, the value of λ is between γ and 1. Under such
178 circumstances, a hierarchical densifying mode mesh will be finally constructed via the iterative procedure (Fig. 1). As
179 γ is fixed at X4, the hierarchical meshes in this study are called X4L1, X4L2, and X4L3 meshes for short. In a
180 polycentric refinement mode, by adding a different refinement center x_{rc2} , the density function of the polycentric
181 refinement mode is defined as:

$$182 \quad \rho(x_i) = \frac{1}{2(1-\gamma)} \left[\tanh\left(\frac{\beta - \|x_{rc1} - x_i\|}{\alpha}\right) + \tanh\left(\frac{\beta - \|x_{rc2} - x_i\|}{\alpha}\right) + 2 \right] + \gamma. \quad (7)$$

183 The geodesic distance between the two refinement centers must satisfy $\|x_{rc1} - x_{rc2}\| > 2\beta$.

184 3 Dry-atmosphere simulations

185 3.1 Single-region refinement

186 The dry-atmosphere test examines the pure numerical solution of the model. It does not include the nonlinear
187 interaction between dynamics, moisture transport and parameterization. To evaluate the performance of the VR model,
188 we first compare the model dynamics with that simulated by a QU configuration, based on the dry baroclinic wave test
189 case of Jablonowski and Williamson (2006). The QU meshes used here include G6 (~120 km; 40,962 cells), G7 (~60
190 km; 163,842 cells), and G8 (~30 km; 655,362 cells). Three types of generators for constructing the VR meshes are
191 obtained based on: (i) icosahedral bisection (G6X4), (ii) final-step trisection (G5B3X4), and (iii) spherical uniform
192 random points (SURX4). The minimum iteration number is 300,000 for G6X4, and 1,000,000 for both G5B3X4 and
193 SURX4. To assess the simulated baroclinic waves in the refined and transition regions, x_{rc} is located at [180° E, 35°
194 N], with $\alpha = \pi/20$ and $\beta = \pi/6$. Figure 2 displays an example of the X4 meshes used for this test.



195 Figure 3 shows the global distribution of surface pressure at day 9 simulated by the nonhydrostatic core in the
196 baroclinic wave test. The location and magnitude of the high- and low-value centers in the three VR simulations (Figs.
197 3d-3f) well match those in the QU-mesh solutions (Figs. 3a-3c). Due to the asymptotic density function used for the
198 VR meshes, we find some grid imprinting patterns in G6X4. In the G5B3X4 case, the distribution of surface pressure
199 in the equatorial region well coincides with that of G8. The grid imprinting pattern in the southern hemisphere still
200 exists, but has been slightly alleviated as compared to G6X4. The distribution of surface pressure in SURX4 is similar
201 to G8 except for the grid imprinting pattern in the coarse region.

202 To quantify the influence of the VR meshes on the solution accuracy, we calculated the global level 2 (l_2) error
203 norms for surface pressure. The error in each simulation is computed against the G8 solution. The evolution of the l_2
204 error in the hydrostatic solver (Fig. 4a) is close to that in the nonhydrostatic one (Fig. 4b). During the first 10 days, the
205 l_2 errors in all VR simulations are close to that of G6. Among the three VR meshes, the l_2 value of the G6X4 test is
206 the largest, but its error in the nonhydrostatic test is slightly smaller than that in the hydrostatic test. In the nonhydrostatic
207 model, the curve of G6X4 almost lies within the uncertainty regime that represents the maximum error norm within the
208 hydrostatic and nonhydrostatic models. In general, the solution accuracy of two solvers is comparable.

209 As for the QU meshes, the l_2 error in the G7 test is overall smaller than G6. An increase in the number of
210 generators greatly eliminates the grid imprinting in the globe. As for the VR meshes, the difference in l_2 errors between
211 SURX4 and G5B3X4 reflects the influence of the distribution of the initial generators on the mesh quality. The SURX4
212 model possesses more grid points, but its solution error is slightly higher than G5B3X4 at all 15 days. This implies that
213 the randomly generated initial points are more likely to be trapped into the local area during the iterative procedure of
214 mesh generation, leading to more local mesh irregularities. Owing to the more well-distributed initial points, the error
215 in the G5B3X4 simulation is mostly smaller than that in the QU G6 during the first 10 days.

216 Figure 5 illustrates the distribution of relative vorticity at the model level near 850 hPa after 10 simulation days.
217 Note that these values are displayed on the original unstructured triangular cell that defines vorticity, so the oscillation
218 at the smallest grid space (more conspicuous for the coarse resolution; e.g., Fig. 5a) reflects the grid shape rather than
219 grid-scale noise (see the QU results interpolated to the regular longitude-latitude grids in Fig. S1). As the resolution
220 increases, the QU model produces stronger vortices with an evident filament structure (Figs. 5a-5c). In the VR model,
221 the two vortices in the west fall within the fine-resolution regions. The structure of the westernmost vortex in the G6X4
222 simulation (Fig. 5d) is close to G8. The G5B3X4 test (Fig. 5e) captures the fine-scale structure of the two vorticity
223 centers. The magnitudes of the westernmost vortex in G5B3X4 are even closer to those in G8 than G7. When more
224 points are used, as shown in SURX4 (Fig. 5f), both the simulated structure and the magnitudes of vorticities are further
225 improved. The easternmost vortex falls within the transition zone across the fine and coarse resolution. The variation
226 of the mesh sizes there has little impact on the shape of the vortex. The structure and intensity of the vortex are better
227 simulated by G5B3X4 as compared to G6X4. Although the mesh resolutions in the transition zone are coarser than the
228 fine-resolution regions, the pattern of the vortices in G5B3X4 and SURX4 is quite close to that in G8.

229 The VR model generally simulates the smooth structure of the vortices in the refined region, as the high-resolution
230 QU model. In the SURX4 test, however, a slight roughness is found on the tails of the vortices. This defect is largely
231 caused by local mesh irregularities. Compared to the icosahedron-based SCVT meshes, the random initial generators
232 degrade the mesh quality and the simulation performance. Nevertheless, the deterioration hardly does harm to the
233 pattern and magnitude of the vorticity. Corresponding to grid imprinting in the surface pressure over the southern
234 hemisphere, the relative vorticity distributions there in the VR simulations are not as uniform as that in the QU models
235 (figure not shown).

