**Table S1: Mesh resolution and timesteps for the dry-atmosphere test.**

| Test case | Mesh | Averaged grid distance (km) | Minimum grid distance (km) | Maximum grid distance (km) | Timesteps for the hydrostatic model (s) | Timesteps for the nonhydrostatic model (s) |
|---|---|---|---|---|---|---|
| Baroclinic wave: base test | G6 QU | 120.17 | 97.93 | 121.76 | 150 | 150 |
| | G7 QU | 60.09 | 47.60 | 60.88 | 90 | 90 |
| | G8 QU | 30.04 | 23.04 | 30.54 | 40 | 40 |
| | G6X4 | 107.93 | 42.95 | 188.08 | 30 | 30 |
| | G5B3X4 | 71.98 | 27.55 | 125.74 | 30 | 30 |
| | SURX4 | 68.27 | 28.34 | 119.65 | 30 | 30 |
| Baroclinic wave: Adding a symmetrical perturbation | G6 QU | 120.17 | 97.93 | 121.76 | Not tested | 40 |
| | G6X4 | 107.93 | 42.95 | 188.08 | Not tested | 40 |
| | G5B3X4 | 71.98 | 27.55 | 125.74 | Not tested | 30 |
| | G8X4L2 | 28.80 | 13.14 | 61.19 | Not tested | 10 |
| | G7X4-polycentric | 53.02 | 26.63 | 114.94 | Not tested | 10 |

**Table S2: Mesh resolution and timesteps for the moist-atmosphere test.**

| Test case | Mesh | Averaged grid distance (km) | Minimum grid distance (km) | Maximum grid distance (km) | Timesteps for the hydrostatic model with no splitting (s) | Timesteps for the nonhydrostatic model with DTP splitting (s) |
|---|---|---|---|---|---|---|
| Tropical cyclone: base test | G6 QU | 120.16 | 97.08 | 121.83 | 300 | 150-600-1200 |
| | G7 QU | 60.08 | 47.09 | 60.92 | 100 | 90-360-720 |
| | G6X4L2 | 113.24 | 40.23 | 166.00 | 40 | 40-120-120 |
| | G7X4L2 | 56.62 | 19.71 | 82.64 | 20 | 15-45-45 |
| Tropical cyclone: sensitivity to the three parameters of the hierarchical refinement mode (various hierarchical refinement meshes based on the G6X4 mesh) | $\alpha_1$   XL ($\lambda$)   $\beta_1$ | | | | | |
| | $\frac{\pi}{90}$   2   $\frac{\pi}{12}$ | 113.81 | 39.40 | 163.52 | 40 | 40-120-120 |
| | $\frac{\pi}{60}$   2   $\frac{\pi}{12}$ | 113.62 | 39.35 | 165.12 | 40 | 40-120-120 |
| | $\frac{\pi}{45}$   2   $\frac{\pi}{12}$ | 113.40 | 39.61 | 165.68 | 40 | 40-120-120 |
| | $\frac{\pi}{36}$   2   $\frac{\pi}{12}$ | 113.24 | 40.23 | 166.00 | 40 | 40-120-120 |
| | $\frac{\pi}{30}$   2   $\frac{\pi}{12}$ | 113.02 | 40.71 | 165.23 | 40 | 40-120-120 |
| | $\frac{\pi}{36}$   1.5   $\frac{\pi}{12}$ | 110.59 | 44.58 | 179.62 | 40 | 40-120-120 |
| | $\frac{\pi}{36}$   2.5   $\frac{\pi}{12}$ | 114.26 | 37.10 | 158.05 | 40 | 40-120-120 |
| | $\frac{\pi}{36}$   3   $\frac{\pi}{12}$ | 114.72 | 36.77 | 153.26 | 40 | 40-120-120 |
| | $\frac{\pi}{36}$   3.5   $\frac{\pi}{12}$ | 114.73 | 36.74 | 150.61 | 40 | 40-120-120 |
| | $\frac{\pi}{36}$   2   $\frac{\pi}{9}$ | 111.42 | 39.30 | 170.05 | 40 | 40-120-120 |
| | $\frac{\pi}{36}$   2   $\frac{5\pi}{36}$ | 109.65 | 40.54 | 177.41 | 40 | 40-120-120 |
| | $\frac{\pi}{36}$   2   $\frac{\pi}{6}$ | 107.82 | 43.03 | 186.09 | 40 | 40-120-120 |
| Tropical cyclone: sensitivity to the Smagorinsky coefficient ($C_s$) | G6 QU | 120.16 | 97.08 | 121.83 | 300 | 150-600-1200 |
| | G7 QU | 60.08 | 47.09 | 60.92 | 100 | 90-360-720 |
| | G6X4L2 | 113.24 | 40.23 | 166.00 | 40 | 40-120-120 |
| | G7X4L2 | 56.62 | 19.71 | 82.64 | 20 | 15-45-45 (except for 10-20-20 when $C_s = 0.015$) |

