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Weather and climate forecasting with neural networks: using GCMs with different complexity as study-ground

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Abstract. Recently, there has been growing interest in the possibility of using neural networks for both weather forecasting and the generation of climate datasets. We use a bottom-up approach for assessing whether it should, in principle, be possible to do this. We use the relatively simple General Circulation Models (GCMs) PUMA and PLASIM as a simplified reality on which we train deep neural networks, which we then use for predicting the model weather at lead times of a few days. We specifically assess how the complexity of the climate model affects the neural network's forecast skill, and how dependent the skill is on the length of the provided training period. Additionally, we show that using the neural networks to reproduce the climate of general circulation models including a seasonal cycle remains challenging - in contrast to earlier promising results on a model without seasonal cycle.

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10 1 Introduction

Synoptic weather forecasting (forecasting the weather at lead times of a few days up to 2 weeks), has for decades been dominated by computer models based on physical equations - the so called Numerical Weather Predictions (NWP) models. The quality of NWP forecasts has been steadily increasing since their inception (Bauer et al., 2015), and these models remain the backbone of virtually all weather forecasts. However, the fundamental nature of the weather forecasting problem can be summarized as: "starting from today's state of the atmosphere we want to predict the state of the atmosphere x days in the future". Thus posed, the problem is a good candidate for supervised machine learning. While long thought unfeasible, the recent success of machine learning techniques in highly complex fields such as image and speech recognition warrants a review of this possibility. Machine learning techniques have already been used to improve certain components of NWP and climate models – mainly parameterization schemes (Krasnopolsky and Fox-Rabinovitz (2006); Rasp et al. (2018); Krasnopolsky et al. (2013); O'Gorman and Dwyer (2018)), to aid real-time decision making McGovern et al. (2017), to exploit observations and targeted high-resolution simulations to enhance earth system models (Schneider et al., 2017), for El-Niño predictions (Nooteboom et al., 2018) and to predict weather forecast uncertainty (Scher and Messori, 2018).

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Recently, the ambition has shifted from using machine-learning to enhance numerical models to using it to tackle weather forecasting itself. The holy grail is to use machine-learning, and especially "deep learning", to completely replacing NWP models, although opinions may diverge on if and when this will happen. Additionally, it is an appealing idea to use neural networks/deep learning to emulate very expensive General Circulation Models (GCMs) for climate research. Both these ideas have been tested with some success for simplified realities (Dueben and Bauer, 2018; Scher, 2018). In Scher (2018), a neural network approach was used to skillfully forecast the "weather" of a simplified climate model, as well as emulate its climate. Dueben and Bauer, based on their success in forecasting reanalysis data regridded to very low resolution, concluded that it is "fundamentally" possible to produce deep-learning based weather forecasts. Here, we build upon the approach from Scher and apply it to a range of climate models with different complexity. We do this in order to assess: 1) how the skill of the neural network weather forecasts depends on the available amount of training data; 2) how this skill depends on the complexity of the climate models; and 3) under which conditions it may be possible to make stable climate simulations with the trained networks, and how this depends on the amount of available training data.

(1) is of special interest for the idea of using historical observations in order to train a neural network for weather forecasting. As the length of historical observations is strongly constrained (~ 100 years for long renalyses assimilating only surface observations, and ~40 years for reanalyses assimilating satellite data), it is crucial to assess how many years of past data one would need to produce meaningful forecasts. (2) Is of interest because it evaluates the feasibility of using climate models as a "simplified reality" for studying weather forecasting with neural networks. Finally, (3) is of interest when one wants to use neural networks not only for weather forecasting, but for the distinct yet related problem of seasonal and longer forecasts, up to climate projections.

To avoid confusion, we use the following naming conventions throughout the paper: "model" always refers to physical models (i.e. the climate models used in this study), and will never refer to a "machine learning model". The neural networks are referred to as "network".

Methods

2.1 Climate models

To create long climate model runs of different complexity, we used the Planet Simulator (PLASIM) intermediate complexity GCM, and its dry dynamical core: the Portable University Model of the Atmosphere (PUMA) (Fraedrich et al., 2005). Each model was run for 830 years with two different horizontal resolutions (T21 and T42, corresponding to ~5.65 and ~2.8 degrees latitude, respectively) and 10 vertical levels. The first 30 years of each run were discarded as spinup, leaving 800 years of daily data for each model. The runs will from now on be referred to as PLASIMT21, PLASIMT42, PUMAT21 and PUMAT42. All the model runs produce a stable climate without drift (fig. S1-S10 in the Supplement). Additionally, we regridded PLASIMT42 and PUMAT42 (with bi-linear interpolation) to a resolution of T21. These will be referred to as PLASIMT42_regridT21 and PUMAT42_regridT21.