236 To have a more quantitative evaluation of the momentum field, the horizontal kinetic energy spectra are
237 investigated. The QU-mesh simulation data have been interpolated to T106 (G6), T213 (G7), and T426 (G8) Gaussian



238 grids for spherical harmonic analysis. The VR data have been interpolated to T106. The total kinetic energy spectra of
239 the nonhydrostatic and hydrostatic cores are quite close to each other (Figs. 6a, 6d). For the high wave numbers, the
240 curves of G5B3X4 and SURX4 are close to G8. This proves that the VR model is able to resolve fine-scale structure in
241 its refined region. Near the wave number of 100, the curve of G5B3X4 coincides well with G8 regarding both the
242 rotational and divergence component. At such high wave numbers, SURX4 is even closer to G8 in the rotational
243 component due to the increased number of grid points. Regarding the divergence component, SURX4 suffers from the
244 tiny fluctuations due to local irregular grids at high wave numbers. However, the entire curve of SURX4 overall matches
245 that of G8.

246 Previous VR modeling based on the shallow water tests has found that the error in the refined region can be
247 significantly reduced, and little spurious wave reflection were found in the transition zone, but the error outside of the
248 refined region is relatively large (Düben and Korn, 2014). To further inspect whether the VR model can achieve quality
249 simulations in the coarse regions, we conduct a comparison by adding a meridional symmetrical perturbation of the
250 zonal wind at the southern hemisphere, as in Gassmann (2013). The initial location and magnitude of perturbation are
251 consistent with that in the north. The values of the simulated relative vorticity in the southern hemisphere have been
252 finally substituted by their opposite numbers to facilitate examining the differences over two hemispheres. Figure 7
253 shows the relative vorticity field at day 10. The QU model (Fig. 7a) produces very similar wave train in each hemisphere,
254 although the mesh is not symmetric across the equator. In the two VR simulations (G6X4 and G5B3X4; Figs. 7b, 7c),
255 the model resolves finer scale structures of the northern vortices. In the southern hemisphere, the pattern in G6X4 is
256 similar to G6, though the intensity of the vortices is weaker due to its coarser resolutions. The G5B3X4 model
257 reproduces an equivalent structure and magnitude as G6. The consistence between the VR and the QU solutions
258 validates the reliability of the VR model for global simulations.

259 3.2 Multi-region refinement

260 The density function used above has the advantage of alleviating additional damage caused by the transition zone
261 due to its gradual refinement. However, this formulation is restricted to a single-refinement region, which has limitation
262 if multiple regions of interest demand higher resolutions. Enlarging the single-refinement region to cover the multiple
263 interested areas can lead to high costs, especially when the areas are far from each other. Two multi-region refinement
264 modes are thus examined to obtain desired resolutions in multiple regions while fix the number of the total grid points.

265 The first one is the hierarchical refinement mode with one refinement center. This mode contains three consecutive
266 uniform sub-regions outside the refinement center: the 1st-refinement region, the 2nd-refinement region and the coarse-
267 resolution region. The 2nd-refinement region provides an intermediate resolution between the 1st-refinement and
268 coarse-resolution regions. To examine the model performance in the higher resolution simulations, we conduct the
269 symmetrical perturbation baroclinic wave test using G8X4 superposed by the hierarchical refinement mode XL2 (i.e.,
270 G8X4L2). The refinement center is identical to the single-region refinement in Sect. 3.1. At day 10, the vortices in the
271 southern hemisphere move into the 2nd-refinement regions (Fig. 8a). The fine-scale structures of these waves can be
272 well simulated by the model, though the mesh resolution there is actually coarser than that in the north. To the east of
273 the 1st-refinement region, the vorticity in the inner transition zone is also well preserved.

274 The hierarchical refinement mode is potentially useful to regional high-resolution modeling. However, some
275 influencing systems far away from the refinement center may require equivalent or even finer resolution for accurate
276 representation. Here, we adopt the second multi-region mode, the polycentric refinement mode with diverse refinement
277 centers. We conduct the symmetrically perturbed baroclinic wave test based on a polycentric-refinement G7X4 mesh
278 with two different refinement centers at [180° E, 35° N] and [180° E, 35° S], respectively. At day 10, the model well
279 simulates the fine-scale structures over two refined regions (Fig. 8b). The vorticity pattern in the southern hemisphere



280 well coincides with that in the north. Since the refinement mode is modified based on the gradual refinement type, the
281 transition between the fine and coarse regions are smooth and stable. This refinement mode may provide an effective
282 way to simultaneously improve the simulations at different regions.

283 4 Moist-atmosphere simulations

284 We use the idealized tropical cyclone test case (Reed and Jablonowski, 2011, 2012) as a prototype for moist-
285 atmosphere modeling, which includes the nonlinear interaction of dynamics, moisture transport and parameterization.
286 This test is useful to examine the VR performance because the solution does not fully converge even at 10 km resolution
287 (e.g., Zhang et al., 2020). The simulated tropical cyclone is very sensitive to the mesh size, and higher-resolution will
288 produce more intense storms. Two groups of numerical tests have been conducted: one group based on the
289 nonhydrostatic solver with the DTP splitting function enabled; and one based on the hydrostatic solver with no DTP
290 splitting (i.e., dycore, tracer transport and physics use the same time step). Most experimental configurations follow
291 those in Zhang et al. (2020) except the time step. Results from these two groups are overall consistent. In the following
292 main text, only the results from the nonhydrostatic model will be shown, and the results from the hydrostatic model are
293 given in the supplement file (Figs. S2-5) for a reference.

294 We investigate the maintenance of the tropical cyclone when it goes across the transition zone in the hierarchical
295 refinement mode. Given the size-controllable formulation of the density function, we evaluate two ways to examine
296 the evolution of the tropical cyclone. The first case is to have more rapid resolution changes in the inner transition zone
297 based on the two parameters, α_1 and λ . α_1 controls the width of the inner transition zone, and λ represents the
298 densifying ratio between the 1st- and 2nd-refinement regions. Either narrowing the width of the transition zone or
299 enlarging the inner densifying ratio generates a more abrupt transition zone. The other case is to have the transition
300 zone affect the tropical cyclone in an earlier stage. The initial cyclone is placed closer to or even partly within the
301 transition zone by increasing β_1 , a parameter that denotes the coverage radius of the 1st-refinement region. In the
302 following experiments, we use $\alpha_1 = \pi/36$, $\lambda = (1/2)^4$, and $\beta_1 = \pi/12$ to generate a G6X4L2 mesh as a reference.
303 The 1st-refinement region in this VR mesh is ~ 40 km resolution. All these meshes are generated with fixed $\alpha_2 = \pi/36$
304 and $\beta_2 = \pi/4$.