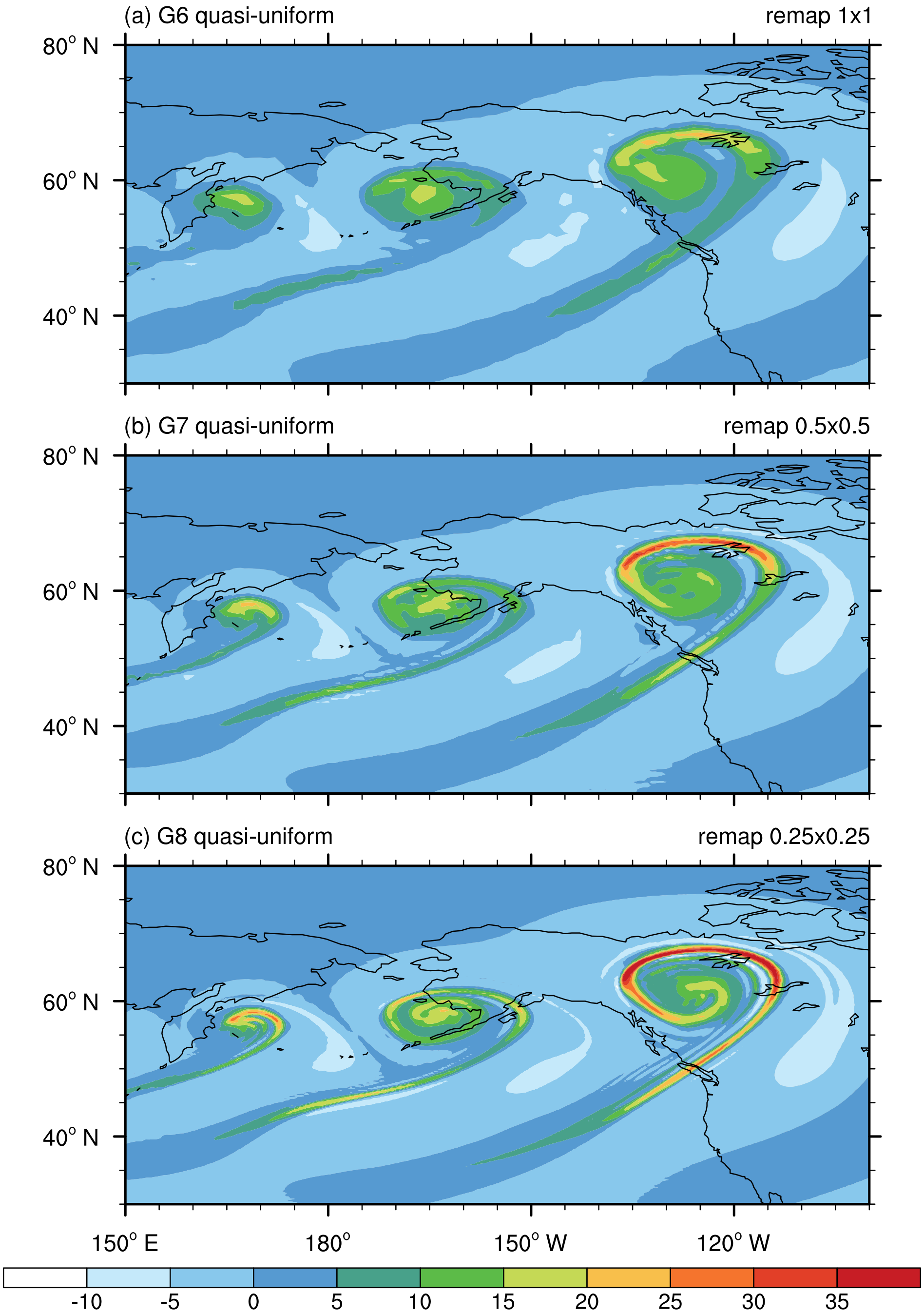


**Figure S1: Baroclinic wave test: relative vorticity ($10^{-5}$ $s^{-1}$) at the model level near 850 hPa after 10 simulation days simulated by the nonhydrostatic model with quasi-uniform meshes. The results are interpolated to global regular latitude longitude grids according to the mesh resolution.**