The PUMA runs do not include ocean and orography. The PLASIM runs include orography, but no ocean-model. An additional

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run was made with PUMA at resolution T21, but with the seasonal cycle switched off (eternal boreal winter). This is the same configuration as used in Scher (2018) and will be referred to as PUMAT21_noseas.

2.2 Complexity

Ranking climate models according to their "complexity" is a non-trivial task, as it is very hard to define what complexity actually means in this context. We note that here we use the term loosely, and do not refer to any of the very precise definitions of "complexity" that exists in various scientific fields (e.g. Johnson (2009)). Intuitively, one might simply rank the models according to their horizontal and vertical resolutions and the number of physical processes they include. However, it is not clear which effects would be more important (e.g. is a model with higher resolution but less components/processes more or less complex than a low-resolution model with a larger number of processes?). Additionally, more physical processes do not necessarily imply a more complex *output*. For example, very simple models like the Lorenz63 model (Lorenz, 1963) display chaotic behaviour, yet it is certainly possible to design a model with more parameters and a deterministic behaviour. To circumvent this conundrum, we adopt a very pragmatic approach based solely on the output of the models, and grounded in dynamical systems theory. We quantify model complexity in terms of the he local dimension d: a measure of the number of degrees of freedom needed to describe the dynamics of a system linearized around a given instantaneous state. In our case, this means that we can compute a value of d for every timestep in a given model simulation. While not a measure of complexity in the strict mathematical or computational senses of the term, d provides an objective indication of a system's dynamics around a given state and, when averaged over a long timeseries, of the system's average attractor dimension. An example of how dmay be computed for climate fields is provided in Faranda et al. (2017), while for a more formal discussion and derivation of d we point the reader to Appendix A in Faranda et al. (2019). The approach is very flexible, and may be applied to individual variables of a system (which represent projections of the full phase-space dynamics onto specific sub-spaces, called Poincaré sections), multiple variables or, with adequate computational resources, to the whole available dataset. The exact algorithm used here is outlined in Appendix A1.

The local dimension was computed for 38 years of each model run as well as for the ERA-Interim reanalysis on a 1x1 degree grid over 1979-2016 (Dee et al., 2011). The choice of 38 years was made because this is the amount of available years in ERA-Interim, and the length of the dataseries can affect the estimate of d (Buschow and Friederichs, 2018). Figure 1 shows the results for 500hPa geopotential height. The complexity of PUMA increases with increasing resolution, whereas both the low and the high resolution PLASIM model have a complexity approaching that of ERA-Interim. Thus - at least by this measure they are comparable to the real atmosphere. The high resolution runs regridded to T21 have nearly the same complexity as the original high resolution runs. The ranking is the same for nearly all variables and levels (fig. S11 in Supplement). For the rest of the paper, the term "complexity" or "complex" always refers to the local dimension.

2.3 Neural networks

Neural networks are in principle a series of non-linear functions with weights determined through training on data. Before the training, one has to decide the architecture of the network. Here, we use the architecture proposed by (Scher, 2018), which is

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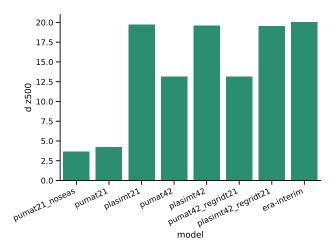


Figure 1. Local dimension d (here used as a data-based measure of "complexity") for the 500hPa geopotential height of the models used in this study and of the ERA-Interim reanalysis.

a convolutional autoencoder, taking as input 3d model fields and outputting 3d model fields of exactly the same dimension. It was designed and tuned in order to work well on PUMAT21 without seasonality (for details see Scher (2018). In order to keep the method comparable, for the main part of this study, no further tuning is done here, and we use the same network layout and hyperparameters as in (Scher, 2018), except for the number of epochs (iterations over the training set) the network is trained. In the original configuration only 10 epochs were used. It turned out that, especially for the more complex models, the training was not saturated after 10 epochs. Therefore, here we train until the skill on the validation data has not increased for 5 epochs, with a maximum of 100 epochs. The layout is depicted in fig. 2. The implications of retuning the network are discussed in section 3.4

For the networks targeted not to forecast the weather, but to create climate simulations (hereafter called climate-networks) we deviate from this setup: here, we include the day of year as additional input to the network, in the form of a separate input channel. To remain consistent with the auto-encoder setup, the output also contains the layer with the day of year. However, when producing the network climate runs, the predicted day of the year is discarded.