305 The tropical cyclone well preserves its shape at day 10, although the transition zone possesses rapid resolution
306 changes (Fig. 9a). The tropical cyclone is initialized at $[180^\circ \text{ E}, 10^\circ \text{ N}]$ in the 2nd-refinement region, near the transition
307 zone between the 1st-refinement and the 2nd-refinement regions. During its movement from the 2nd-refinement into
308 the 1st-refinement region, the change in the grid size leads to little distortion on the tropical cyclone in each experiment.
309 The wind speed distributions of the tropical cyclone across the abrupt transition zone in several tests using different
310 meshes (Figs. 9b-9d) are quite close to that in Fig. 9a. In the control run, a minor cyclonic disturbance appears in the
311 2nd-refinement region. When the inner transition zone becomes narrower, this disturbance gets stronger due to the
312 larger area of this QU region. The location of the minor cyclonic disturbance is affected by the mesh distribution around
313 the transition zone. In contrast, the major cyclone maintains its strength and shape regardless of the mesh variation.

314 To verify this result, we examine the model sensitivity by conducting a series of experiments based on the three
315 parameters (Fig. 10). Only one of the three parameters is altered for each experiment, as shown in the legends. The
316 minimum surface pressure (Figs. 10a-10c) and maximum wind speed at 850 hPa (Figs. 10d-10f) are used as two proxies
317 to quantify the intensity of the tropical cyclone. The tropical cyclone rapidly strengthens in the first two days before it
318 turns into a gently developing stage in each experiment. The evolution of the intensity is diverse when each of the three
319 parameters changes. However, the final intensity of the cyclone in the experiments is overall comparable, suggesting
320 that the VR solutions are consistent.



321 Ringler et al. (2011) suggested that the global solution error is controlled primarily by the coarsest-resolution
322 region in a shallow-water VR model. Our tests imply that for the VR model, the simulation quality of the fine-scale
323 fluid structures is largely determined by the finest resolution, and other configurations only have a minor role in this
324 regard. It should be also noted that the tropical cyclone is overall stronger in each VR simulation than in the QU case that
325 has the same number of points. The tropical cyclone is initialized in the 2nd-refinement region of the VR mesh. The
326 rapid deepening of tropical cyclone in its early development stage benefits from the higher resolution of the 2nd-
327 refinement region than the QU mesh. The cyclone still develops faster than the uniform one when it moves across the
328 inner transition zone.

329 To further investigate the fine-scale resolving ability of the VR model under higher resolution, we conduct this
330 test case based on the G7X4L2 mesh. The mesh parameters are as same as G6X4L2. The 1st-refinement region in the
331 G7X4L2 mesh is at ~20 km resolution. Fig. 11 shows the horizontal (850 hPa; Figs. 11a-11d) and vertical distribution
332 (Figs. 11e-11h) of the horizontal wind speed in QU and VR simulations. The tropical cyclone strengthens after it moves
333 through the transition zone of the G6X4L2 mesh. Compared to the QU-mesh model, the wind speed distribution is more
334 symmetric in the VR model. The maximum wind band of the cyclone converges towards its center, almost within 1
335 degree from the center. Moving to G7, the VR model based on the G7X4L2 mesh produces stronger storms. The
336 meridional distribution of its wind speed is nearly symmetric. The tropical cyclone expands to higher vertical levels
337 and its intensity increases. The VR model well captures the intensity and structure of the tropical cyclone in its higher-
338 resolution region. The hydrostatic model (Fig. S4) are overall consistent with the nonhydrostatic model, with a slightly
339 stronger tropical cyclone.

340 Previous studies have shown that in the QU-mesh models, the tropical cyclone reduces its area coverage while
341 strengthens its intensity as the resolution increases (e.g., Fig. 3 in Zarzycki et al., 2014; Zhang et al., 2020). Our results
342 present a similar feature that the tropical cyclone possesses a smaller area coverage and higher intensity in the refined
343 regions, as compared to the QU model that has the same number of grid points. Moreover, we have found that the
344 intensity of the tropical cyclone is also influenced by the Smagorinsky horizontal diffusion coefficient (Fig. 12). For
345 the QU G6 mesh (Figs. 12a, 12b), as the coefficient becomes smaller, the minimum surface pressure decreases and the
346 maximum wind speed at 850 hPa increases. Although G6X4L2 shows similar variations as the coefficient decreases
347 (Figs. 12e, 12f), it exhibits higher sensitivity to the coefficient. Such difference still exists when the grid points increases
348 up to G7 (Figs. 12c, 12d, 12g, 12h). This highlights that for a VR model, a proper model configuration with well-tuned
349 parameters is more important to achieve good performance.

350 5 Conclusions

351 In this study, a global unstructured-mesh model is systematically tested and evaluated based on its VR
352 configuration. We propose various refinement approaches and evaluate the model behaviors with the aid of dry and
353 moist idealized test cases. The major conclusion is summarized as follows.

354 (i) Regarding the single-region refinement mode, the VR model possesses comparable accuracy compared to the
355 QU one. The G5B3X4 VR-mesh, which has about half number of grid points in the QU mesh G7, can capture
356 the fine-scale structure of vorticity in the fine-resolution region, closer to G8 than G7. In the transition zone,
357 the vorticity is not adversely affected by the variation of the mesh resolution. In the coarse region, G5B3X4
358 reproduces an equivalent distribution of vorticity as the lower-resolution G6. The multi-region refinement
359 modes provide a more flexible way to achieve desired resolution of multiple regions of interest. In these modes,
360 the model also simulates the general pattern well, with the ability to resolve fine-scale filament structures of
361 the vortices.



362 (ii) For moist-atmosphere modeling, the VR model exhibits stability and robustness. In the hierarchical refinement
363 mode, a series of sensitivity tests based on the three refinement parameters validates the maintenance of the
364 tropical cyclone across the transition zone under the diverse refinement environment. The simulation of the
365 fine-scale structure is mainly controlled by the finest-resolution region. The tropical cyclone rapidly develops
366 in the vicinity of the inner mesh transition zone. Compared to the QU model with the same number of grid
367 points, the VR model simulates a stronger cyclone, with its maximum wind band converging towards the
368 cyclone center. Such a variation is similar to resolution sensitivity supported by the QU-mesh models. The
369 difference between the VR and QU models lies in their sensitivity to the Smagorinsky horizontal diffusion
370 coefficient: a higher coefficient tends to weaken the storm, and the VR model exhibits higher sensitivity than
371 the QU-mesh model in this regard.

372 **Code and data availability:** GRIST is available at <https://github.com/grist-dev>, in private repositories. A way is
373 provided for the editor and reviewers to access the code, which does not compromise their anonymity (to our best
374 effort). A version of the model code, running and postprocessing scripts for supporting this paper are available at:
375 <https://zenodo.org/record/3930643>. The grid data used to enable the tests are located at
376 <https://zenodo.org/record/3817060>. The entire model code is in an active development stage subject to several
377 projects. Full access is available to a member or a collaborator. The authority request may be sent to
378 grist_dev@163.com.

