Geosci. Model Dev. Discuss.,
https://doi.org/10.5194/gmd-2019-284-AC1, 2020

Geoscientific Model Development Discussions

Interactive comment on "On the calculation of normalized viscous-plastic sea ice stresses" *by* Jean-François Lemieux and Frédéric Dupont

Jean-François Lemieux and Frédéric Dupont

jean-francois.lemieux@canada.ca

Received and published: 28 January 2020

Response to reviewer 1

We would like to thank reviewer 1 for his/her very helpful comments. Based on the reviewers' comments we realized that some clarification was needed. We have therefore added two new sections ("The normalized yield curve" and "Broader considerations") and we have also written explicitly the steps that should be followed for calculating and plotting the normalized VP stresses. We have adressed these comments with the goal of keeping the manuscript relatively short. Indeed, we want this manuscript to be a "quick" guide for sea ice modelers. Below, the comments from the reviewers (1) are in normal character. Our responses (2) are in bold

while changes to the manuscript (3) mentioned here are also in bold and in quotes. Note that modifications in the revised manuscript are shown in magenta.

REVIEWER 1

(1) The note "On the calculation of normalized viscous-plastic sea ice stresses" by Lemieux and Dupont describes how to compute normalised viscous-plastic sea ice stress properly. They also describe two common traps one can fall into when computing this quantity. This is a valuable (small) contribution that would have saved me from trying to figure out things myself (and wasting a lot of time on that). The text is clearly written, there are a few small comments to consider, see below. The representation is convincing and the explanation of the procedure and the common errors are clear.

I have one small issue. I would like the authors to revisit the derivation of their equation (6). First, one needs eqs(1,3,4,5) (and not just 1 and 5) and Delta to arrive at an expression like this; second, it only works if P_p in eq(1) is replaced by the replacement pressure P (that's not immediately clear from the text). If one does not want to use the replacement pressure P (and there are reasons to do so), the derivation ends up with P_p instead of P on the rhs, because in eq(1) P_p is on the rhs. This is important because eq(10) with then have a "1" instead of P/P_p and in eq(16) it would be P_p/P instead of "1". This has implications for the interpretation (but not for the general conclusions, as far as I can see). Adding a treatment of the no-replacement pressure case would be very helpful for the generality of the paper, so I recommend that the paper be published only after addressing this issue.

(2) Ok it is now mentioned in the revised manuscript that equations (1,3,4,5) are needed.

Setting $P = P_p$ leads to the following equation

$$\left(\frac{\Delta}{\Delta^*}\right)^2 = \left[\sigma_{p1}^n + \sigma_{p2}^n + \frac{P_p}{P_p}\right]^2 + e^2\left[\sigma_{p1}^n - \sigma_{p2}^n\right]^2. \quad (1)$$

This slightly different equation (compared to equation(10) in the manuscript) does not affect the recommendations given here. The ratio $\frac{P_p}{P_p}$ or $\frac{P}{P_p}$ (equation(10)) defines the center of the ellipse. As mentionned by Geiger et al. 1998, normalized states of stress in the viscous regime are positionned on concentric ellipses all centered at $(-0.5, 0.5)$ when $P = P_p$ while the center of an ellipse when using the replacement pressure depends on the ratio $\frac{P}{P_p}$. The following sentence has been added to the revised manuscript (in the new section called "Broader considerations").

(3) "If one does not use a replacement pressure, the stresses in step 2 should be calculated the same way with $P = P_p$. Instead of lying on ellipses defined by equation (10), the normalized viscous states of stress would lie on concentric ellipses centered at $\sigma_{p1}^n = \sigma_{p2}^n = -0.5$ (Geiger et al. 1998)."

Minor comments:

(1) page 1 l21 large spatial

(2) Done.

(1) l24 Unfortunately,...I would add how that leads to misunderstandings in order to formulate a "problem statement". If we all assume we know what we are doing then there's no problem. E.g., Subtle mistakes in calculating stresses can lead to a complete misinterpretation of the state of convergence. Or similar...

(2) We have added the following sentence:

(3)"As demonstrated here, care must be taken when calculating the normalized stresses as two potential mistakes can lead to a misinterpretation of modeling results."

(1) page 2 l40: I prefer to write Δ as $((e_{11}+e_{22})^2 + e^{-2}((e_{11}-e_{22})^2 + 4e_{12}^2))^{\frac{1}{2}}$, because it is also more straightforward to implement

(2) We agree. It has been changed.

(1) page 3 l63: such as a Picard solver...or with a Newton solver

(2) Done.

(1) l65: Kimmritz et al 2015 use the terminology of "modified" EVP. "revised" EVP was used by Bouillon et al 2013.

(2) Ok it has been corrected.

(1) l76 a Picard solver

(2) We decided to keep the sentence as is because the words solve, solution and solver are already present...

(1) page 5 l117: remove: that could be done by modelers

(2) Done.

(1) l122: truely?

(2) We don't think we need to add 'truely'.

(1) page 6 l140: remove "that could be made by modelers"

(2) Done.

(1) l145 rephrase sentence: This is the equation of an ellipse we obtain if the principal stresses are normalized by the replacement pressure.

(2) Ok it has been rephrased.