The last 10% samples of the training data are used for validation. This allows to monitor the training progress, control overfitting (the situation where the network works very well on the training data, but very poorly on the test data), and potentially limit the maximum number of training epochs. As input to the neural networks 4 variables (u, v, t and z) at 10 pressure levels are used, whereas each variable at each level is represented as a separate input layer (channel). All networks are trained to make 1-day forecasts. Longer forecasts are made by iteratively feeding back the forecast into the network. We did not train the network directly on longer lead-times, based on the finding of Dueben and Bauer (2018) that it is easier to make multiple short forecasts compared to a single long one. Due to the availability of model data and in keeping with Scher (2018) we chose 1-day forecasts as opposed to the shorter forecast step (1 hour) in Dueben and Bauer (2018).

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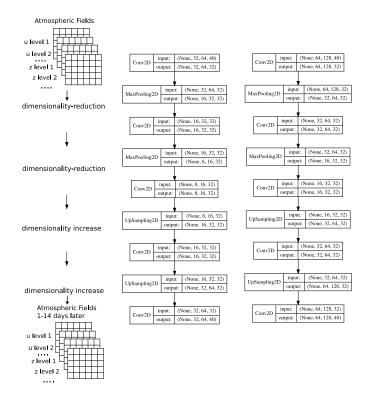


Figure 2. Architecture of the neural network for the models with resolution T21 (left) and T42 (right). Figure based on Fig. S1 from (Scher, 2018)

For each model, the network was trained with a set of 1, 2, 5, 10, 20, 50, 100, 200, 400 and 800 years. Since with little training data the network is less constrained, and the training success might strongly depend on exactly which short period out of the model run is chosen, the training up to and including 20 years were repeated 4 times, shifting the start of the training data by 10, 20, 30 and 40 years. The impact of the exact choice of training period will be discussed where appropriate.

All the analyses shown in this paper are performed on the forecasts made on the first 30 years of the model run, which were never used during training and therefore provide objective scores (the 'test' dataset).

2.4 Metrics

As validation for the network forecasts, we use two commonly used forecast verification metrics, namely the Root Mean Square Error (RMSE) and the Anomaly Correlation Coefficient (ACC). The RMSE is defined as

$$10 \quad RMSE = \sqrt{\overline{\left(prediction - truth\right)^2}} \tag{1}$$

where the overbar denotes a mean over space and time (for global measures), or over time only (for single gridpoints).

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The ACC measures the spatial correlation of the forecast anomaly fields with the true anomaly fields for a single forecast. The anomalies are computed with respect to a 30-day running climatology, computed on 30 years of model data (similar to how ECMWF computes it's scores for forecast validation).

5 $ACC_t = correlation$

$$([truth_{1,1} - clim_{1,1},, truth_{nlat,nlon} - clim_{nlat,nlon}], [prediction - clim_{1,1},, prediction - clim_{nlat,nlon}])$$

$$(2)$$

To compute a score over the whole period, the ACC for all individual forecasts are simply averaged:

$$ACC = \overline{[ACC_t, ..., ACC_{nforecast}]}$$
(3)

10 The ACC ranges from -1 to 1, with 1 indicating perfect correlation of the anomaly fields, and 0 no correlation at all.