379 **Supplement:** `vr_supplement.pdf` contains Table S1, S2; Figs. S1-S5.

380 **Author contribution:** Y. Zhou performed mesh generation, numerical experiments, data analysis, with inputs from Y.
381 Zhang, and wrote the initial manuscript. Y. Zhang developed the model, designed this study and revised the manuscript.
382 J. Li supervised the project of model development. R. Yu supervised the team member and provided impetus and
383 resources. Z. Liu is responsible for parallel computing, and customized the parallel mesh generation software for GRIST.
384 All the authors continuously discussed the model development and the results of the manuscript.

385 **Competing interests.** The authors declare that they have no conflict of interest.

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390 **References**

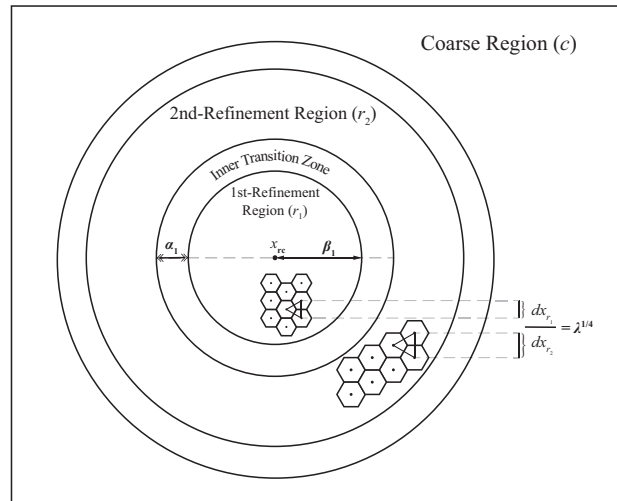
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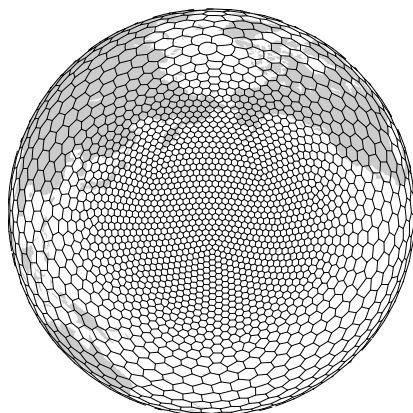
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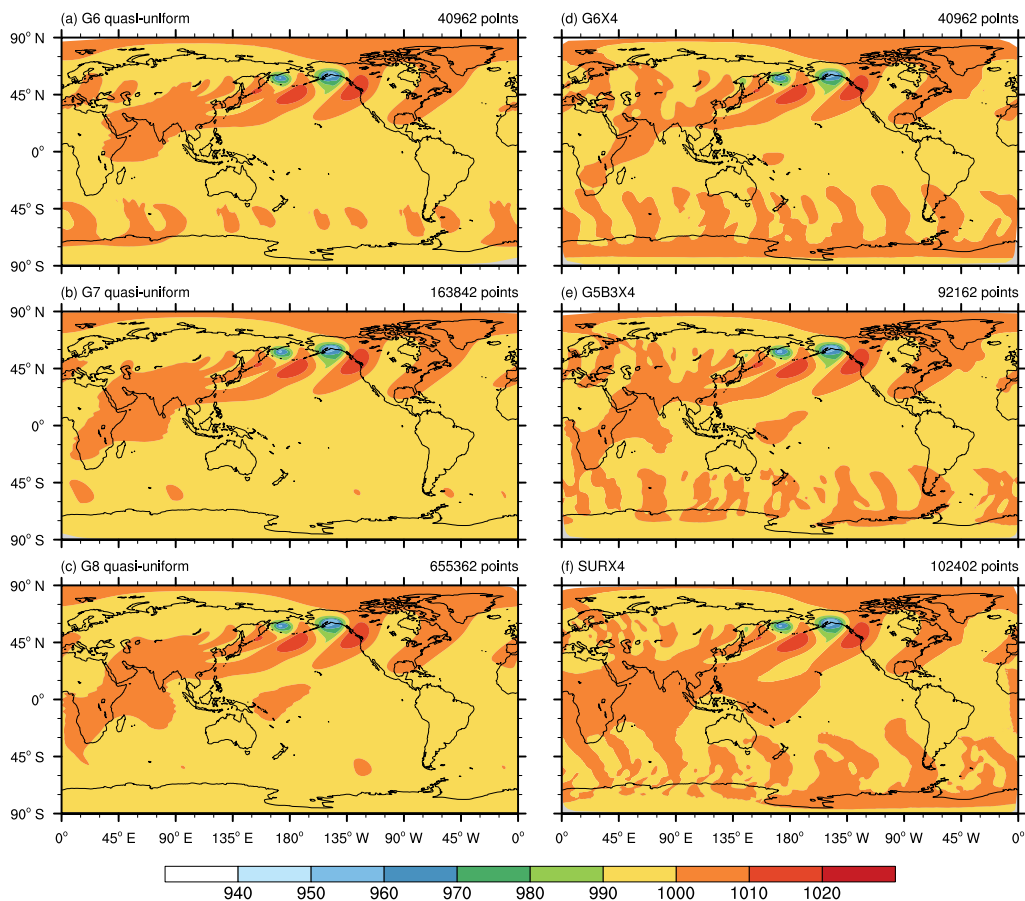
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Figure 1: A schematic diagram of the hierarchical refinement mesh, illustrating the function of three parameters of the density function. α_1 controls the width of the inner transition zone; λ represents the inner densifying ratio between the 1st-refinement and 2nd-refinement resolution regions; β_1 denotes the coverage radius of the 1st-refinement region.

