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Geoscientific Model Development Discussions

Interactive comment on "Quasi-hydrostatic equations for climate models and the study on linear instability" *by* Robert Nigmatulin and Xiulin Xu

Ilias Sibgatullin

sibgat@imec.msu.ru

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Dear Authors,

For your convenience I've regrouped the equations and put the numbers. You may find easier to answer the quations in this message.

The major incorrectness in the paper is the theorem, which states that as the vertical acceleration approaches zero, the hydrostatic approximation (which admits the finite vertical velocity) is asymptotically exact. A proof of this controversial statement is not given in the paper. This statement is responsible for the strange scales, for which

hydrostatic approximation is applied in the paper.

Now I will point to just another two incorrectness, due to which the system of equations is also incorrect for weather prediction at *any* scales.

The Authors take the traditional L.F.Richardson's framework (Kasahara 1966) without citation, and simplify the expression for the total time derivative of pressure $\frac{dp}{dt}$ by omitting the horizontal pressure advection:

$$p = g\int_z \rho dz \tag{1}$$

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + \vec{v}\nabla p$$

$$\begin{aligned}
\frac{dp}{dt} &= g\int_z \frac{\partial \rho}{\partial t}dz + \quad \vec{v}_{hor}\nabla_{hor}p + v_z\nabla_z p\\
\frac{dp}{dt} &= g\int_z [-div(\rho\vec{v})]dz + \quad \vec{v}_{hor}\nabla_{hor}p - v_z\rho g\\
\frac{dp}{dt} &= g\int_z [-div_{hor}(\rho\vec{v}_{hor})]dz + v_z\rho g + \quad \vec{v}_{hor}\nabla_{hor}p - v_z\rho g\\
\frac{dp}{dt} &= g\textstyle\int_z [-div_{hor}(\rho\vec{v}_{hor})]dz + \quad \vec{v}_{hor}\nabla_{hor}p
\end{aligned} \tag{2}$$

After that the Authors neglect the second term in the last expression above, based on the very strange scale analysis:

1. They estimate the scale of the divergence of first term by the scale of only one component. But the divergence itself is by order(s) of magnitude less than scale its components.

2. They estimate the $\nabla_{hor}p$ *dynamically* as U^2/L. This estimation could be correct if for example we had initially a layer at the state of rest, and then the large horizontal

scale perturbation of pressure would produce the waves (propagating with about the speed of sound). But this estimation is absolutely incorrect for weather prediction in the real atmosphere which is set in motion after cyclogenesis with zonal winds, motions of air masses and weather fronts. Indeed, the pressure advection can be sometimes very small, as well as the divergence, or sometimes the total derivative of pressure is small. But for a general case the neglect of pressure advection in favor of the divergence of mass flux may result in accumulation of additional vertical velocity.

The pressure advection is very cheap from the computations point of view, if we know the pressure and velocity. But if there was a meaningful reason to get rid of it, the much less ambiguous way would be just to further expand the last expression (2) above:

$$\frac{dp}{dt} = g\int_z [-div_{hor}(\rho\vec{v}_{hor})]dz \qquad + \qquad g\vec{v}_{hor}\int_z \nabla_{hor}\rho dz \tag{3}$$

$$\frac{dp}{dt} = -g\int_z \rho\, div_{hor}(\vec{v}_{hor})dz \quad -g\int_z \vec{v}_{hor}\nabla_{hor}\rho dz + \quad g\vec{v}_{hor}\int_z \nabla_{hor}\rho dz \tag{4}$$

The 2-nd and 3-d term compensate each other if velocity does not depend on z. So this expression

$$\frac{dp}{dt} = -g\int_z \rho\, div_{hor}(\vec{v}_{hor})dz \tag{5}$$

is exact if v_{hor} does not depend on z, for such a case it expresses an obvious fact that

$$\frac{dp}{dt} = -p\, div_{hor}(\vec{v}_{hor}),$$

when the pressure is the height of the air column. And it can be a fair approximation, ***in contrast to omitting the pressure advection, which is not actually an approximation but an unbalancing of the expression for the total derivative of pressure.***

I will illustrate it with the motion of a wagon with sand (see the attached picture) which corresponds to the horizontal bulk transport of the masses of air. Of course you will never see such a pure motion in the real atmosphere but here I separate it to illustrate the formula.

The pressure corresponds to the height of the sand, and obviously, when the wagon moves as a whole $\frac{dp}{dt} = 0$ for any column of sand. So in the expression of the total derivative $\frac{dp}{dt} = \frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} = 0$ and $\frac{\partial p}{\partial t} = -v\frac{\partial p}{\partial x}$.

Now if we "neglect" the pressure advection $v\frac{\partial p}{\partial x}$ we will get some finite, and may be very big value of $\frac{dp}{dt}$, instead of ZERO.

Returning to the atmosphere, ***this is why the additional vertical velocity may be accumulated***, and ***this is why the energy conservation is violated***, since $\frac{dp}{dt}$ is a part of the expression for the vertical velocity and change of the full energy.

I hope that the Authors will answer this detailed claim for the mistakes, or finally retract all these mathematical inconsistencies.

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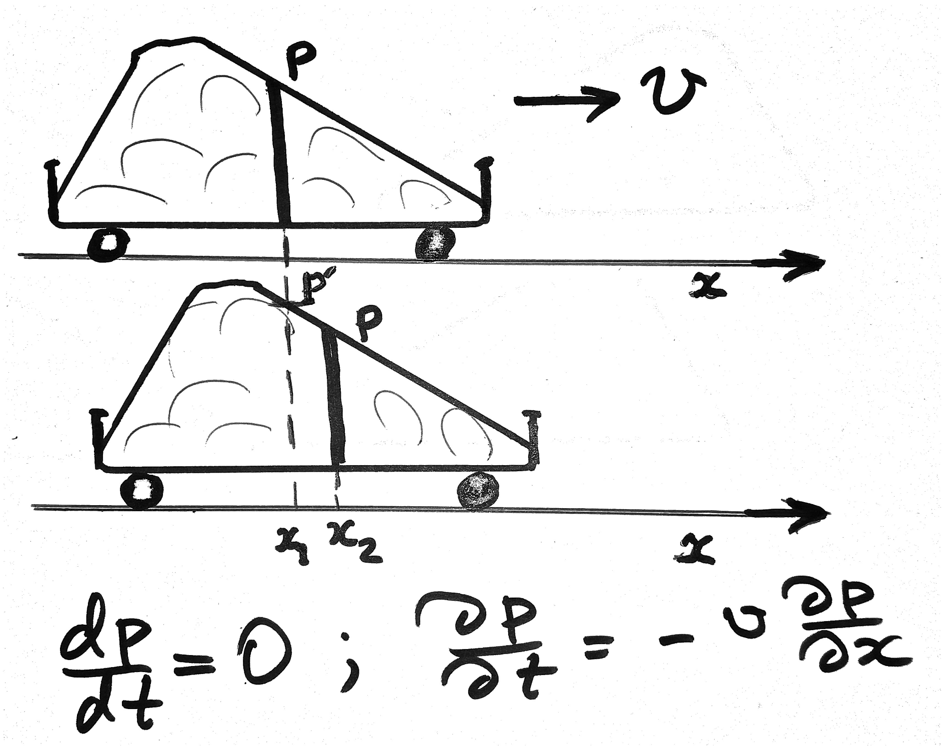

ρ
$\rightarrow v$
x
ρ'
ρ
x_1 x_2
x
$\frac{d\rho}{dt} = 0 \; ; \; \frac{\partial \rho}{\partial t} = - v \frac{\partial \rho}{\partial x}$

Fig. 1.