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Geoscientific Model Development Discussions

Interactive comment on "Slate: extending Firedrake's domain-specific abstraction to hybridized solvers for geoscience and beyond" *by* Thomas H. Gibson et al.

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1. GENERAL COMMENTS

This is an excellent, well-written paper describing an important contribution to finite element software. Among numerical analysts and practitioners, there has been increasing interest in hybridization, static condensation, and post-processing of mixed and (more recently) discontinuous Galerkin finite element methods. However, these methods have been difficult to implement for users of standard finite element software. The authors have developed and documented a high-level domain specific language, called Slate, along with an implementation that makes these techniques widely accessible within the Firedrake finite element package. This is comparable to how the Unified

Form Language (UFL), underlying both Firedrake and FEniCS, made it possible for finite element variational problems to be described at a high level. The authors include numerical examples illustrating the application of Slate, and the numerical results agree with theory.

2. SPECIFIC COMMENTS

There are only a few minor revisions that I would suggest.

2.1. There are a few inaccuracies and omissions in the historical references for hybridization of mixed finite element methods, particularly in the introduction.

2.1.1. The bibliographic reference to Brezzi and Fortin's book (from 1991, not 2012) is incorrect. The correct reference is:

F. Brezzi and M. Fortin, Mixed and hybrid finite element methods, vol. 15 of Springer Series in Computational Mathematics, Springer-Verlag, New York, 1991.

The authors may also wish to cite the following, which is essentially an updated version of the Brezzi and Fortin book:

D. Boffi, F. Brezzi, and M. Fortin, Mixed finite element methods and applications, vol. 44 of Springer Series in Computational Mathematics, Springer, Heidelberg, 2013.

2.1.2. Hybridization and static condensation of mixed methods was apparently first introduced in the following reference:

B. M. Fraejis de Veubeke, Displacement and equilibrium models in the finite element method, in Stress Analysis, O. Zienkiewicz and G. Holister, eds., Wiley, New York, 1965. Reprinted in Internat. J. Numer. Methods Engrg., 52 (2001), pp. 287–342.

2.1.3. Local post-processing appeared in Arnold and Brezzi (1985) for the hybridized RT method and in Brezzi, Douglas, and Marini (1985) for the hybridized BDM method. The authors frequently cite the former but not the latter when mentioning hybridization and post-processing of mixed methods. Also, the 1991 paper of Stenberg on post-

processing is cited elsewhere in the paper but not in the introduction. I believe that these three papers should be cited in the relevant part of the introduction (p. 2, l. 18).

2.2. Section 2.1 is somewhat confusing and could be improved.

2.2.1. The notation $a(\mathbf{c}; \mathbf{v})$ originally made me think that $a(\cdot; \cdot)$ was a bilinear form. It took a few times through before I made sense of the notation and realized how linear, bilinear, etc., forms are specified. It might be helpful to include one or two concrete examples before introducing a "general form." This is done nicely in Section 2.1 of Alnaes et al. (2014), which I suggest that the authors emulate.

2.2.2. The notation for the $\mathcal{I}^c$ and $\mathcal{I}^f$ integrals is also confusing. Presumably c stands for "cell" and f for "facet," but at first I thought c was the coefficient function $\mathbf{c}$ and f was some source function, as in equation (10). Since the authors have been using $\mathcal{T}_h$ for cells and $\mathcal{E}_h$ for facets, perhaps a clearer notation would be to call these $\mathcal{I}^{\mathcal{T}}$, $\mathcal{I}^{\mathcal{E},\circ}$, and $\mathcal{I}^{\mathcal{E},\partial}$. A short sentence mentioning that the three $\mathcal{I}$s correspond to the contributions from the cells, internal facets, and boundary facets, respectively, would make this easier for the reader to understand.

3. TECHNICAL CORRECTIONS

I have no technical corrections to suggest, other than to correct the bibliographic reference to the book by Brezzi and Fortin, as described above.

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