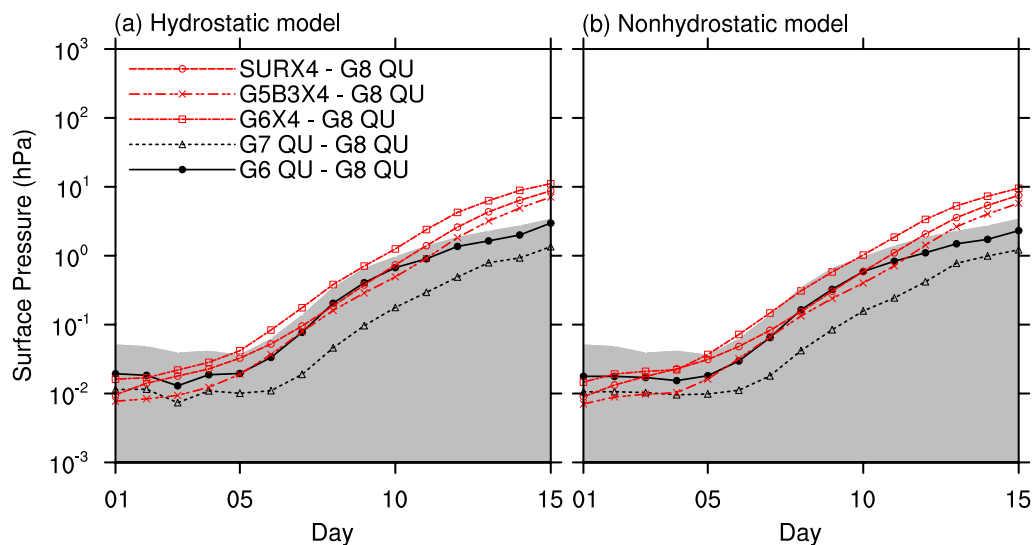


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503 **Figure 2: An illustration of the X4 variable-resolution mesh used for the baroclinic wave test with the refinement center x_{rc}**
504 **at [180° E, 35° N], $\alpha = \pi/20$, and $\beta = \pi/6$. The mesh sizes are scaled up here for a clear vision of each grid cell based on**
505 **reduced generators.**
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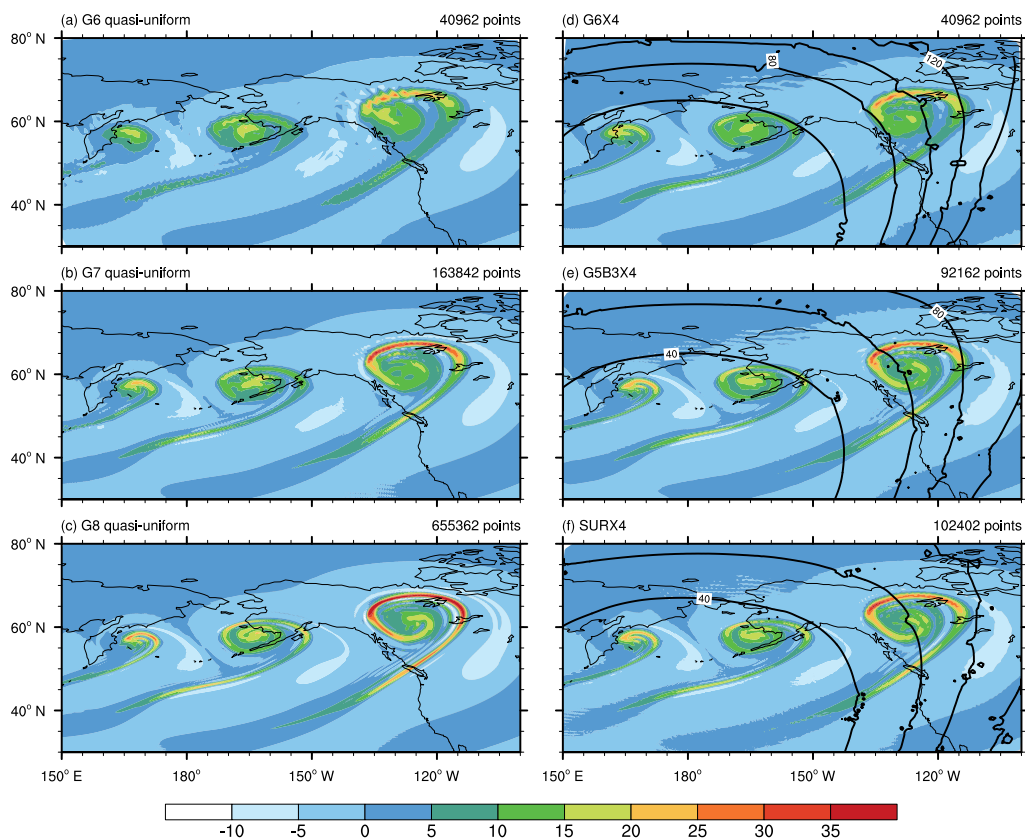
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Figure 3: Baroclinic wave test: surface pressure (unit: hPa) at day 9 simulated by the nonhydrostatic model with (a-c) quasi-uniform and (d-f) variable-resolution meshes.



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Figure 4: Baroclinic wave test: the L_2 error norms of surface pressure as a function of time for (a) the hydrostatic and (b) the nonhydrostatic dynamical core. The error of the low-resolution quasi-uniform mesh (black) and the variable-resolution mesh (red) is computed against the high-resolution quasi-uniform G8 mesh. The gray area denotes the uncertainty in the reference solutions, which selects the maximum L_2 error norms from four curves (the quasi-uniform G7 and G8 cases considering the hydrostatic and nonhydrostatic dynamical core as two models), following Jablonowski and Williamson (2006).