(1) l149, but why only for the elliptical yield curve and not for the Coulombic and Diamond yield curves?

(2) This is a good question...We argue that the recommendations in our manuscript apply to all the yield curves. We do not have the diamond nor the modified coulombic yield curve implemented in the McGill model but we have recently coded a standard Mohr-Coulomb yield curve with compressive capping (i.e., a triangle). This other constitutive formulation is obtained by writting ζ, η and P as

$$\zeta = \frac{P_p}{2|\dot{\epsilon}_I^*|}, \tag{2}$$

$$P = 2\zeta|\dot{\epsilon}_I|, \tag{3}$$

$$\eta = \frac{\left(\frac{P}{2} - \zeta\dot{\epsilon}_I\right)\sin\phi}{2\dot{\epsilon}_s^*} \tag{4}$$

where $\dot{\epsilon}_I$ is the divergence, $|\dot{\epsilon}_I^*| = \max(|\dot{\epsilon}_I|, d_{min})$ with d_{min} a small number similar to Δ_{min}, ϕ is the angle of friction, $\dot{\epsilon}_s^* = \max(\dot{\epsilon}_s, s_{min})$ with $\dot{\epsilon}_s$ the maximum shear strain rate and s_{min} another small number, set equal to d_{min}.

Note that this formulation of Mohr-Coulomb assumes a pure shear flow rule. Divergence (larger than d_{min}) can only occur at the tip of the triangle and convergence when the sea ice pressure is equal to P_p.

It is observed that with this new rheology, the Picard solver really struggles to obtain a numerically converged solution. With $P* = 27.5 \times 10^3$ Nm^{-2} , $d_{min} = 2 \times 10^{-9}$ s^{-1} and $sin\phi = 0.5$ (i.e., $\phi = 30°$), the solver does not converge. When calculating the normalized stresses the proper way, there are states of stress outside the yield curve. Similar to the results obtained with the elliptical yield curve, the stresses (shown in Fig.1 in this document) appear to have converged if only u^k is used to calculate the normalized stresses.

To obtain a fully converged solution, some rheology parameters were modified from the values given above; $P* = 5 \times 10^2$ Nm^{-2}, $sin\phi = 0.01$ and d_{min}=$s_{min} = 2 \times 10^{-8}$

s^{-1}. Consistent with the results obtained with the ellipse, the fully converged stresses normalized by P_p are either on or inside the yield curve (shown in Fig.2).

Fig.3 shows the fully converged stress invariants when normalized by the replacement pressure P. As for the ellipse, the solution is not realistic as there are no stresses in the viscous regime. Strangely, there are no states of stress on the long side of the triangle. This can understood when considering the normalized first stress invariant ($\sigma_I = (\sigma_{11} + \sigma_{22})/2$). It is easy to show that

$$\sigma_I = \zeta \left(\dot{\epsilon}_I - |\dot{\epsilon}_I| \right). \tag{5}$$

Normalizing by $P = 2\zeta|\dot{\epsilon}_I|$, the normalized σ_I (i.e. σ_I^n) is

$$\sigma_I^n = \frac{1}{2} \left(\frac{\dot{\epsilon}_I}{|\dot{\epsilon}_I|} - 1 \right). \tag{6}$$

Consistent with what is observed in Fig.3, σ_I^n can take only two possible values: $\sigma_I^n = 0$ if $\dot{\epsilon}_I > 0$ (divergence) or $\sigma_I^n = -1$ if $\dot{\epsilon}_I < 0$ (convergence).

To further support our conclusions, we have added some of this material in the revised manuscript (in the new section "Broader considerations".

Going back to the results of Wang and Wang 2009, it seems that the diamond yield curve does not use a replacement pressure (see their p.3). We don't know, however, why some states of stress are inside the yield curve for the modified Coulomb. Is it possible it was correctly normalized by the ice strength? This is not discussed in the manuscript but we speculate that Wang and Wang also made the other mistake: they calculated the normalized stresses only using the latest iterate. This problably explains

why the solution seems to have converged. Consistent with our new results in our revised manuscriopt when using a Mohr-Coulomb, Ringeisen et al. 2019 were not able to get numerical convergence with the modified Coulomb (see their Fig. 12). Anyway, we think that mentioning this figure from Wang and Wang only adds more confusion as it is not clear what they did exactly. We have removed the reference to it in the revised manuscript.

(1) page 7 l166: gives

(2) Done.

(1) page 10 Figure 2: I think the caption is misleading. It should start with the statement that sigma is computed based on u^k only.

(2) Done.

Jean-François Lemieux

REFERENCES

Geiger,C.A., W.D. Hibler and S.F. Ackley, "Large-scale sea ice drift and deformation' Comparison between models and observations in the western Weddell Sea during 1992", J. Geophys. Res, 103, 21893-21913, 1998.

Ringeisen, D., M. Losch, B. Tremblay and N. Hutter, "Simulating intersection angles between conjugate faults in sea ice with different viscous-plastic rheologies", The Cryosphere, 13, 1167-1186, 2019.

C9

norm_stress_inv_MC_73.png

C10

norm_stress_inv_MC_71.png

C11

norm_stress_inv_MC_72.png

C12