

# ***Interactive comment on “On the calculation of normalized viscous-plastic sea ice stresses” by Jean-François Lemieux and Frédéric Dupont***

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## **Response to reviewer 2**

We would like to thank reviewer 2 for his/her very helpful comments. Based on the reviewers' comments we realized that some clarification was needed. We have therefore added two new sections (“The normalized yield curve” and “Broader considerations”) and we have also written explicitly the steps that should be followed for calculating and plotting the normalized VP stresses. We have addressed these comments with the goal of keeping the manuscript relatively short. Indeed, we want this manuscript to be a “quick” guide for sea ice modelers. Below, the comments from the reviewers (1) are in normal character. Our responses (2) are in bold

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while changes to the manuscript (3) mentioned here are also in bold and in quotes. Note that modifications in the revised manuscript are shown in magenta.

## REVIEWER 2

(1) Topic of the manuscript is the correct evaluation of the normalized viscous-plastic sea ice stresses. The model under consideration is the viscous-plastic sea ice model which was formulated by Hibler in 1979 with a replacement pressure introduced by Kreyscher et al. in 2000. The manuscript focuses on the evaluation of the normal stress that are archived with a Picard solver. Two error sources that may occur using the diagnostic are described.

Main issues:

(1) I miss a more detailed discussion of the term numerical convergence of the VP solution and a more careful use of the term numerical convergence. Sometimes you describe by numerical convergence that all stress states are on/in the ellipse ( physical consistency) sometimes you use the term the numerical convergence for the convergence of the sea ice velocity. Please distinguish better between this two cases. Applications might think that the diagnostic implies numerical convergence of the solution VP solution (sea ice velocity). Explanation/motivation why plotting the normalized stresses is a suitable diagnostic to evaluate numerical convergence of the VP solution in a first step. Clarify that being physical consistent does not imply that one has a convergent approximation of the sea ice velocity. Maybe add a paragraph to the introduction how this diagnostic needs to be used.

**(2) The stresses are function of the velocity vector. A numerically converged velocity therefore leads to numerically converged stresses (consistent with the physics of the problem). We have improved the text and introduce the nonlinear residual vector to**

better explain what we mean by numerical convergence. We have also added in section 4 the sentence:

(3)“A ‘fully’ converged solution for  $u$  is characterized by a small residual. As the stresses are function of  $u$ , a fully converged velocity vector leads to states of stress that are either on (plastic) or inside (viscous) the yield curve.”

(2) We also better explain in the introduction why the normalized stresses are useful to assess convergence and physical consistency. The last paragraph of the introduction now starts with the following sentences:

(3)”Calculating and plotting the normalized states of stress with respect to the yield curve is a useful diagnostic for assessing the physical consistency and numerical convergence of a VP solution. Indeed, this method can confirm whether a sea ice rheology is properly implemented in a model. The method is also helpful for evaluating numerical convergence. This is especially true for the explicit elastic-VP (EVP) solver (e.g., Hunke, 2001) which does not include a measure of convergence such as a residual.“

(1) Can you please explain how the diagnostic should be evaluated for Newton-like solvers? I don’t think that it is straight forward. Using your 1D example a fully implicit discretized rheology reads as  $\sigma = Pp/(2|\epsilon_k|\epsilon_k) - Pp/2 = -Pp$ . Does this mean that the diagnostic is unnecessary? I do not think so as Newton-type methods also introduce some form of linearization...

(2) The same method should be followed for Newton solvers. Indeed, they are also based on a linearization with  $u^{k-1}$  for the calculation of the Jacobian matrix and the residual vector.

(1) Please provide the explicit formulation of the yield curve that you use to plot the figures.

**(2) the formulation has been written in the captions of Fig.1-3. The normalized yield curve is also better explained in section 3 of the revised manuscript. The following text has been added after equation (10):**

**(3) ”...which describes a family of ellipses that depend on the ratio  $\Delta/\Delta^*$  for their size and on the ratio  $P/P_p$  for their center. Equation (10) with  $\Delta/\Delta^* = P/P_p = 1$  defines what we refer to as the normalized yield curve in principal stress space. Hence, according to our rheology, normalized plastic stresses should fall on the normalized yield curve while normalized viscous stresses should lie on smaller ellipses inside the normalized yield curve (Geiger et al. 1998).“**

(1) Is the diagnostic effected if other limitations are used in (4)? How to deal with different linearization?

**(2) Good points. We have added in the revised manuscript a section called “Broader considerations” where we argue that our conclusions remain the same for other approaches for the limitation (e.g. Kreyscher et al. 2000, Lemieux and Tremblay 2009):**

**(3) “The recommendations given above remain the same if another approach is used for limiting the viscous coefficients (see equation 4). Numerical experiments with the approach of Kreyscher et al. 2000 or with the hyperbolic tangent of Lemieux and Tremblay 2009 allow one to draw the same conclusions (not shown).”**

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**(2) We have also added a sentence to explain how the normalized stresses should be calculated when using another kind of linearization (e.g. Lemieux and Tremblay 2009):**

**(3) “While it is not recommended to linearize the rheology term with the previous two iterates (as done by Lemieux and Tremblay 2009) the stresses in step 2 (see beginning of section 5) should in this case be obtained from  $\sigma_{ij} = 2\eta(\mathbf{u}_l)\dot{\epsilon}_{ij}(\mathbf{u}^k) + [\zeta(\mathbf{u}_l) - \eta(\mathbf{u}_l)]\dot{\epsilon}_{kk}(\mathbf{u}^k)\delta_{ij} - P(\mathbf{u}_l)\delta_{ij}/2$  with  $\mathbf{u}_l = (\mathbf{u}^{k-1} + \mathbf{u}^{k-2})/2$ .”**

(1) I recommend that the paper be published only after addressing these issues.

Minor issues:

(1) L. 5 -8 The first example is true for approximations calculated with Picard solver. What about Newton and EVP? The 2 sentences can be misleading.

**(2) Thanks for pointing this out. The same conclusions apply for both Picard and Newton solvers (see comment above). We have, however, clarified the text in the abstract and in the conclusion to clearly state that the first “mistake” can only be made for implicit solvers while the second one applies to implicit and explicit solvers.**

(1) L106 Here numerical solution describes the numerical convergence of v. In line 90 the term numerical convergence is used to describe that the stress states are in/on the ellipse (which is the physical consistency). Be more specific when using the term numerical convergence.

**(2) See our response to your first main comment above.**

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(1) L106 The residual of the momentum equation? Which residual?

**(2) The residual  $F = Au - b$  and the criterion for the nonlinear convergence have been introduced in the revised manuscript. This sentence has been modified.**

(1) L107-110 I think this point must be emphasized and moved to the introduction ( see main issue 1))

**(2) This is a conclusion of Lemieux and Tremblay 2009. We prefer not to repeat it elsewhere in the manuscript. However, the sentence was not well formulated and we have therefore decided to rephrase it:**

**(3) “Note that, in general, the fact that states of stress are on or inside the yield curve does not imply full convergence; the final positions (on and inside the yield curve) are obtained once  $u^k$  is the fully converged solution (Lemieux and Tremblay 2009).”**

(1) L 121 The solution of the momentum equation?

**(2) See our response to your first main comment above.**

(1) L154 Please be more specific how numerical convergence can be assessed.

**(2) See our other responses above.**

**Jean-François Lemieux**

## REFERENCES

**Geiger,C.A., W.D. Hibler and S.F. Ackley, “Large-scale sea ice drift and deformation’ Comparison between models and observations in the western Weddell Sea during 1992”, J. Geophys. Res, 103, 21893-21913, 1998.**

**GMDD**

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