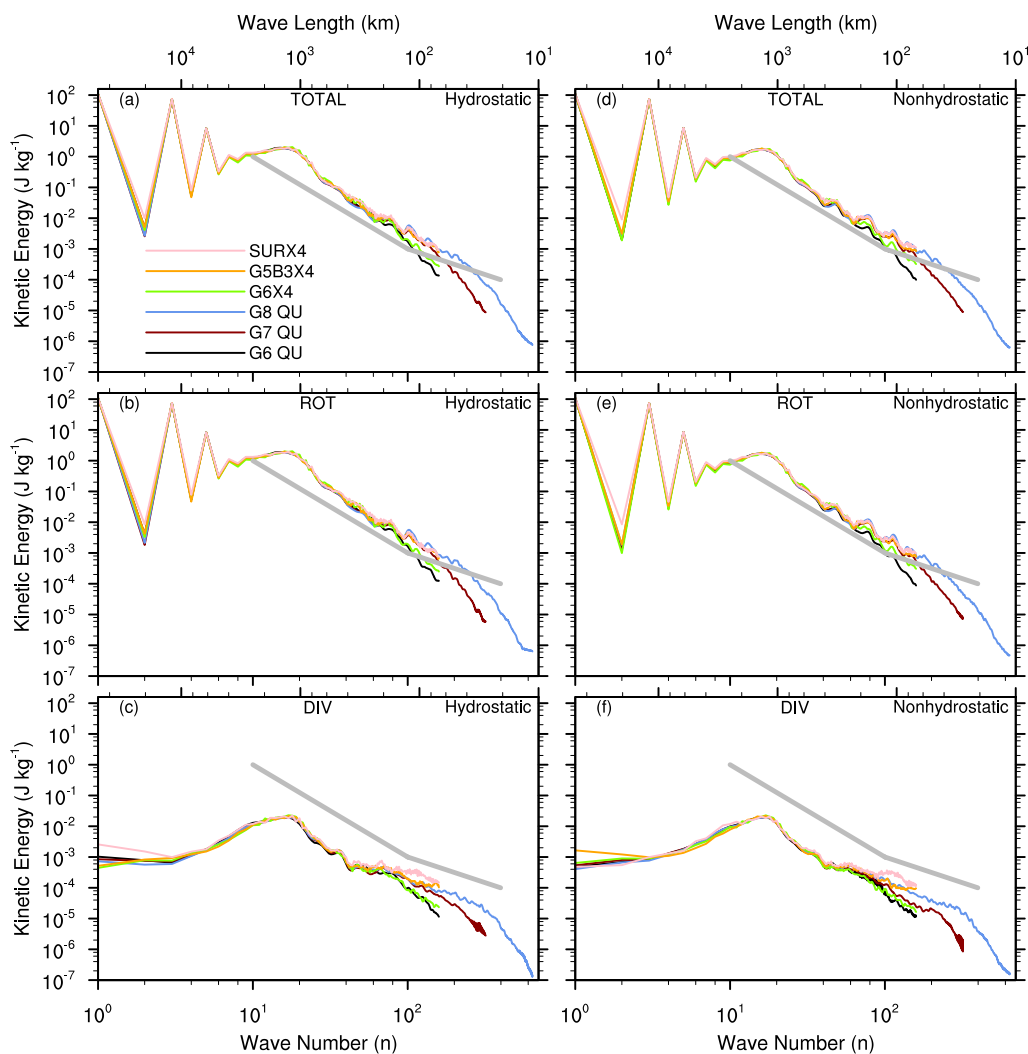
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Figure 5: As in Fig. 3, but for relative vorticity (10^{-5} s^{-1}) at the model level near 850 hPa after 10 simulation days. The contour lines denote the mesh resolutions (in kilometer). The vorticity is defined and displayed on the raw triangular grid (also true for Figs. 7 and 8). See Fig. S1 for the remapped QU-mesh solutions on the regular latitude longitude grid.



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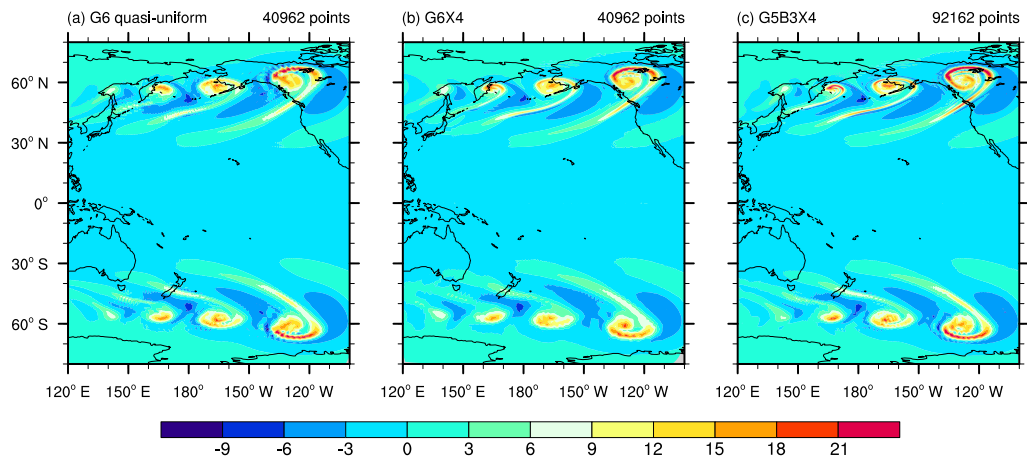
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Figure 6: Baroclinic wave test: horizontal kinetic energy spectra at the model level near 850 hPa at day 10 in terms of (a) the total kinetic energy, (b) the rotational component, and (c) the divergence component, based on quasi-uniform and variable-resolution meshes. Results from the hydrostatic (left column) and nonhydrostatic (right column) dynamical core are shown. The thick gray lines denote the -3 and $-5/3$ slopes, respectively. Units: J kg^{-1} .



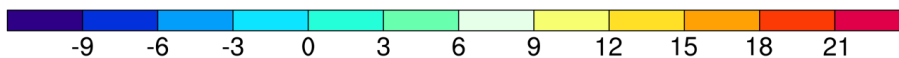
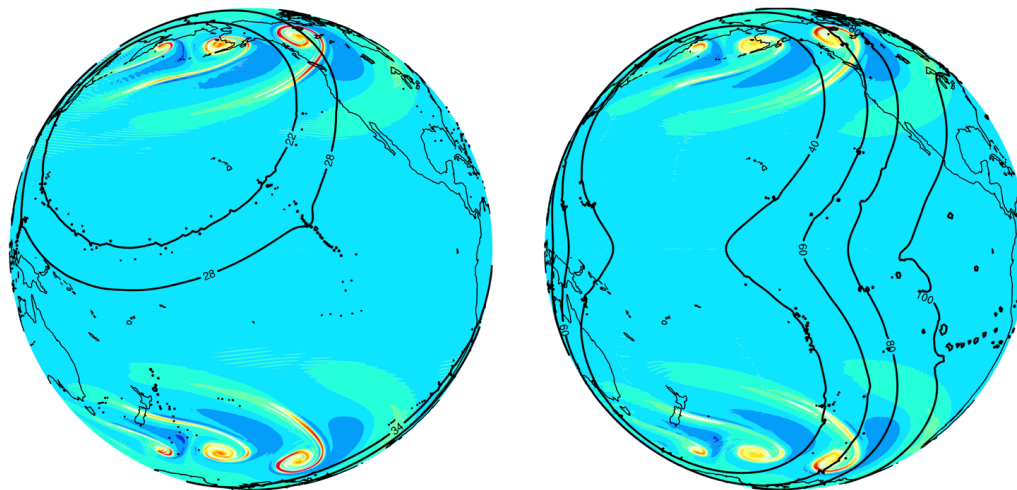
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Figure 7: Adding a symmetrical perturbation in the southern hemisphere for the baroclinic wave test: relative vorticity (10^{-5} s^{-1}) at the model level near 850 hPa at day 10 simulated by the dry dynamical core with (a) quasi-uniform G6, (b) variable-resolution G6X4, and (c) G5B3X4 meshes. The values in the southern hemisphere are substituted by their opposite numbers for a clear comparison. Data are displayed on the raw triangular grid that defines vorticity.