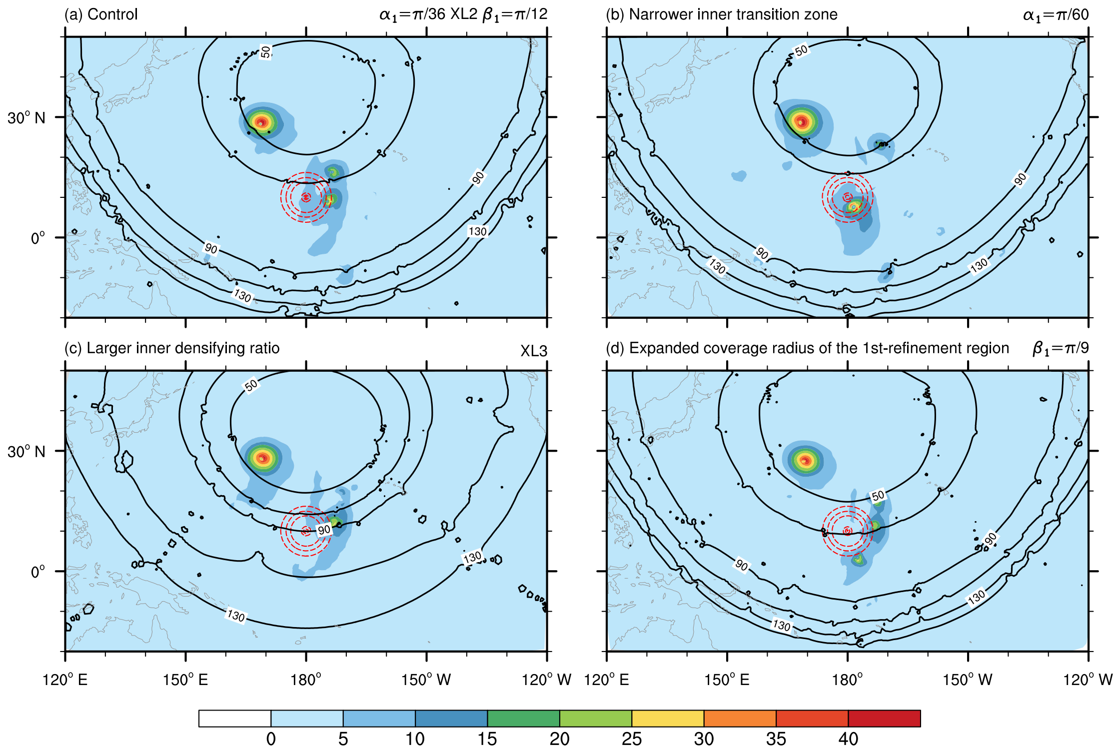


**Figure S2: As in Fig. 9, but for the hydrostatic model with no DTP splitting.**

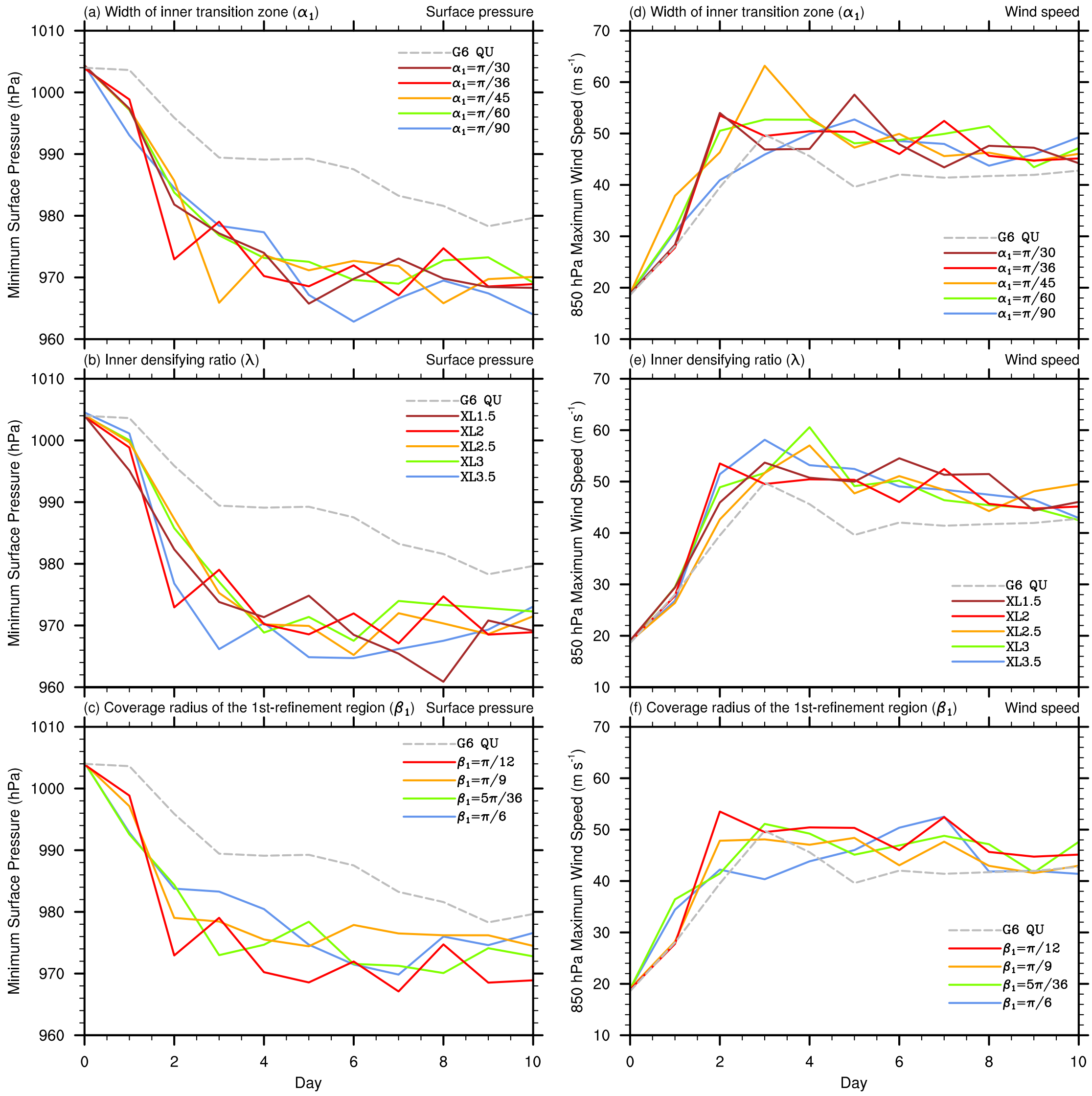


