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Geoscientific Model Development Discussions

Interactive comment on "Quasi-hydrostatic equations for climate models and the study on linear instability" *by* Robert Nigmatulin and Xiulin Xu

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I was pointed to a misprint in my previous comment

...

$$\frac{dp}{dt} = -g \int_z \rho \, div_{hor}(\vec{v}_{hor}) dz$$

is exact if v_{hor} does not depend on z, for such a case it expresses an obvious fact that

$$\frac{dp}{dt} = -p \, div_{hor}(\vec{v}_{hor}),$$

when the pressure is the height of the air column ...

Of course, I meant the weight of a column of air. There is a direct analogy to shallow water equations here, so I've made a misprint.

Another question was about "my" dimensional analysis of the total derivative of pressure, if I do not like the analysis by the Authors.

The analysis below is given to show that, in my opinion, primitive scale analysis of the primitive equations is not enough for definite conclusions whether the pressure advection can be dropped during the time integration of the equations. Such analysis is correct as an overall estimate, but it can not predict the importance of zonal or bulk flows. Also I've put a review at

https://pubpeer.com/publications/CDC804462F2FC7E0826F4E0E09BB4E

where the major concern is a proof of the Theorem, from which it follows, that short waves have to transform to long waves, as acceleration becomes smaller.

First, I would recall the expression for the total derivative of pressure in hydrostatic approximation before any simplifications (3)

$$\frac{dp}{dt} = g\int_z [-div_{hor}(\rho\vec{v}_{hor})]dz + g\vec{v}_{hor}\int_z \nabla_{hor}\rho dz$$

The pressure advection is the second term, which can be estimated as

$$g\,U\,\frac{\Delta\rho}{L_{hor}}\tilde{H},$$

where $\Delta\rho$ is the change of ρ over the horizontal scale, $\tilde{H} = H - z$.

The first term could be very overroughly estimated as :

$$g\int_z \frac{\partial\rho}{\partial t}dz = g\,\frac{\Delta\rho}{\tau}\tilde{H} = g\,U\frac{\Delta\rho}{L_{hor}}\tilde{H},$$

where τ is the time scale, so $\tau = L_{hor}/U$. Here the vertical convection is neglected in the same manner as the Authors do not account for vertical motion, when they try to estimate pressure advection as $\rho U^2/L_{hor}$. For such a case the orders of the first and second terms are equal, so pressure advection can not be neglected.

But let's use the most direct approach for estimation of the first term as

$$g\frac{\hat{\rho}DIV(v_{hor})}{L_{hor}}\tilde{H},$$

where $DIV(v_{hor})$ – estimation of the horizontal divergence.

Hence, the ratio of the pressure advection and the first term can be estimated as

$$\frac{U}{DIV(v_{hor})}\frac{\Delta\rho}{\rho} = \frac{\Delta\rho}{\alpha\rho},$$

where α is the factor by which the divergence of the velocity is less than the velocity itself. For the synoptic scale motions such a factor α is about $1/10$ (Charney, 1948). On the other hand, the pressure (and density) changes routinely by about $\frac{1}{100}$ or even up to $\frac{1}{20}$ over synoptic scale (see an attached example of the pressure map over Europe), so: $\frac{\Delta\rho}{\rho} \approx \frac{1}{100}$ or sometimes even $\approx \frac{1}{50}$.

The result so far is that by the most direct approach above, the ratio of the pressure advection to the first term in (3) for synoptic scale motions $\frac{\Delta\rho}{\alpha\rho}$ can be estimated by about $\frac{1}{10}$ or even $\frac{1}{5}$.

Such a ratio may be enough to neglect pressure advection in tendency equation, when one wants to roughly know the pressure for tomorrow. But for time integration of the primitive equations with small but quite important vertical velocity expressed with the help of $\frac{dp}{dt}$, such a one-sided deficiency may provoke the accumulation of the residual, and result in additional vertical velocity and violation of the conservation of the full energy.

So even such a direct scale analysis does not give me a definite answer whether pressure advection can be neglected for time integration of the primitive equation. Decomposition of the velocity to geostrophic and ageostrophic also does not give a definite answer, since both pressure advection and divergence vanish in purely geostrophic limit. So I'm not sure that such a conclusion is possible to formulate as a theorem. It would be interesting to find a manuscript where the ratio of the pressure advection to the divergence term is estimated for motions at different scales, and at different latitudes. Even not so much from practical point of view, as for qualitative understanding. May be indeed there are situations at some latitudes, when pressure advection can be always ignored during time-integration of primitive equations, I am ignorant about that.