(a) Hierarchical refinement G8X4L2

(b) Polycentric refinement G7X4



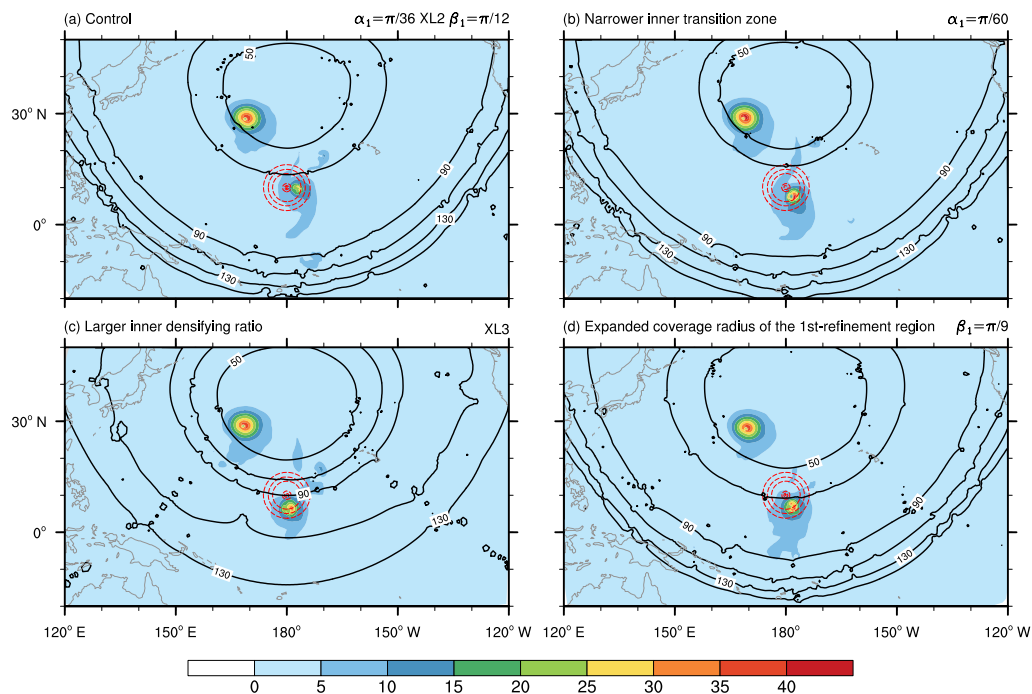
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Figure 8: As in Fig. 7, but for the two multi-region refinement meshes: (a) the hierarchical refinement mesh G8X4L2 and (b) the polycentric refinement mesh based on G7X4. The contour lines denote the mesh resolutions (km).



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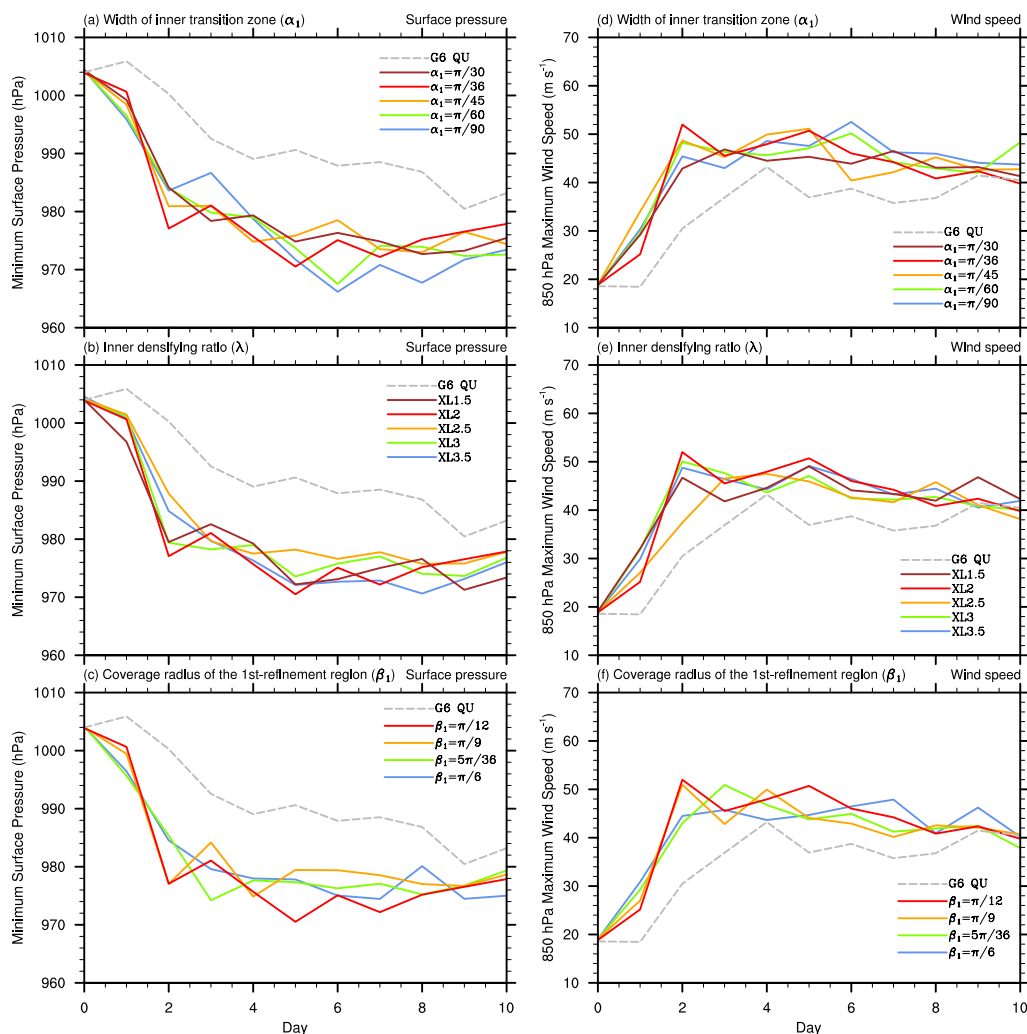
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Figure 9: Idealized tropical cyclone test: the horizontal wind speed (m s^{-1}) at 850 hPa after 10 simulation days based on hierarchical refinement meshes with (a) the control, (b) reduced α_1 and (c) higher λ for more rapid changes in the mesh resolution of the transition zone, and (d) larger β_1 to make the transition zone affect the tropical cyclone in an earlier stage. The red dashed lines mark the initial location of the tropical cyclone. The black contour lines denote the mesh resolutions (km).



