

We would like to thank the reviewer for the comments provided which helped to significantly improve the clarity of the manuscript. Below you will find the answers to the specific comments.

Reviewers comments are indicated in bold letters. Changes in the manuscript are [indicated in blue](#).

1) What is the model for the parameter flow in the ETKF? In page 4 line 30, the authors mentioned that "a persistence model is assumed for the model parameters (i.e $\theta_t = \theta_{t-1}$)". This would mean that the model for the parameter flow is $\theta_t = \theta_{t-1}$, and the ensemble of the parameters will only shrink, which does not agree with the plots in Figure 4, where the ensemble oscillates as if $\theta_t = c\theta_{t-1} + W_t$.

We agree with the reviewer that this point was not clear in the text. Parameters are assumed to remain constant during model integration. However after assimilation we performed multiplicative inflation so that the analysis parameter ensemble spread equals the first-guess parameter ensemble spread. In this way we avoid the collapse of the ensemble of parameters and can estimate their time evolution. We add included this in Page 8, Lines 26-28.

“The relaxation to prior spread inflation approach (RTPS, Whitaker and Hamill (2012)) with a parameter of $\alpha=0.5$ is applied to the state variables to reduce the impact of sampling error. [For the parameters, the ensemble spread is inflated back to its original value after assimilating the observations \(similar to the conditional inflation approach of Aksoy et al. \(2006\). This is equivalent to assume that the parameter uncertainty is time-independent, thus preventing the parameter ensemble spread from collapsing.](#)”

2) How is the ensemble of the parameter generated at the initial time?

We thank the reviewer for pointing this out, we add some clarification to the text, in Page 8, Line 24 and Page 9, Line 4-5.

“The ensemble at the first assimilation cycle is initialized using zero ash concentrations for all members and a set of parameters that are sampled randomly from a Gaussian distribution [whose mean and variance for each experiment are detailed below.](#)”

...

“All presented data assimilation and parameter estimation experiments are summarized in Table 2 [including the statistical properties of the initial parameter ensemble](#). Finally, a set of simulation experiments are carried out using a larger domain to evaluate the impact of the optimized parameters upon the simulation of the ash cloud farther from the vent.”

3) How is the physical constraint (page 8 line 15) ensured in the spread of the ensemble at all times?

We thank the reviewer for pointing this out. The explanation given in the text was not clear. The check of the parameter values is performed individually for each ensemble member. An improved description of the algorithm is provided in Page 9, Line 6-10.

“In the case of eruption source parameters, nonphysical values are checked individually for each ensemble member and are replaced with a random realization from a Gaussian distribution with the same mean and standard deviation as the analysis ensemble. If the randomly generated value is outside the physically meaningful range for the parameter, the process is repeated until the randomly generated value is within the physically meaningful range. The physically meaningful range for model parameters is set to 0-20 km and 0-15 for h and A-Suzuki respectively.”

4) The physically meaningful range (page 8 line 15) is 0-20 km and 0-15, but the spreads in tests are 500m and 0.5 (page 8 line 24). Is there any specific reason for such a relatively small spread?

We thank the reviewer for raising this point. Following the reviewer’s comment we add a short discussion about this in Page 11, Lines 16-20.

“Figure 7 (b) shows the spatially averaged ash concentration ensemble spread. One way to assess if the current parameter ensemble spread is well tuned is to compare the ash concentration forecast error and spread. If these are similar then we can assume that our uncertainty is well represented in the ensemble. In this case, the uncertainty in the ash concentration is mainly associated with the uncertainty in the source parameters. As observed, the spread values are close to the RMSE values in Figure 7 (a), which indicates that after convergence of the parameters, the ensemble spread is closely representing the magnitude of the errors.”

5) Is it possible to describe how does the FALL3D model depend on the parameters?