3 Results

3.1 Forecast skill in a hierarchy of models

We start by analyzing the RMSE and ACC of two of the most important variables: 500hPa gepotential height (hereafter named zg500) and 800hPa temperature (hereafter called ta800). The first is one of the most commonly used validation metrics in NWP; the second is very close to temperature at 850hPa, which is another commonly used validation metric. We focus on the networks trained with 100 years of training data, which is the same length as in Scher (2018), and is of special interest because it is roughly the same length as is available in current century-long reanalyses like ERA-20C and the NCEP/NCAR 20CR (Compo et al., 2011; Poli et al., 2016). Figure 3 shows the global mean RMSE of network forecasts at lead times of up to 14 days for all models for both zg500 and ta800. As expected, the skill of the forecasts decreases monotonically with lead-time. Unsurprisingly, PUMAT21_noseas - the least complex model - has the highest skill (lowest error) for all lead-times for both variables, followed by PUMAT21. PUMAT42, which is more complex than PUMAT21, but less complex than the two PLASIM runs, lies as expected in between. Interestingly, at a lead time of 1 day, both PLASIM runs have very similar skill, but PLASIM42 has higher errors at longer lead-times, despite their very similar complexity. When looking at the ACC instead (higher values better), the picture is very similar, except that for zg500, PLASIMT42 has slightly lower skill than PLASIMT21. For the T42 runs that were regridded to T21 before the training the results are as follows: for PUMA, the skill of the network in predicting the regridded version of the T42 is very similar to the skill on the original T42 run. For PLASIM the skill on the regridded T42 run is comparable to both the skill on T42 and on T21 runs, albeit closer to the latter. Indeed, the skills on the original PLASIM T42 and T21 runs are much closer to each other than for PUMA. Regridding the network predictions of the two T42 runs to the T21 grid results in only very small changes relative to the difference between the models, especially at longer lead-times (not shown).

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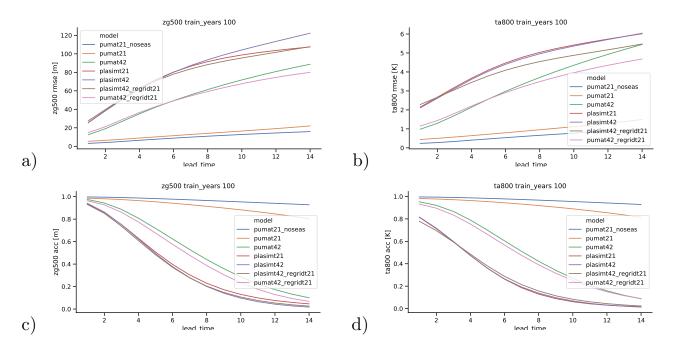


Figure 3. Root Mean Square Error (RMSE) and Anomaly Correlation Coefficient (ACC) of 500hPa gepotential height (a,c) and 800hPa temperature (b,d) for network forecasts for all models for different lead times (in days). All forecasts are based on 100 years of training data.

We next turn our attention to the spatial characteristics of the forecast error. Figure 4 shows geographical plots of the RMSE for 6-day forecasts of the networks trained with 100 years of data (the same training length as in fig. 3). In agreement with the global mean RMSE analyzed before, PUMAT21_noseas has lowest errors everywhere (fig. S12 in Supplement), followed by PUMAT21. PLASIMT21 and PLASIMT42 have a more complicated spatial error structure, and the mid-latitude storm-tracks emerge clearly as high-error regions. The zonally non-uniform distribution is likely caused by the influence of orography (present in the PLASIM runs but not in the PUMA runs). The smaller but sill present zonal non-uniformity in PUMAT42 is probably related to the fact that the neural network used here does not wrap around the boundaries.

3.2 Dependence on amount of training years

A key issue is the extent to which the above results, and more generally the skill of the network forecasts, depend on the length of the training period used. Fig. 5 shows the skill of the network forecasts for 500hPa geopotential height for different training lengths, for a lead-time of 1 day (a,c) and 6 days (b,d). As mentioned in the methods section, the networks with short training periods were trained several times with different samples from the model runs. Figure 5 displays the mean skill; the shading represents the uncertainty of the mean, which is negligibly small. For the 1-day forecasts, the results are as expected: the skill increases with an increasing number of training years, both in terms of RMSE and ACC. This increase is strongly nonlinear and, beyond ~200 years, the skill benefit of increasing the length of the training set is limited. This suggests that the complete

