and Kipp, 1980; Vuyst and Vignjevic, 2013[TS40]; Williams, 2019). Comparisons with observed fracture behaviour has indicated the predictive value of these schemes (Xu et al., 2010; Husek et al., 2016[TS41]). We combine the various approaches to best fit the dynamical multi-phase mass movement model that is developed. Following Grady and Kipp (1980), we simulate a degradation of strength parameters. Our material consists of a soil and rock matrix. We assume fracturing occurs along the inter-granular or inter-rock contacts and bonds (see also Cohen et al., 2009). Thus, cohesive strength is lost for any fractured contacts. We simulate degradation of cohesive strength according to a volume strain criteria. When the stress state lies on the yield surface (the set of critical stress states within the six-dimensional stress-space), during plastic deformation, strain is assumed to contribute to fracturing[CE10]. A critical volume strain is taken as a material property, and the breaking of cohesive bonds occurs based on the relative volume strain. Following Grady and Kipp (1980) and Vuyst and Vignjevic (2013)[TS42], we assume that the degradation behaviour of the strength parameter is distributed according to a probability density distribution. Commonly, a Weibull distribution is used (Williams, 2019). Here, for simplicity we use a uniform distribution of cohesive strength between 0 and $2c_0$, although any other distribution can be substituted. Thus, the expression governing cohesive strength becomes Eq. (26).

$$\frac{\partial c}{\partial t} = \begin{cases} -c_0 \frac{1}{2} \frac{\left(\frac{\epsilon_{\mathrm{v}}}{\epsilon_{\mathrm{v}0}}\right)}{\epsilon_{\mathrm{c}}} & f(I_1, J_2) \geq 0, c > 0 \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

Here c_0 is the initial cohesive strength of the material, $\epsilon_{\mathrm{v}0}$ is the initial volume, $\left(\frac{\epsilon_{\mathrm{v}}}{\epsilon_{\mathrm{v}0}}\right)$ is the fractional volumetric strain rate and ϵ_{c} is the critical fractional volume strain for fracturing.

2.2.3 Water partitioning

During the movement of the mixed mass, the solids can thus be present as a structured matrix. Within such a matrix, a fluid volume can be contained (e.g. as originating from a groundwater content in the original landslide material). These fluids are typically described as groundwater flow following Darcy's law, which poses a linear relationship between pressure gradients and flow velocity through a soil matrix. In our case, we assumed the relative velocity of water flow within the granular solid matrix as very small compared to both solid velocities and the velocities of the free fluids. As an initial condition of the material, some fraction of the water is contained within the soil matrix (f_{fc}). Additionally, for loss of cohesive structure within the solid phase, we transfer the related fraction of fluids contained within that solid structure to the free fluids.

$$\frac{\partial f_{\mathrm{fc}}}{\partial t} = -\frac{\partial(1-f_{\mathrm{fc}})}{\partial t} = \begin{cases} -f_{\mathrm{fc}} \frac{c_0}{c} \frac{\max(0.0, \dot{\epsilon}_{\mathrm{v}})}{\epsilon_{\mathrm{f}}} & f(I_1, J_2) \geq 0, c > 0 \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

$$\frac{\partial f_{\mathrm{sc}}}{\partial t} = -\frac{\partial(1-f_{\mathrm{sc}})}{\partial t} = \begin{cases} -f_{\mathrm{sc}} \frac{c_0}{c} \frac{\max(0.0, \dot{\epsilon}_{\mathrm{v}})}{\epsilon_{\mathrm{f}}} & f(I_1, J_2) \geq 0, c > 0 \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

Beyond changes in f_{fc} through fracturing of structured solid materials, no dynamics are simulated for influx or outflux of fluids from the solid matrix. The initial volume fraction of fluids in the solid matrix defined by ff_{fc} and $s f_{\mathrm{sc}}$ remains constant throughout the simulation. The validity of this assumption can be based on the slow typical fluid velocities in a solid matrix relative to fragmented mixed fluid–solid flow velocities (Kern, 1995; Saxton and Rawls, 2006). While the addition of evolving saturation would extend the validity of the model, it would require implementation of pre-transfer functions for evolving material properties, which is beyond the scope of this work. An important note on the points made above is the manner in which fluids are re-partitioned after fragmentation. All fluids in fragmented solids are released, but this does not equate to free movement of the fluids or a disconnection from the solids that confined them. Instead, the equations continue to connect the solids and fluids through drag, viscous and virtual mass forces. Finally, the density of the fragmented solids is assumed to be the initially set solid density. Any strain-induced density changes are assumed small relative to the initial solid density ($\frac{\rho_{\mathrm{c}}}{\rho_{\mathrm{s}}} \ll 1$).

2.2.4 Fluid stresses

The fluid stress tensor is determined by the pressure and the viscous terms (Eqs. 29 and 30). Confined solids are assumed to be saturated and constant during the flow.

$$\mathbf{T}_u = p_{\mathrm{f}}\mathbf{I} + \boldsymbol{\tau}_{\mathrm{f}} \tag{29}$$

$$\boldsymbol{\tau}_{\mathrm{f}} = \eta_{\mathrm{f}}\left[\nabla\mathbf{u}_{\mathrm{u}} + (\nabla\mathbf{u}_{\mathrm{c}})^t\right] - \frac{\eta_{\mathrm{f}}}{\alpha_{\mathrm{u}}}\mathcal{A}(\alpha_{\mathrm{u}})\left(\nabla\alpha_{\mathrm{c}}(\mathbf{u}_{\mathrm{u}} - \mathbf{u}_{\mathrm{c}}) + (\mathbf{u}_{\mathrm{c}} - \mathbf{u}_{\mathrm{u}})\nabla\alpha_{\mathrm{c}}\right) \tag{30}$$

Here $\mathbf{I}$ is the identity tensor, $\boldsymbol{\tau}_{\mathrm{f}}$ is the viscous stress tensor for fluids , p_{f} is the fluid pressure, η_{f} is the dynamic viscosity of the fluids and $\mathcal{A}$ is the mobility of the fluids at the interface with the solids that acts as a phenomenological parameter (Pudasaini, 2012[TS43]).

The fluid pressure acts only on the free fluids here, as the confined fluids are moved together with the solids. In Eq. (30), the second term is related to the non-Newtonian viscous force induced by gradients in solid concentration. The effect as described by Pudasaini (2012)[TS44] is induced by a solid concentration gradient. In the case of unconfined fluids and unstructured solids ($f_{\mathrm{sf}} = 1$, $f_{\mathrm{sc}} = 1$)[CE11]. Within our