**Figure S3: As in Fig. 10, but for the hydrostatic model with no DTP splitting.**

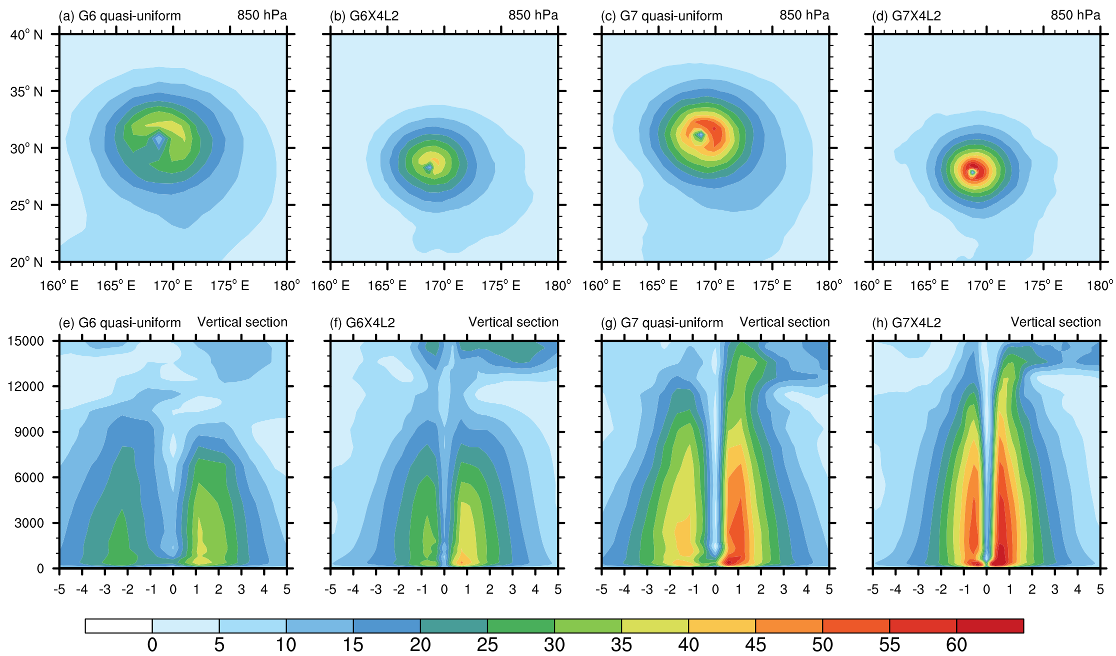


**Figure S4: As in Fig. 11, but for the