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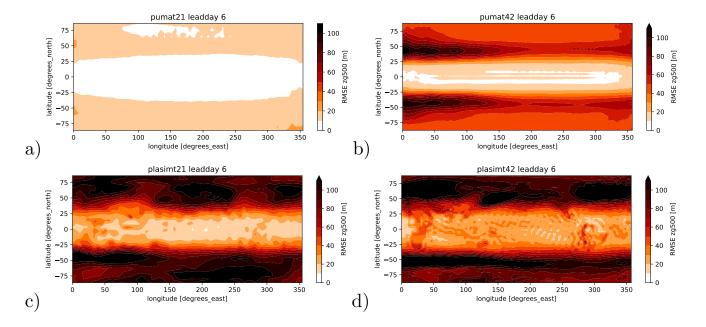


Figure 4. Maps of RMSE of the 6-day network forecasts, for the networks trained with 100 years for PUMAT21 (a), PUMAT42 (b), PLASIMT21 (c) and PLASIMT42 (d).

model-space is already encompassed by around 200 years of daily data. More years will not provide new information to the network. However, it might also be the case that there is in fact more information in more years, but that the network is not able to utilize this additional information. For the 6-day predictions, the PLASIMT21 networks display a counterintuitive behaviour: the skill (in terms of RMSE) does not monotonically increase with increasing length of the training period, but decreases from 100 years to 200 years, while for >200 years it increases again. A similar result - albeit less pronounced - is also seen for the PLASIMT42_regridT21 networks, and also - in a slightly different form - for the skill measured via the ACC. To interpret this, one has to remember that the networks are all trained on 1-day forecasts. For 1-day forecasts - as mentioned above - the skill is indeed increasing with increasing training length, and the PUMAT21 network trained on 200 years makes better forecasts than the one trained on 100 years. Intuitively one would assume this to translate to increased skill of the consecutive forecasts used to create the 6 day forecasts. The fact that this is not the case here might be caused by non-linear error growth. Some networks might produce slightly lower errors at lead-day 1, but the particular errors they have could be faster-growing than those of a network with larger day-1 errors.

3.3 Climate runs with the networks

The trained networks are not limited to forecasting the model "weather", but can also be used to generate a climate run starting from an arbitrary state of the climate model. For this, we use the climate networks that also include the day of year as input (see methods section). Of special interest is the question of whether the climate time-series obtained by the network is stable.

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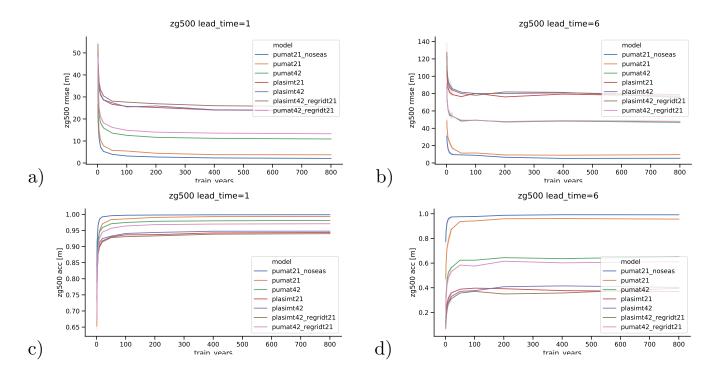


Figure 5. Dependence of network forecast skill on the length of the training period. Shown is the Root Mean Square Error (RMSE) and Anomaly Correlation Coefficient (ACC) of the network-forecasted 500hPa gepotential height for networks trained on different amounts of training years. Each line represents one model. The shading on the left side of the plots represents the 5-95 uncertainty range of the mean RMSE/ACC, estimated over networks with different training samples.)

In Scher (2018), the network climate for PUMAT21_noseas (trained on 100 years of data) was stable and produced reasonable statistics compared to the climate model. We trained our climate networks both on 30 years and 100 years of data for all models with seasonal cycle. While the networks were all stable, they do not produce particularly realistic representations of the model climates. After some time, the storm tracks look unrealistic, and the seasonal cycle is also poorly represented (e.g. in some years, some seasons are skipped – see videos S1-S8 in the supplement that show the evolution of zg500 and zonal windspeed at 300hPa for the network climate runs). Figure 6 a) shows the evolution of ta800 at a single gridpoint at 76 °N for PUMAT21 and the climate network trained on 30 years, started from the same (randomly selected) initial state (results for different randomly selected initial states are very similar, not shown). The network is stable, but the variance is too high and some summers are left out. Surprisingly, the network for 30 years of PLASIMT21 (panel b) produced a more realistic climate. Training on 100 years instead of 30 years does not necessarily improve the quality of the climate-runs (panels c,d). For PLASIMT21, in fact, the network climate trained on 100 years is even more problematic.