The scale analysis above does not prove, that advection of pressure is always substantial. It shows only, that such an analysis is not enough for neglect of pressure advection in integration of the equations, and more delicate approach is needed. The approach undertaken by the Author, which is based on the *estimation of the mass flux divergence by scale of only one component, and estimation of pressure gradient as* $\rho U^2/L$ *is wrong at any scales of weather prediction problems* in hydrostatic approximation (I wonder if someone can give a counterexample, assuming the initial conditions are not special), so it can lead to *useless* models.

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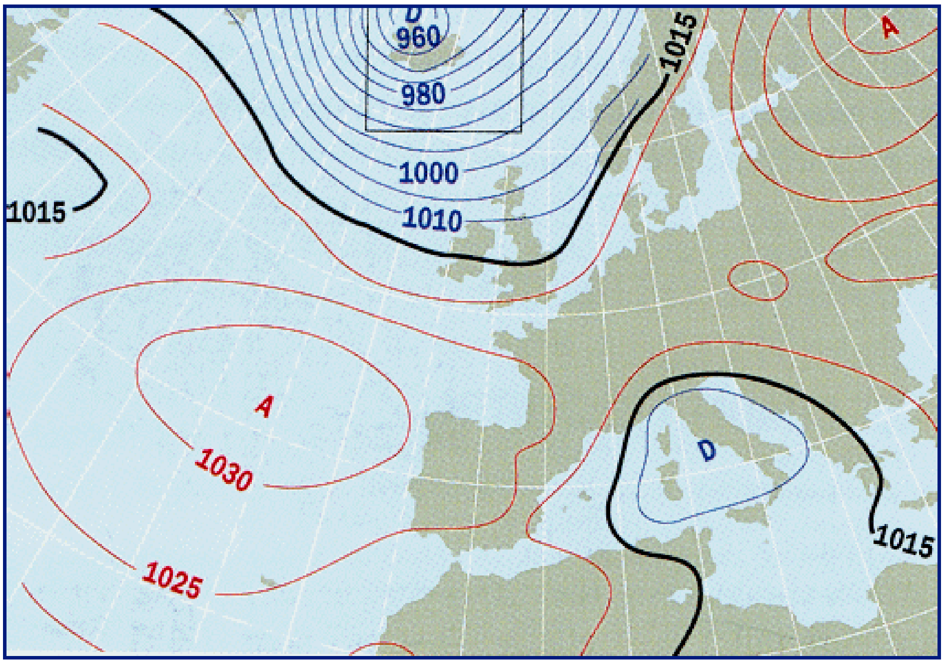

Fig. 1.

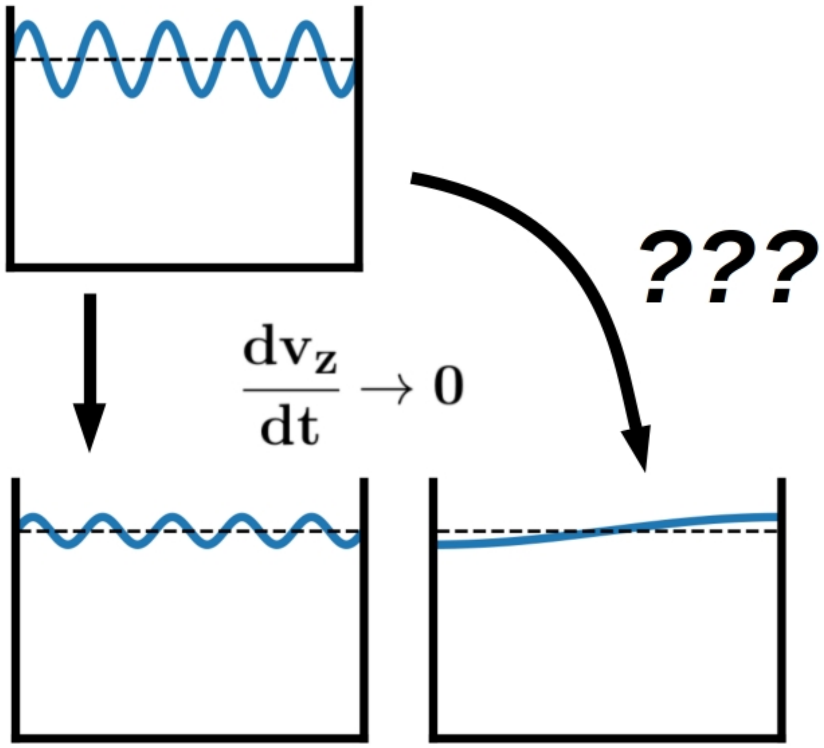

Fig. 2.