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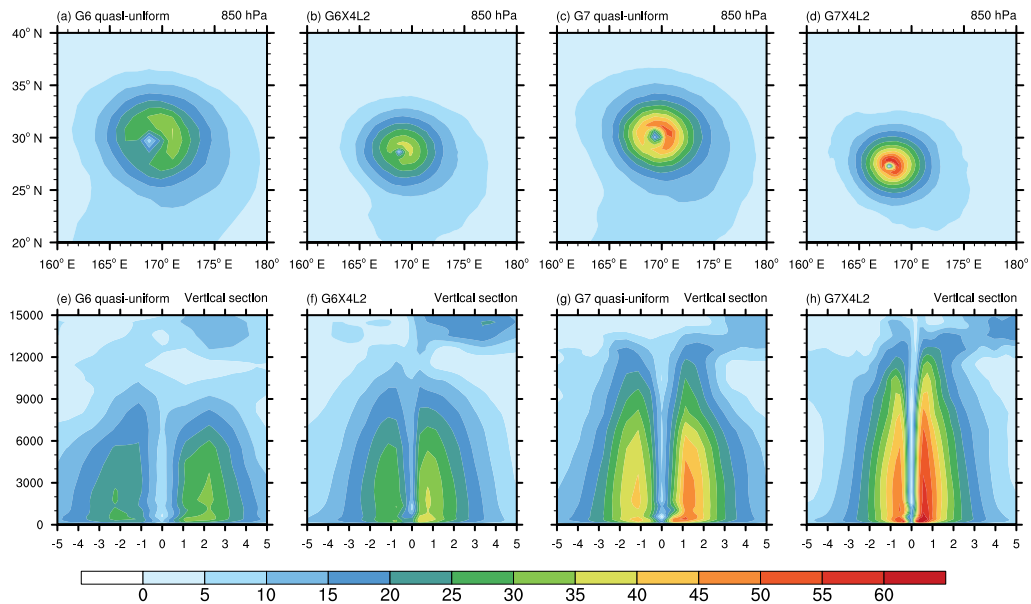
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Figure 10: Idealized tropical cyclone test: temporal evolution of minimum surface pressure and maximum 850-hPa horizontal wind speed on the quasi-uniform G6 and hierarchical refinement meshes with (a, d) α_1 , (b, e) λ , and (c, f) β_1 changed in the sensitivity tests. As one parameter is changed, the other two parameters are fixed according to the control run with $\alpha_1 = \pi/36$, $\lambda = (1/2)^4$ (i.e., XL2), and $\beta_1 = \pi/12$. The three mesh parameters denote the width of the inner transition zone, the inner densifying ratio, and the coverage radius of the 1st-refinement region, respectively.



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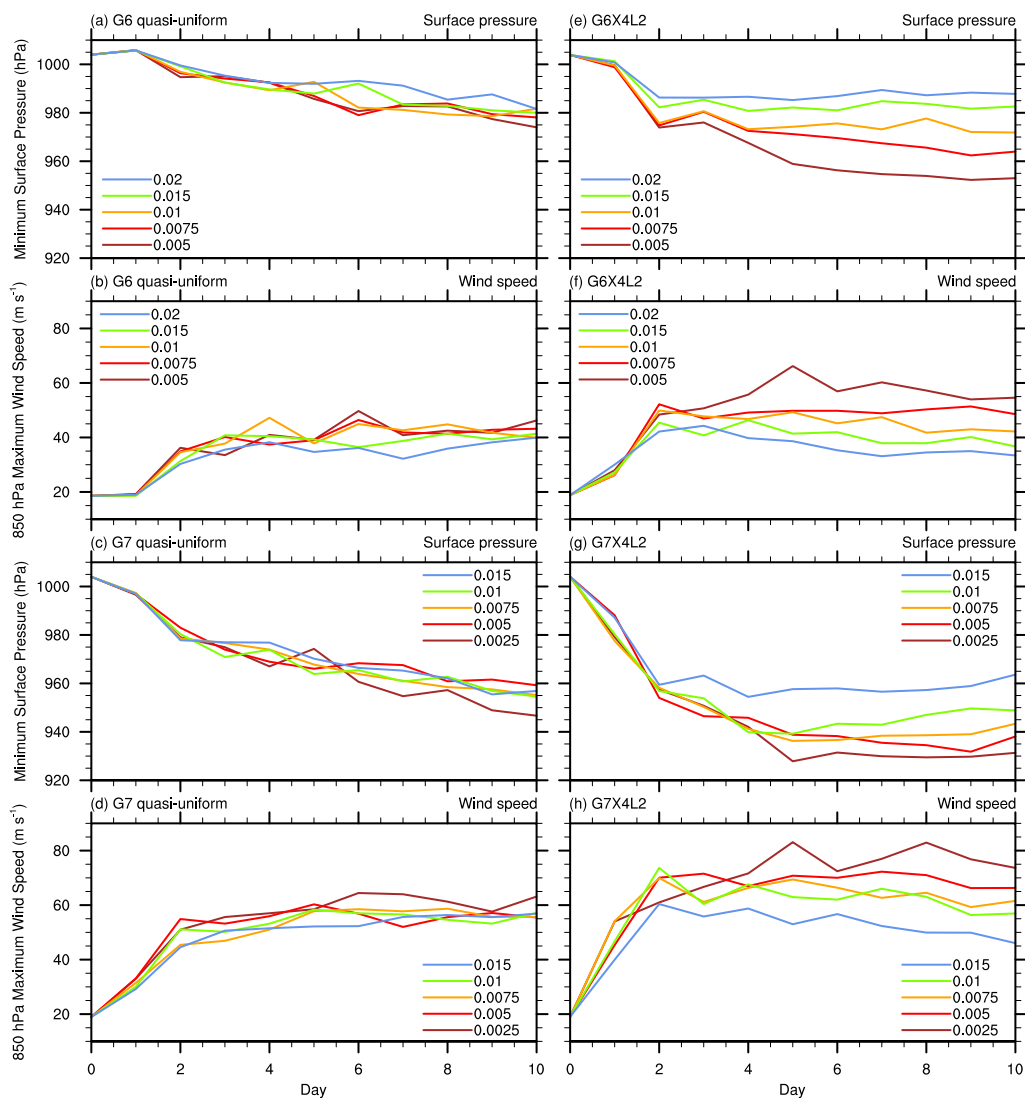
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Figure 11: Idealized tropical cyclone test: (a-d) the horizontal wind speed (m s^{-1}) at 850 hPa after 10 simulation days based on quasi-uniform and variable-resolution meshes and (e-h) the corresponding vertical cross section of the wind speed with a meridional range of ± 5 -degree apart from the center of the tropical cyclone. The vertical coordinate of the cross section denotes the height (m).



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 555 **Figure 12: Idealized tropical cyclone test: temporal evolution of minimum surface pressure and maximum 850-hPa**
 556 **horizontal wind speed based on the quasi-uniform meshes (left column) and hierarchical refinement meshes (right column)**
 557 **with decreasing Smagorinsky horizontal diffusion coefficients marked by colors.**