Interestingly, the mean climate of the PLASIMT21 network is reasonably realistic (fig. 7), whereas the mean climates for PLASIMT42 and PUMAT42 have large biases (see fig S11-17 in the Supplement).

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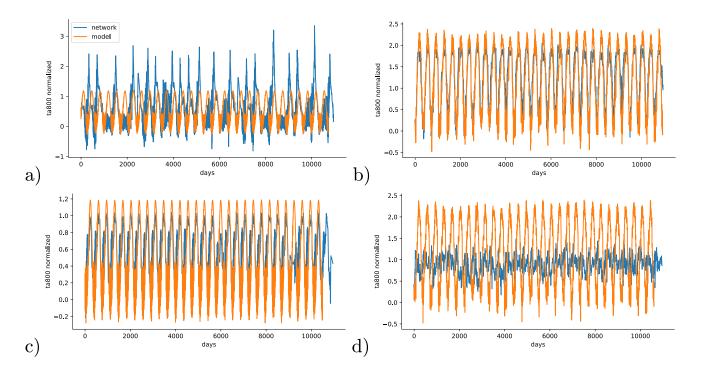


Figure 6. Evolution of daily ta800 at a single grid-point at 76°N in the GCM (orange) and in the climate network trained on the GCM (blue), started from the same initial state. The networks were trained on: 30 years of PUMAT21 (a), 30 years of PLASIMT21 (b), 100 years of PUMAT21 (c) and 100 years of PLASIMT21 (d)

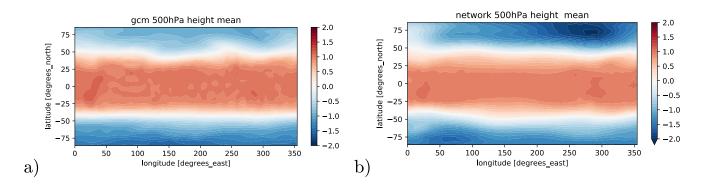


Figure 7. 30 year mean of normalized 500hPa geopotential height for PLASIMT21 (a) and the network trained on 100 years of PLASIMT21(b).

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3.4 Impact of re-tuning

The design of this study was to use an already established neural network architecture - namely one tuned on a very simple model - and apply it to to more complex models. However, it is of interest to know how much tuning the network architecture to the more complex models might increase forecast skill. Therefore, the same tuning procedure as in Scher (2018) for PUMAT21_noseas was repeated for PLASIMT21. Interestingly, the resulting configuration for PLASIMT21 was exactly the same as for for PUMAT21_noseas. Thus, even with re-tuning the results would be the same. As a caveat, we note that tuning neural networks is an intricate process, and many arbitrary choices have to be made. Notably, one has to pre-define the different configurations that are tried out in the tuning (the "tuning space"). It is possible that with a different tuning space for PLASIMT21, a different configuration would be chosen than for PUMAT21_noseas. However, at least within the tuning space we used, we can conclude that a setup working well for a very simple model (PUMAT21_noseas) is also a good choice for a more complex model like PLASIMT21.

4 Conclusions

We have tested the use of neural networks for forecasting the 'weather' in a range of simple climate models with different complexity. For this we have used a deep convolutional auto-encoder architecture that Scher (2018) developed for a very simple general circulation model without seasonal cycle. The network is trained on the model in order to forecast the model state 1 day ahead. This process is then iterated to obtain forecasts at longer lead times. One of the major aims of this study was to assess whether it is possible to use a simplified reality - in this case the most simple GCM without seasonal cycle - to develop a method that also works on more complex GCMs. We showed that, for the problem of forecasting the model 'weather', this seems to be the case: the network architecture also worked on the more complex models, albeit with lower skill. The latter point is hardly surprising, as one would expect the time-evolution of the more complex models to be harder to predict. The fact that we can successfully forecast the weather in a range of simple GCMs a couple of days ahead is an encouraging result for the idea of weather forecasting with neural networks. We also tried to re-tune the network architecture from Scher (2018) to one of our more complex models. Surprisingly, the best network configuration that came out of the tuning procedure was exactly the same as the one obtained for the simpler model in (Scher, 2018). This further supports the idea that methods developed on simpler models may be fruitfully applied in more complex settings.