hydrostatic model with no DTP splitting.**

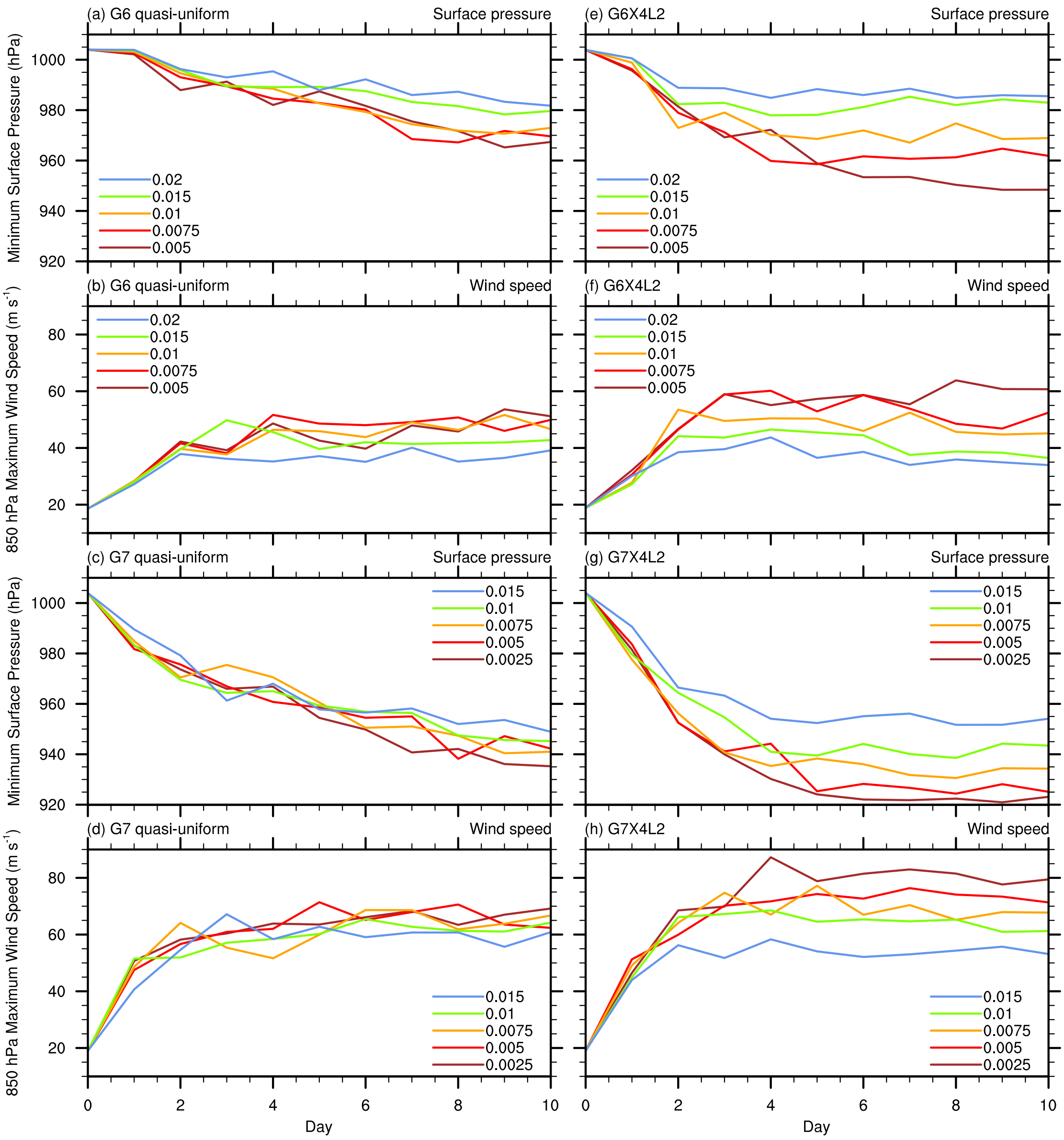


**Figure S5: As in Fig. 12, but for the hydrostatic model with no DTP splitting.**