Following this comment that is also shared by Reviewer 1, we added a line citing an article that makes a deep discussion on the sensitivity of the model to different parameters. Also we extended the explanation of Suzuki source term and how it affects the mass distribution in the vertical profile. These changes are included in Page 4, between Lines 2 and 17 and a new Figure (Figure 1) has been prepared which shows the sensitivity of the vertical ash emission profile to parameters h and A (which are the ones estimated in this work):

“For simplicity and without loss of generality, we will assume here a MER given by the Mastin et al., (2009) scheme, which depends on the fourth power of the top height of the eruptive column and does not account for wind effects, and a Suzuki vertical mass distribution (Pfeiffer et al., 2005) that is an empirical vertical ash mass eruption rate distribution that assumes no interactions with the surrounding atmosphere (e.g. effects of wind shear or stratification upon the eruptive column), also it is assumed that the shape of the vertical flow rate is the same for all particle sizes and is given by:

$$S(z) = (1 - \frac{1}{h} \exp[A \frac{z}{h} - 1])^\lambda \quad (1)$$

where $S(z)$ is the mass eruption rate distribution function, z is the altitude above the vent, h is the top height of the eruptive column, A and λ are two dimensionless parameters. Figure 1 shows the sensitivity of the vertical emission profile to different values of h and A . It is important to recall that h not only controls the maximum height of the eruptive column, but also the total mass emitted (Fig. 1 a). Parameter A do not modify the total amount of mass being emitted but significantly affects the level at which the maximum emission takes place (Fig. 1 b) which can significantly affect the posterior evolution of the ash plume particularly if cases in which there is strong vertical wind shear. The parameter λ is a measure of how concentrated is the emission around the maximum (not shown). A previous sensitivity test (Osoreo, 2018) has shown that the two FALL3D model parameters that affect most the model results are the eruption column height h and the parameter A in the Suzuki distribution. For this reason, these two parameters will be used in the following sections to define the ETKF-FALL3D system experiments. The sensitivity of the FALL3D model to these parameters in terms of the deposit, has been documented by (e.g., Scollo et al., 2008) . "

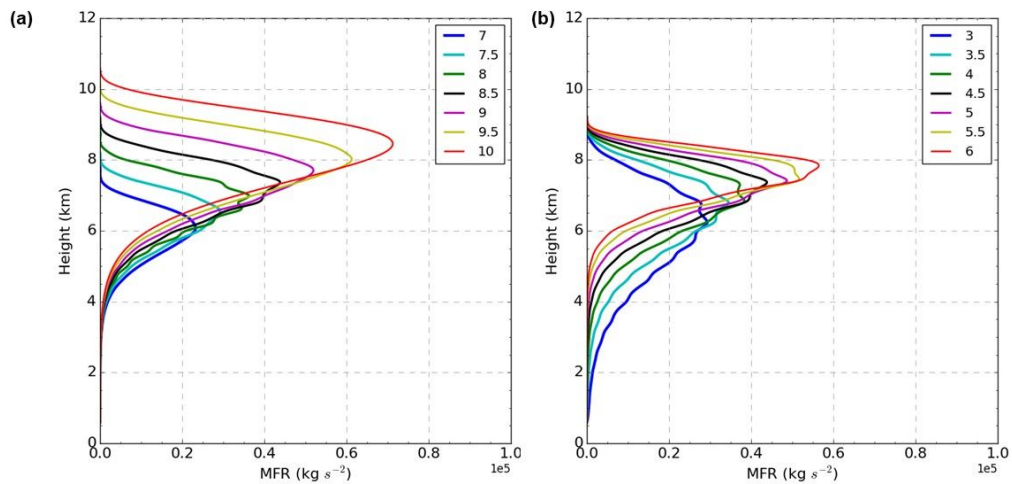


Figure 1. Vertical mass distribution for different (a) eruptive column top heights and (b) A-Suzuki parameters

6) Are there some parameters lead to instability or unphysical state values?

This is a good point. Parameters optimized in this work are related to the eruptive source. Because of this there are no instabilities associated to the parameters. However, unphysical state values can result for example from negative parameter values (e.g. negative column height or A-Suzuki parameter).