Geosci. Model Dev. Discuss.,
https://doi.org/10.5194/gmd-2020-146-AC2, 2020
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Geoscientific Model Development Discussions

Interactive comment on "Quasi-hydrostatic equations for climate models and the study on linear instability" *by* Robert Nigmatulin and Xiulin Xu

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Received and published: 14 September 2020

Dear Sir or Madam:

Thank you very much for reviewing our manuscript. In the following text, we hope to answer the questions raised in this review. Comments of Referee #1 are in italic; our answers are in regular letters.

Line 19: Please contrast the quasi-hydrostatic equations against other systems with filtered sound waves, e.g. anelastic and pseudo-incompressible equations. Also it would be advantageous to give other solutions for avoiding the problem of sound waves, such

as implicit-explicit temporal integrators.

Answer: One can use the pseudo-incompressible approximation ($div\overrightarrow{u} = 0$) or the anelastic approximation ($div\rho\overrightarrow{u} = 0$) to filter sound waves by neglecting the time differential term of density in the continuity equation, but the conservation of mass is violated and replaced by the "conservation of volume". In comparison, the quasi-hydrostatic approximation filters the sound wave by neglecting the inertial terms in the vertical momentum equation. The comparison of acoustic-wave-filtered systems is also made in Durran (2008).

The quasi-hydrostatic approximation is valid for long-term and large-scale processes, if we take the vertical length scale to 1 km, and the time scale to 1 hour:

$\frac{\partial v_z}{\partial t} \sim \frac{L_z}{\tau^2} \sim \frac{10^3}{3600^2} \ll g\ (9.8\frac{m}{s^2})$.

Even if we include the term $\partial v_z/\partial t$ in the vertical momentum equation, the numerical error would be compatible with its real value.

Besides, in Davies (2003) it writes,"Anelastic equation sets are the principal basis of many theoretical and modeling studies of small-scale dynamics, for which they play an analogous role to that of the hydrostatic primitive equations for planetary-scale dynamics."

In fully compressible (unapproximated) equations, one can avoid the problem of sound waves by numerical techniques, like the variational approach Rõõm (1998), implicit or semi-implicit time stepping Tanguay (1990).

Line 21: "Most global climate models are based on a system of dynamic equations in quasi-hydrostatic approximation." I believe this statement to be incorrect. Most global atmospheric models either use the fully non-hydrostatic equations or the hydrostatic equations with shallow atmosphere approximation. To the best of the reviewers knowledge only the UK Met Office model has an option for the quasi-hydrostatic equations. See, for example, Ullrich et al. (2017).

Answer: We would modify this statement to "The quasi-hydrostatic approximation is adopted in the atmospheric models for studying global long-term non-extreme climatic processes.". The reason why we focus on the hydrostatic equations is that in non-hydrostatic equations, an additional time spacing restriction should be added in the vertical direction:

$\Delta t < K\frac{\Delta z}{V_{ver}}$.

As $\Delta z \ll \Delta x, \Delta y$, much smaller time steps for non-hydrostatic equations are needed.

However, non-hydrostatic effects may be important for processes of smaller scales, for instance, when the cloud microphysics is considered.

Line 36: "Almost the entire mass of the atmosphere is located in the layer with thickness H of order 10km. So the atmospheric dynamics outside polar zones can be considered in quasi-Cartesian coordinate system..." Why does the relative thinness of the atmosphere affect the use of a Cartesian coordinate system?

Answer: The continuity equation for the one-dimensional spherically symmetric motion (purely radial flow) in the spherical coordinate writes:

$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + \frac{2\rho v}{r} = 0.$

In the case of $\Delta r = H\ (\sim 10\ km) \ll r(\sim\ 6000\ km)$, the third term of the continuity equation is negligibly small in comparison to the second term.

Line 36-38: The authors should be more clear that they are using a planar approximation of the equations. Otherwise there isn't an explanation for neglecting the curvature terms in equations (1.2)-(1.4) below.

Answer: The curvature terms (like the third term $\frac{\rho v}{r}$ in the equation above) can be neglected, since the thickness of the atmosphere is much less than the Earth's radius and then the curvature terms are negligibly small in comparison to the gradient terms.

Line 138: Also see Kasahara and Washington (1966). You may also wish to refer to

DeMaria (1995) equation (2.13), which is an example of recent mention of this equation.

