Geosci. Model Dev. Discuss.,
https://doi.org/10.5194/gmd-2020-146-AC3, 2020

Geoscientific Model Development Discussions

Interactive comment on "Quasi-hydrostatic equations for climate models and the study on linear instability" *by* Robert Nigmatulin and Xiulin Xu

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Received and published: 30 October 2020

To finalize the discussion part, we have the following statements regarding the questions that occurred in the discussion.

1. This work aims to study the shortwave stability property of the vertically quasi-hydrostatic system of equations using the linearized equations for perturbations. This system of equations is applicable and used for almost all large-scale climatic and meteorological calculations with characteristic time greater than $\tau \geq 10^2$ s, spatial scales over $L \geq 10^3$ m and velocity scale $V \sim L/\tau \leq 10$ m/s. Four

types of evaluation for vertical velocity (or pressure) are compared on the base of vertically quasi-hydrostatic approximation:

   (a) The asymptotically exact equation (fourth equation of (2.23))
   (b) Holton's approximation (4.23)
   (c) Quasi-incompressibility of air particle (4.28)
   (d) Quasi-incompressibility in space point (Marchuk's approximation) (4.30)

2. We note that Sibgatullin's criticism aims at the asymptotically exact equation (a), published in the paper of the first co-author in 2018, and it is not the subject of our manuscript. In the manuscript it is shown that in the equation for vertical velocity, the term with the substantial pressure derivative divided by pressure ($p^{-1}\mathrm{d}p/\mathrm{d}t$) consists of four components

$$\frac{1}{p}\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{1}{p}\frac{\partial p}{\partial t} + \frac{1}{p}v_x\frac{\partial p}{\partial x} + \frac{1}{p}v_y\frac{\partial p}{\partial y} + \frac{1}{p}v_z\frac{\partial p}{\partial y}$$

In the paper 2018 it was shown that the terms, connected with horizontal relative pressure transfer (the second and the third components in the right side of the last expression) tend to zero asymptotically. We have also answered Ilias Sibgatullin many times, both at seminars in Russia and during the manuscript discussion. He does not want to understand our straightforward arguments repeated in this discussion. In particular, we have answered Sibgatullin on how to conduct the estimation

$$\frac{1}{p}v_y\frac{\partial p}{\partial y}, \frac{1}{p}v_x\frac{\partial p}{\partial x} \sim \frac{\mathrm{M}^2}{\tau} \sim \frac{\varepsilon}{\tau} \to 0, \quad \mathrm{M} = \frac{V}{C} \leq 10^{-3}, \quad C = \frac{\gamma p}{\rho} \sim 300\mathrm{m/s}$$
$$(see\, SC16\, and\, SC8),$$

where M and C are Mach Number and sound speed. It means that horizontal relative pressure transfer asymptotically tends to zero for $M \sim \epsilon \to 0$. Instead of this estimation, Sibgatulin permanently considers the horizontal pressure transfer terms (without $1/p$)

$$v_x \frac{\partial p}{\partial x},\, v_y \frac{\partial p}{\partial y} \quad (see\, SC7).$$

Therefore, further discussion on this issue makes no sense. After all, equation (a) (or the fourth equation of (2.23)) may be considered as one of the approximations.

3. It is shown that the shortwave perturbations with a large ratio of horizontal wavenumbers to the vertical wavenumber will make the solutions (even the resting state solutions) of quasi-hydrostatic systems (A, B, C, D) unstable. As shortwave perturbations occur during the numerical calculation of the original differential equations, and the wavelengths of the perturbations are proportional to the grid sizes of meshing, the result of shortwave stability can be used for appropriate meshing to achieve stable numerical calculation.

4. The dispersion relation for gravity waves is different from that in our result (from anonymous referee #2, see RC2). We state all the differences between our set of equations and the equations used in RC2 in the reply SC13. The perturbations in RC2 are assumed to be in harmonic form with constant amplitude, while we allow the amplitude of perturbations to change by time, as in the manuscript (3.11).

5. More doubts about the quasi-hydrostatic equations' applicability occur from anonymous referee #1 (see RC1). As described in the manuscript, the quasi-hydrostatic approximation is only valid for long-term climatic processes (when all inertia forces are negligibly small compared with the gravity force). It is important to realize that the neglect of inertial forces appears only in the equation of vertical velocity. And it makes the system of equations (2.23) non-hyperbolic with

an infinite speed of sound, then its solution is unstable to harmonic shortwave perturbations when the solution evolves forward in time. Taking into account the small vertical inertia force makes the numerical meteorological and climatic calculations non-realistic because of extremely small time steps. The energy of the system is conserved as the thermodynamic equation is used to evaluate the vertical velocity. However, the perturbations' energy is not constrained because they are caused by numerical error and filled in the entire space. A detailed response to anonymous referee #1 is in AC2.

We are trying to improve the manuscript to make it clear with points 3, 4 and 5 marked by anonymous referees # 1 and # 2. We are very grateful to them for their remarks.

Sincerely,

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