The second problem we addressed was using the trained networks to create very long integrations of model weather, namely a "climate" run. For this, the network is started with a random initial state from the climate model run, and then creates a run of daily fields for several decades. Scher (2018) found this generated a stable climate for the simplest model considered here, which does not have a seasonal cycle. We further find that it is to some extent also possible for more complex models. However, even when training on relatively long periods (100 years), the climates produced by the networks have some unrealistic properties, such as reproducing the seasonal cycle somewhat poorly, having significant biases in the long-term mean values and often unrealistic storm tracks. The fact that these problems don't occur for the simplest GCM without seasonal cycle, but do occur for the same GCM with seasonal cycle, indicates that seasonality considerably complicates the problem. While not

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a solution for creating climate runs, this suggests that for the weather forecasting problem it might be wise to train separate networks for different times of the year (e.g. one for each month).

Our results further strengthen the idea - already proposed by Dueben and Bauer (2018) and Scher (2018) - of training a neural network on the large amounts of existing climate model simulations, feeding the trained network with today's analysis of a NWP model and using the network in order to make a weather forecast. The high computational efficiency of such neural network forecasts would open up new possibilities, for example of creating ensemble forecasts with much large ensemble sizes than the ones available today. Therefore, this approach would provide an interesting addition to current weather forecasting practice, and also a good example of exploiting the large amount of existing climate model data for new purposes.

Code and data availability. The code developed and used for this study is available in the accompanying repository at http://doi.org/10.5281/
2 zenodo.2572863. All external libraries used here are open source. The trained networks and the data underlying the all plots are available in the repository. The model runs can be recreated with the control files (available in the repository) and the source code of PUMA/PLASIM, which is freely available at https://www.mi.uni-hamburg.de/en/arbeitsgruppen/theoretische-meteorologie/modelle/PLASIM.html ERA-Interim data is freely available at https://apps.ecmwf.int/datasets/data/interim-full-daily/levtype=pl/

Appendix A

15 A1 Computation of local dimension

Here, we outline very briefly how the local dimension d is computed. To foster easy reproducibility, we present the computation in an algorithm-like fashion, as opposed to formal mathematical notation. For a more rigorous theoretical explanation the reader is referred to Faranda et al. (2019). The code is available in the repository accompanying this paper.

First, we define the distance between the 2-D atmospheric fields at times t_1 and t_2 as:

20
$$dist_{t1,t2} = sqrt\left(\left(x_{t_1,1} - x_{t_2,1}\right)^2 + \left(x_{t_1,2} - x_{t_2,2}\right)^2 + \dots + \left(x_{t_1,N_j} - x_{t_2,N_j}\right)^2\right)$$
 (A1)

where j is the linear gridpoint index and N_j is the total number of gridpoints. To compute d_t , namely the local dimension of a field at time t, we first take the negative natural logarithm of the distances between t and all other timesteps t_i :

$$g_{t,t_i} = -\ln(dist_{t,t_i}) \tag{A2}$$

and then retain only the distances that are above the 98th percentile of g_{t,t_i} :

25
$$exceedances = g_{t,t_i} - 98^{th}percentile(g_{t,t_i}) \forall g_{t,t_i} > 98^{th}percentile(g_{t,t_i})$$
 (A3)

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These are effectively logarithmic returns in phase space, corresponding to cases where the field x_t is very close to the field x_t . According to the Freitas-Freitas-Todd theorem (Freitas et al., 2010), modified in (Lucarini et al., 2012), the probability of such logarithmic returns is a generalized Pareto distribution (Pickands III et al., 1975). The local dimension d_t can then be obtained as the inverse of the distribution's scale parameter, which can also be expressed as the inverse of the mean of the exceedances:

$$d_t = 1/mean(exceedances) \tag{A4}$$

The local dimension is an instantaneous metric, and Faranda et al. (2019) have shown that time-averaging of the data can lead to counter-intuitive effects. The most robust approach is therefore to compute d on instantaneous fields. Here, in order to use the same data as for the machine learning, we have used daily means. We have verified that, at least in ERA-Interim, this has a negligible effect on the average d value for all variables except for geopotential at 100hPa and 200hPa (not shown).

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Competing interests. The Authors declare that they have no competing interests

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