Answer: Different from (2.13) in Kasahara and Washington (1966), we evaluate each term in equation (2.13) of current work and get an asymptotic equation (2.20) for $\varepsilon \to 0$.

In DeMaria (1995), equation (2.13) and thermodynamic equation (2.4) are used as diagnostic equations. Instead of these two equations, in the equation set of the current work, we adopt the equation of vertical velocity (2.20) and the vertical hydrostatic equation (last equation of 2.23), then the temperature is obtained by the state equation (2.24).

Line 140: There has been some work recently showing that these non-hydrostatic terms may be more important in a moist context. See, for example, Gao et al. (2017) or Yang et al. (2017).

Answer: We totally agree with referee #1 that the non-hydrostatic terms should be taken into account in the models with moist, because the time scale for microphysics of clouds is much smaller than the time scale in the current work, and the vertical velocity of air can influence the diffusion of water vapor. In our model moist is not considered by far.

Line 145 (equation 2.23): Do the quasi-hydrostatic equations here satisfy any sort of energy principle? If the system exhibits instability for certain ratios of horizontal and vertical grid spacing, then the diagnostic vertical velocity equation must be responsible for the addition of energy to the system. Presumably one should be able to show which terms are responsible for this violation.

Answer: The thermodynamic equation (1.6) in current work is identical with equation (2.4) in DeMaria (1995). It is not included as a diagnostic equation in current work, but the vertical velocity (2.20) is obtained by the thermodynamic equation (1.6) and the continuity equation (1.1). The vertical velocity is not neglected in the horizontal momentum equations. Therefore, the thermodynamic equation (1.6) is valid while using (2.20).

Besides, the energy density takes the form

$e = \frac{v_x^2}{2} + \frac{v_y^2}{2} + \frac{v_z^2}{2} + c_p T,$

If we take the scales: $V_{hor} \sim 10\frac{m}{s}, V_{ver} \sim 1\frac{m}{s},\ c_p \sim 10^3\ J/(kg\ K),\ \Delta T \sim 10\ K$, *then the kinetic energy is negligibly small,* $e_k \ll e$*. Despite this fact, in the current work, the horizontal velocity is obtained by the momentum equations, and the vertical velocity is evaluated by (2.20).*

Line 220: It would be advantageous to show how the quasi-hydrostatic equations diverge from the unapproximated equations when it comes to instability. One should be able to show agreement between the quasi-hydrostatic equations and unapproximated equations for a certain regime of k_hor and k_ver

Answer: As we apply only the quasi-hydrostatic approximation, the unapproximated equations are identical to the Navier-Stocks equations, which are hyperbolic and automatically stable under shortwave perturbations.

Line 460: If I'm understanding the authors correctly, this instability is present regardless of the values of kappa2. Even for small values of kappa2 the system will eventually go unstable without some external control. So wouldn't a better solution be to use an equation set that actually satisfies a closed energy principle?

Answer: As described in (3.11), the shortwave instability is associated with the positive increment of perturbation amplitude ϖ_{**}, in particular, the higher value of ϖ_{**} corresponds with stronger instability. As shown in Figure 2, k_ver = 15 corresponds with a higher value of kappa2; thus, in such a case it is more unstable as ϖ_{**} is larger in comparison with the case k_ver = 150. Also, small positive value of kappa2 leads to small value of ϖ_{**}, it is easy to eliminate such instability using pseudo viscosity introduced in current work or other techniques introduced in Ullrich et al. (2017).

Line 475: Can anything be said about the accuracy of these equations, analogous to Davies et al. (2003)?

Answer: Equations (2.1) – (2.4) with hydrostatic approximation in Davies et al. (2003) are equivalent to the second, third, sixth and fourth equations of (2.23) in the current work, respectively. As temperature and density are linked by the state equation, in our set of equations, the continuity equation (first equation of (2.23)) is used instead of the thermodynamic equation (2.5) in Davies et al. (2003). Another difference is that we introduce a new variable $\dot{M}$ (analogous to $\frac{D\pi}{Dt}$ in Davies et al. (2003)) for the closure of the equations set.

In terms of accuracy of these equations, we can thus conclude as in Davies et al. (2003), that the hydrostatic equations misrepresent the vertical modes at small horizontal scale. But such a problem does not exist for large horizontal scale.

We will definitely change the manuscript according to your comments and suggestions.

Thank you again for your precious time in reviewing our manuscript!

Sincerely,

Robert Nigmatulin and Xiulin Xu

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